Constructing GGE for QFTs

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PRA 91 (2015) and more

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Problem

- Thermalization of a closed quantum system
- Low dimensions → extra conservation laws can coexists with nontrivial dynamics
- After a long time system thermalizes to the GGE (Rigol, Dunjko, Olshanii)

$$\rho_{\mathrm{GE}} = \exp\left(-\beta H\right) \qquad \rho_{\mathrm{GGE}} = \exp\left(-\sum_n \lambda_n I_n\right)$$

How to choose conserved charges?

Integrable field theories

- Scattering is purely elastic (Zamolodchikov-Fadeev algebra)
- Momentum modes are conserved
- Construct the GGE with them

$$\rho_{\mathrm{GGE}} = \exp\left(-\int \frac{dk}{2\pi}\lambda(k)N(k)\right)$$

It can work! (Fioretto, Mussardo 2010)

$$\lim_{t\to 0}\langle\Psi(t)|\mathcal{O}|\Psi(t)\rangle = \mathrm{tr}\left(\rho_{\mathrm{GGE}}\mathcal{O}\right)$$

Two problems and one solution

- Only works for an infinite system
- Is in the momentum space

- Solution → Find better conserved charges

- Require: $$N(k) \leftrightarrow I_n$$

Outline

1. Construction of the GGE

2. Making sense out of it

Part 1: Construction of the GGE

Ising field theory

Two interacting Majorana fermions

$$H = \int \mathrm{d}x \, \frac{i}{2} \left[R(x)\partial_x R(x) - L(x)\partial_x L(x)\right] + imR(x)L(x)$$

Bogoliubov transformation

$$H = \int \frac{\mathrm{d}k}{2\pi} \omega(k) N(k), \qquad \omega(k) = \sqrt{m^2 + k^2}.$$

Mode occupation operator

$$N(k) = Z^\dagger(k) Z(k), \qquad \{Z^\dagger(k) Z(q)\} = 2\pi\delta(k - q)$$

Ultra-local conserved charges

In the real space

$$I_n^+ = \frac{i}{2}\int \mathrm{d}x\,\left[R(x)\partial_x^{2n+1}R(x) + L(x)\partial_x^{2n+1}L(x)\right]$$

$$I_n^- = \frac{i}{2}\int \mathrm{d}x\,\left[R(x)\partial_x^{2n+1}R(x) - L(x)\partial_x^{2n+1}L(x) + 2mR(x)\partial_x^{2n}L(x)\right]$$

Ultra-local conserved charges

In the momentum space:

$$I_n^{\pm} = \int \frac{\mathrm{d}k}{2\pi}\, \epsilon_n^{\pm}(k) N(k),$$

$$\epsilon_n^{+}(k) = \omega(k) k^{2n},$$

$$\epsilon_n^{-}(k) = k^{2n+1}$$

Incomplete basis:

$$I_n^{\pm} = \int \frac{\mathrm{d}k}{2\pi}\, \epsilon_n^{\pm}(k) \left[f(k) + N(k)\right],$$

$$N(k) \leftrightarrow I_n$$

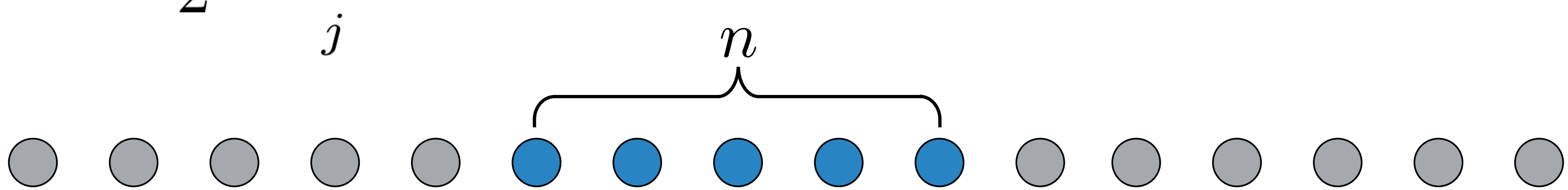
Lattice Ising model

$$H_{\mathrm{lattice}} = \frac{iJ}{2} \sum_j a_{2j} \left[a_{2j+1} - h a_{2j-1} \right]$$

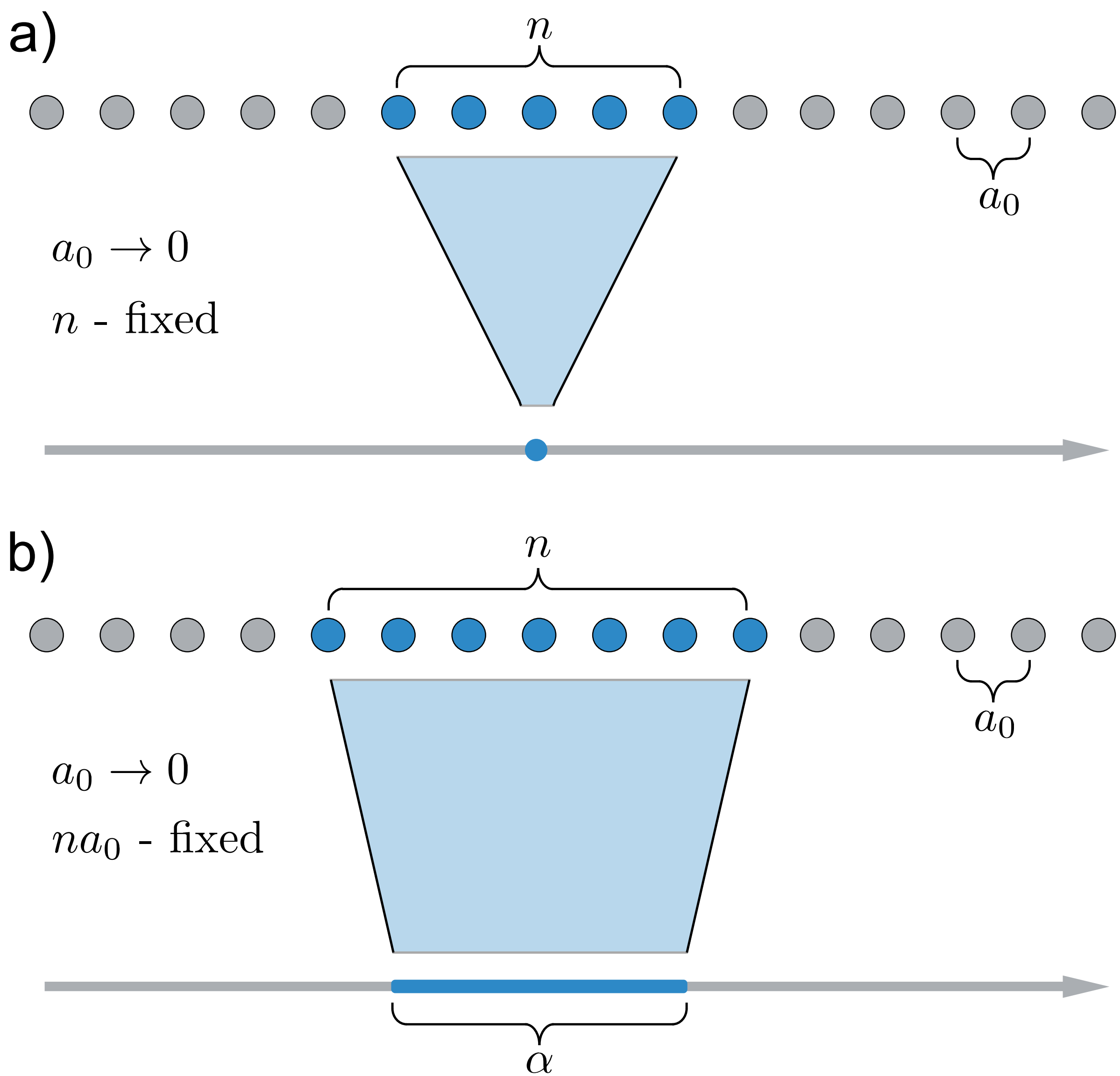
Conserved charges

$$\mathcal{I}_n^+ = \frac{iJ}{2} \sum_{j,\sigma=\pm 1} a_{2j} \left[a_{2j+2n\sigma+1} - h a_{2j+2n\sigma-1} \right],$$

$$\mathcal{I}_n^- = \frac{iJ}{2} \sum_j a_{2j} a_{2j+2n} - a_{2j-1} a_{2j+2n}$$



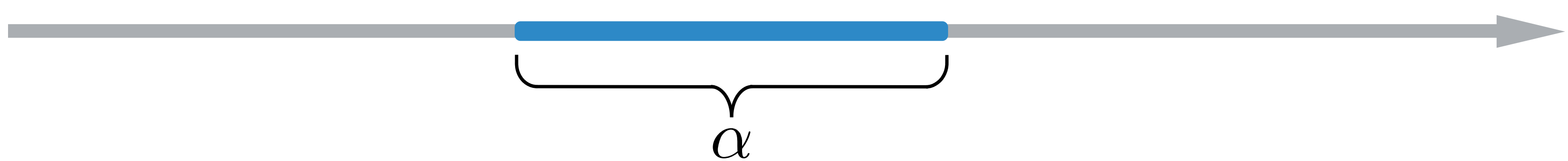
Continuum limit redone



Quasi-local charges

$$I^+(\alpha) = \frac{i}{4}\int \mathrm{d}x \left[R(x) + L(x)\right](\partial_x - m) \times \left[R(x+\alpha) - L(x+\alpha) + (\alpha \to -\alpha)\right]$$

$$I^-(\alpha) = \frac{i}{2}\int \mathrm{d}x \left[R(x)R(x+\alpha) + L(x)L(x+\alpha)\right]$$



Quasi-local charges

$$I^{\pm}(\alpha) = \int \frac{\mathrm{d}k}{2\pi}\, \epsilon^{\pm}(k,\alpha) N(k),$$

$$\epsilon_n^{+}(k,\alpha) = \omega(k)\cos(k\alpha),$$
$$\epsilon_n^{-}(k,\alpha) = \sin(k\alpha)$$

$$N(k) \leftrightarrow I(\alpha)$$

Correct GGE:

$$\rho_{\mathrm{GGE}} = \exp\left(-\sum_{\sigma=\pm}\int d\alpha\, \lambda^{\sigma}(\alpha) I^{\sigma}(\alpha)\right)$$

Summary 1/2

- Ultra-local charges in QFT's cannot be used to construct the GGE, are incomplete.

- Derived a complete set of quasi-local charges for the Ising

- Also for the Lieb-Liniger (PRA 91, 2015)

Part 2: making sense out of the GGE

Problem

Physical theory requiring continuum of information!

Solution

Give less information!

Mass quench

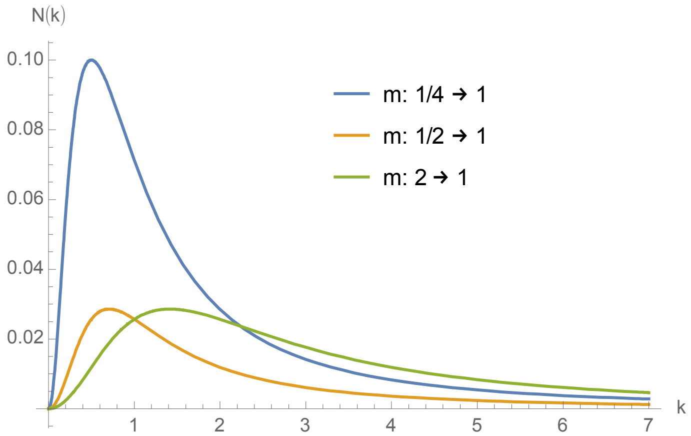
- Initial state (Rossini et al)

$$|\Psi(0)\rangle = \exp\left(-i\int_0^\infty \frac{dk}{2\pi} K(k) Z^\dagger(-k) Z^\dagger(k)\right)|0\rangle$$

$$K(k) = \tan\left(\frac{1}{2}\arctan(k/m) - \frac{1}{2}\arctan(k/m_0)\right)$$

- Momentum distribution

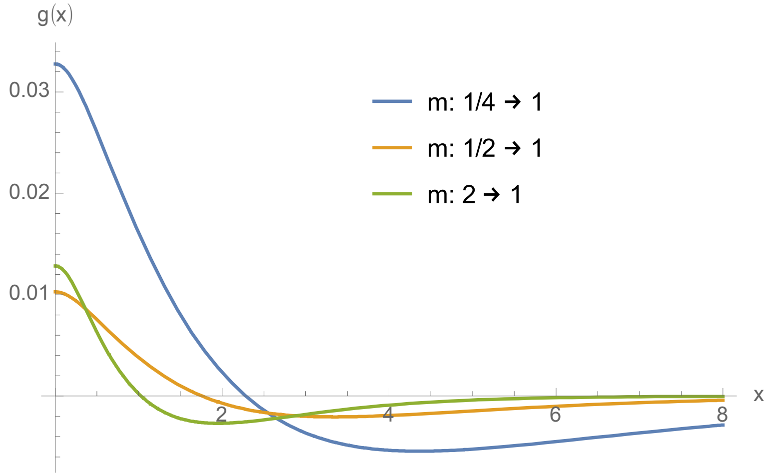
$$N(k) = \frac{K^2(k)}{1+K^2(k)}$$



Local correlation function

- In the real space

$$g(x) = \langle (R(x) - iL(x)) \, (R(0) + iL(0)) \rangle$$



- Through mode occupation numbers

$$g(x) = \int \frac{dk}{2\pi} \frac{N(k)}{2\omega(k)} \cos(kx)$$

Truncated GGE

$$\rho_{\mathrm{GGE}} = \exp\left(- \int_0^{\infty} d\alpha \, \lambda^{\sigma}(\alpha) I^{+}(\alpha) \right)$$

- Finite interval

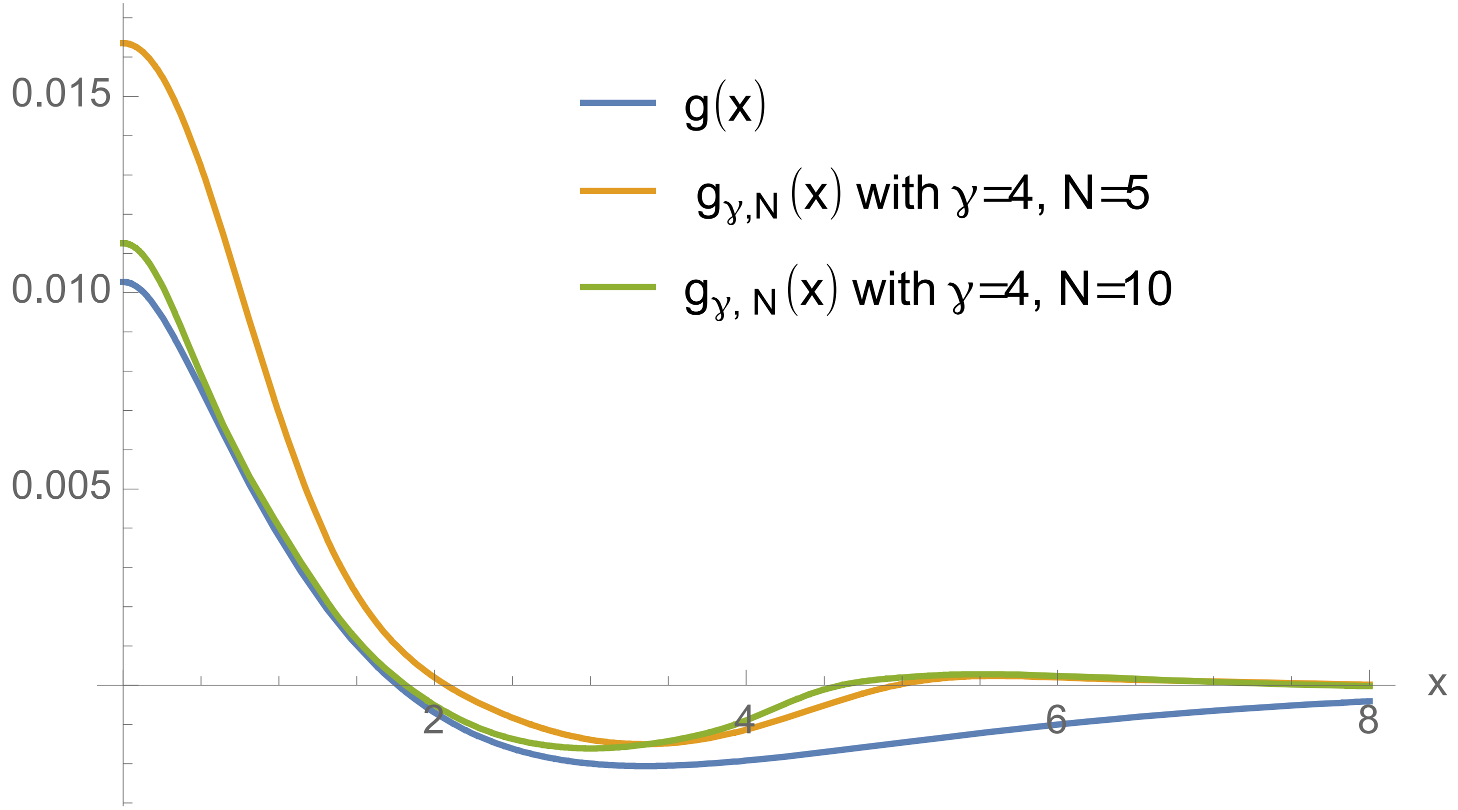
$$\rho_{\mathrm{GGE}} = \exp\left(- \int_0^{\gamma} \mathrm{d}\alpha \, \lambda^{+}(\alpha) I^{+}(\alpha) \right)$$

- Discretize

$$\rho_{\mathrm{GGE}} = \exp\left(- \sum_{m=1}^{N} \lambda^{+}(\alpha_m) I^{+}(\alpha_m) \right)$$

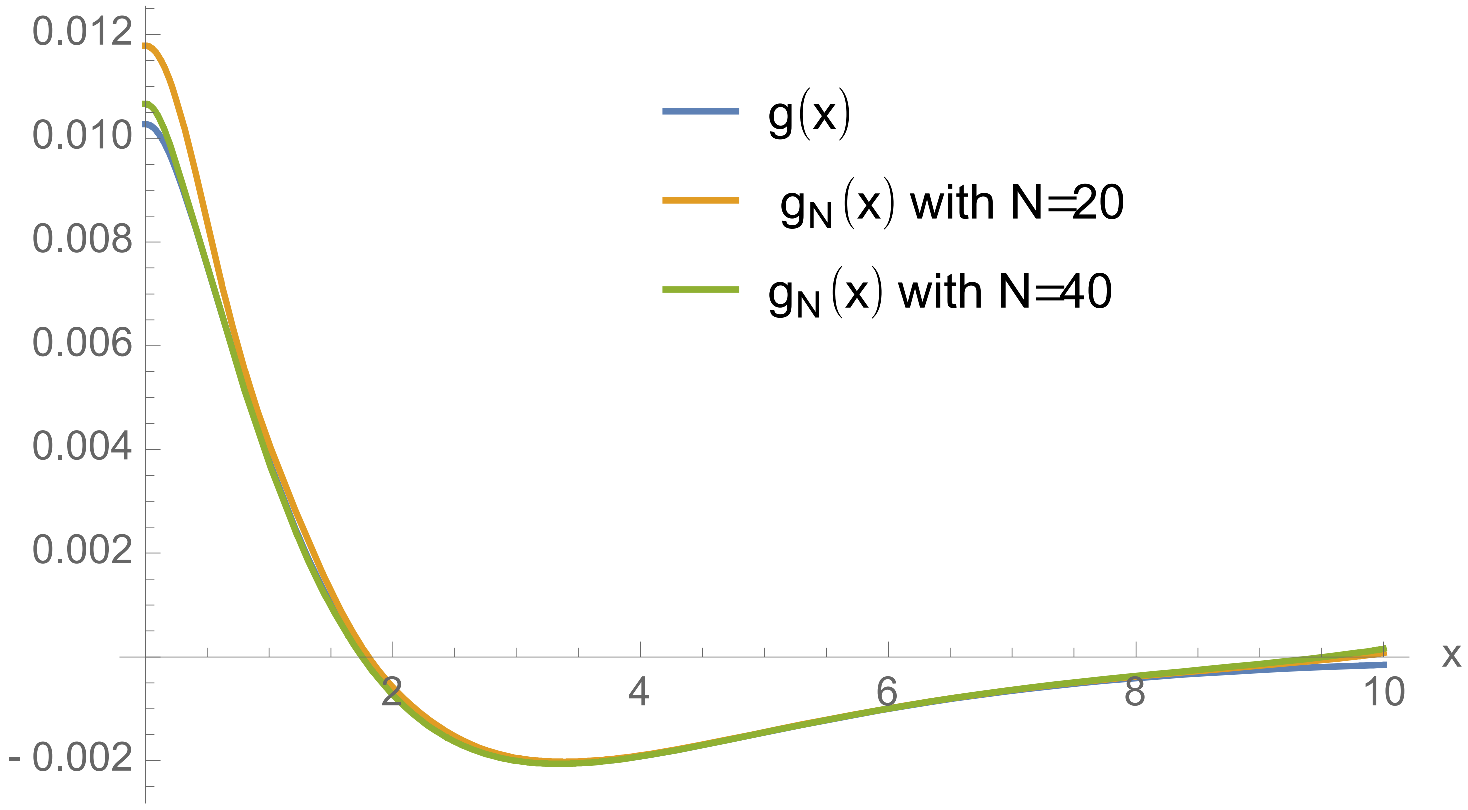
Motivated by M. Fagotti and F. Essler

Results



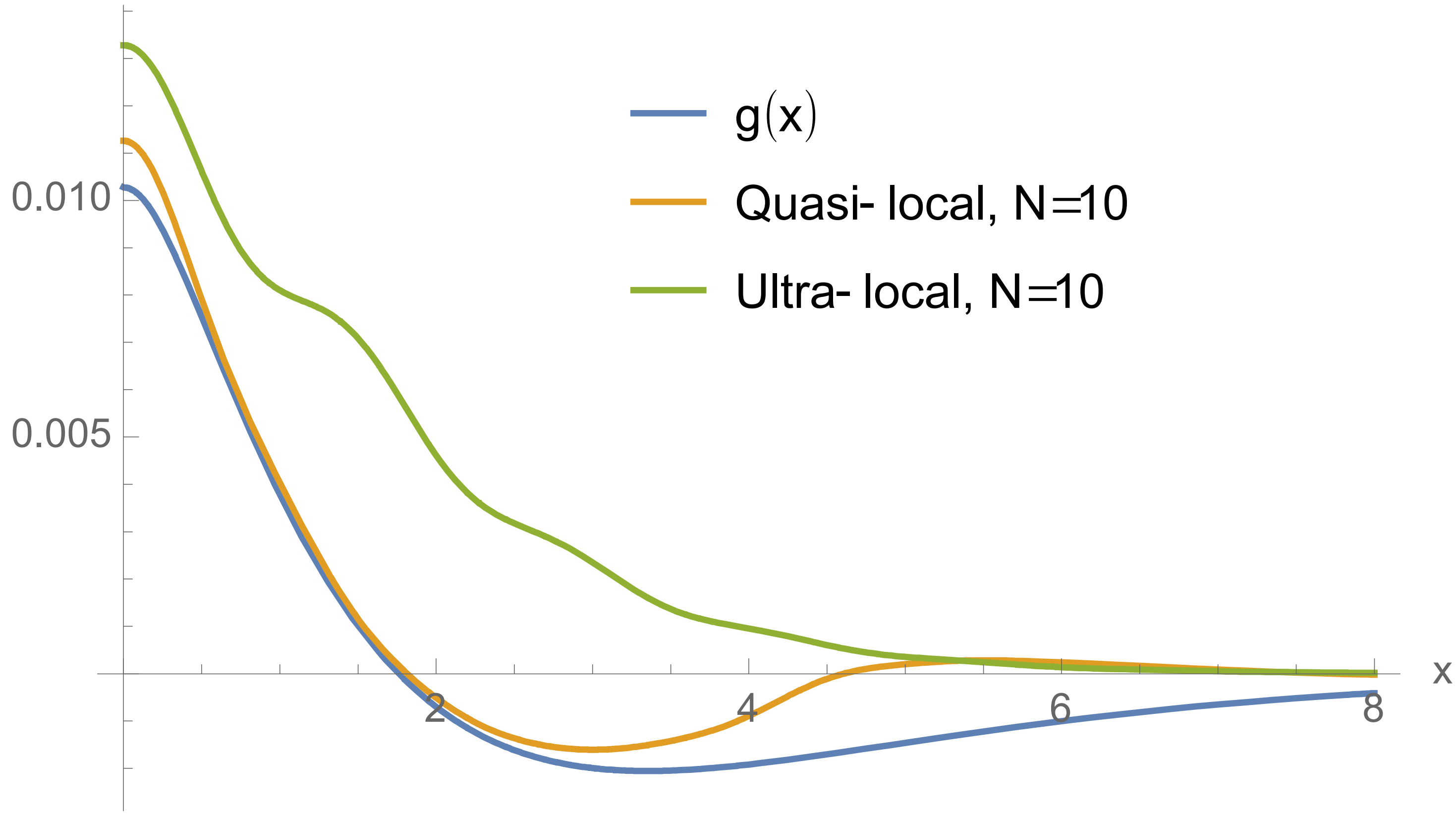
0.015
0.010
0.005
2
4
6
8
x
g(x)
$g_{\gamma,N}(x)$ with $\gamma=4$, N=5
$g_{\gamma,N}(x)$ with $\gamma=4$, N=10

Results



0.012
0.010
0.008
0.006
0.004
0.002
- 0.002
2
4
6
8
10
x
g(x)
g_N(x) with N=20
g_N(x) with N=40

Ultra-local charges



Summary

- Setting up the GGE might be tricky
- In the field theory, ultra-local charges are not enough
- Quasi-local charges are a way to go
- Real space representation of the quasi-local charges for more complicated QFT's ?

Thank you!