École
« L'abécédaire de SIDE (ASIDE) »
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School
"Abecedarian of SIDE (ASIDE)"
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# SESSION—Cluster algebras and discrete integrable systems. Dynamics on graphs and combinatorics

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Titre/Title : "Introduction to cluster algebras (part 1)"

Cluster algebras, introduced by Fomin and Zelevinsky around 2000, are certain commutative rings defined recursively via a dynamical process known as seed mutation. Part 1 of our introduction to cluster algebras will focus on motivation, definitions, and concrete examples.

Titre/Title : "Introduction to cluster algebras (part 2)"

We continue this introduction to cluster algebras with an overview of their main structural properties. Two highlights are the Lauren phenomenon (any sequence of mutations results in a Laurent polynomial of the initial conditions rather than a general rational function) and the positivity conjecture (the coefficients of these Laurent polynomials are positive integers).

Titre/Title : "Compatible Poisson structures and quantization of cluster algebras"

Shortly after their initial introduction, it was discovered by Gekhtman, Shapiro, and Vainshtein that cluster algebras carry a Poisson structure which interacts nicely with mutations. These compatible Poisson structures naturally lead to a quantization of cluster algebras first investigated by Berenstein and Zelevinsky. In this lecture I will introduce these ideas and illustrate with basic examples coming from Lie theory.

Titre/Title : "Integrability in cluster algebras"

Mutations in cluster algebras and Y-patterns provide a rich family of discrete dynamical systems with the potential to exhibit integrability. Example of classical integrable systems that are now understood to lie in this framework include Q-systems and the octahedron (i.e. discrete Hirota bilinear) recurrence. I will explore this general point of view, using the pentagram map of Richard Schwartz as a running example.