

An exactly solvable quantum four-body problem associated with the symmetries of an octacube

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In this Letter, I show that eigenenergies and eigenstates of a system consisting of four one-dimensional hard-core particles with masses $6m$, $2m$, m , and $3m$ in a hard-wall box can be found exactly using Bethe ansatz. The ansatz is based on the exceptional affine reflection group \tilde{F}_4 associated with the symmetries and tiling properties of an octacube—a Platonic solid unique to four dimensions, with no three-dimensional analogues.

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