École
« L'abécédaire de SIDE (ASIDE) »
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School
"Abecedarian of SIDE (ASIDE)"
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SESSION—Continuous symmetries of discrete equations. Structure preserving discretization of differential equations and numerical methods

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Titre/Title : "Symmetry-preserving numerical schemes"

Most fundamental differential equations of mathematical physics admit a certain symmetry group of point transformations. When considering the discrete counterpart of a differential equation, there are many reasons why one might be interested in preserving the symmetries of the original equation. From the conceptual point of view, the symmetry-preserving discrete equations should share physically meaningful solutions and properties of the original equation. In addition, when symmetries are preserved, it is possible to use Lie group techniques to obtain exact solutions and conservation laws. From a practical perspective, symmetry-preserving numerical schemes should, like other geometric numerical integrators, provide numerical results that are better than, or at least as good as, non-invariant numerical schemes.

In these lectures, we will review two procedures for constructing symmetry-preserving finite difference numerical schemes. The first procedure is based on Lie's infinitesimal symmetry generators, while the second approach uses the new theory of equivariant moving frames. Both approaches have advantages and disadvantages. These will be discussed and illustrated using important differential equations from mathematics and physics. Numerical simulations will be presented, and innovative techniques for obtaining better invariant numerical schemes will be discussed.