Atelier
« Méthodes probabilistes en géométrie spectrale et EDP »
22—26 août 2016

Workshop
"Probabilistic Methods in Spectral Geometry and PDE"
August 22—26, 2016

Nodal sets of high-energy arithmetic random waves

Maurizia Rossi*

maurizia.rossi@uni.lu

Arithmetic random waves are Gaussian Laplace eigenfunctions on the two-dimensional torus. Their zero sets consist of the union of smooth curves, the so-called nodal lines. In this talk, we study the behaviour of the latter, in the high-energy limit. In the first part, we focus on their length. The expected nodal length was found by Rudnick and Wigman (2008) to be proportional to the square root of the corresponding eigenvalue, whereas the asymptotic variance was investigated by Khrishnapur, Kurlberg and Wigman (2013). They proved that it is non-universal, and is intimately related to the arithmetic of lattice points lying on a circle with radius corresponding to the energy. We focus on the asymptotic distribution of the nodal length: we find that it converges to a non-universal (non-Gaussian) limiting distribution, depending on the angular distribution of lattice points lying on circles—as for the variance. Our argument has two main ingredients. An explicit derivation of the Wiener-Ito chaos expansion for the nodal length shows that it is dominated by its 4th order projection 4th order chaos component (in particular, somewhat surprisingly, the 2nd order chaos component vanishes). The rest of the argument relies on a careful analysis of the 4th order chaotic component. This is based on joint work with Marinucci, Peccati and Wigman (2016). A natural further step in the understanding of nodal sets consists of studying the intersections of nodal lines corresponding to two independent arithmetic random waves. Note that this is equivalent to investigate zeroes of complex arithmetic random waves, so-called phase singularities. In the second part of this talk, we focus on their total number, giving the expected value, the asymptotic variance and distribution, the latter both in the high-frequency limit. Even if such results mirror those for the nodal length, we have to face different difficulties due to the discrete nature of the problem which force us to develop some novel techniques.

This talk is based on joint work with Marinucci, Peccati and Wigman and also on a work in progress jointly with Dalmao, Nourdin and Peccati.

*UR Mathématiques-Faculté des Sciences, Université du Luxembourg. 6, rue Coudenhove-Kalergi, L-1359 Luxembourg-Kirchberg, LUXEMBOURG