Quantum quenches in the sine-Gordon model: a semiclassical study

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Introduction

Quantum quenches: out of equilibrium time evolution of closed quantum systems

- Thermalization, nature of asymptotic steady state.
  unanswered questions: e.g. GGE
  M. Kormos et al., Phys. Rev. B **88**, 205131 (2013); F.H.L. Essler, G. Mussrdo, M. Panfil, Phys. Rev. A **91**, 051602 (2015)
  B. Wouters et al., Phys. Rev. Lett. **113**, 117203 (2014), B. Pozsgay et al., Phys. Rev. Lett. **113**, 117203 (2014).
- Time evolution, approach to steady state
  - numerical studies are constrained to small system sizes or short times, especially for continuum models
  - analytic description:
    - models that can be mapped to free particles
      (Luttinger liquid, XY and Ising spin chains, Tonks-Girardeau gas)
    - integrable systems? need overlaps and form factors
      promising direction: quench action approach
      J.-S. Caux and F. H. L. Essler, Phys. Rev. Lett. 110, 257203 (2013).

→ B. Bertini, D. Schuricht, and F. H. L. Essler, J. Stat. Mech. P10035 (2014)

The sine-Gordon model

$$\mathcal{S} = \frac{c}{16\pi}\int \mathrm{d}x\mathrm{d}t\left[\frac{1}{c^2}(\partial_t\Phi)^2 - (\partial_x\Phi)^2 + \lambda\cos(\beta\Phi)\right]$$

- $1/\sqrt{2} < \beta < 1$

  gapped, repulsive regime with soliton and antisoliton that interpolate between minima of the potential $\Phi = n\frac{2\pi}{\beta}, \quad n \in \mathbb{Z}$
- quench defined by the initial state

$$|\psi_0\rangle = \exp\left\{\int_0^\infty \frac{\mathrm{d}\theta}{2\pi} K_{ab}(\theta)\hat{Z}_a^\dagger(-\theta)\hat{Z}_b^\dagger(\theta)\right\}|0\rangle$$

  - superposition of independent pairs of kinks
  - natural for small quenches from $\Phi = 0$:
    a local perturbation creates a soliton-antisoliton pair
  - pair structure is ubiquitous
    (free bosons, Ising chain, LL, Lieb-Liniger, XXZ spin chain)

Review: form factor results

B. Bertini, D. Schuricht, and F. H. L. Essler, J. Stat. Mech. P10035 (2014)

small quench:

$$\rho_{ab} \approx \int_0^\infty \frac{\mathrm{d}\theta}{2\pi} mc\cosh(\theta)|K_{ab}(\theta)|^2 \ll 1$$

Decay of a special vertex operator

$$\left\langle e^{i\beta\Phi(x,t)/2} \right\rangle = \mathcal{G}_{\beta/2}\ \ e^{-t/\tau} \qquad \tau^{-1} = 2m \int_0^\infty \frac{\mathrm{d}\theta}{\pi} \sum_{ab} |K_{ab}(\theta)|^2 \sinh\theta + \mathcal{O}(K_{ab}^4)$$

Generic vertex operator: short time expansion

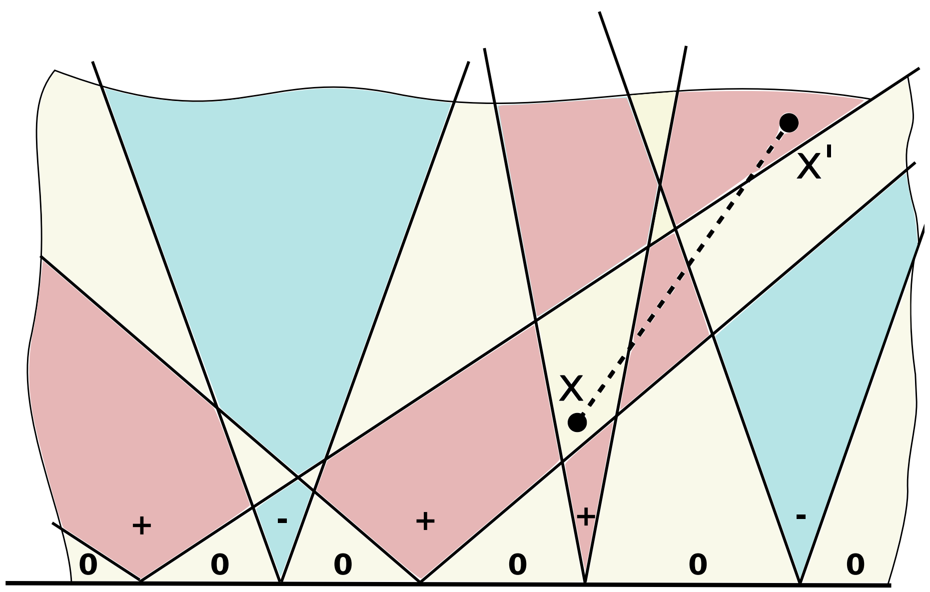
$$\frac{\left\langle e^{i\alpha\Phi(x,t)} \right\rangle}{\mathcal{G}_\alpha} = 1 - \sin^2(\pi\alpha/\beta)\,\frac{t}{\tau} + \mathcal{O}(t^2)$$

Decay to zero at a different rate?

Semiclassical approach $\rightarrow$ understand the real input

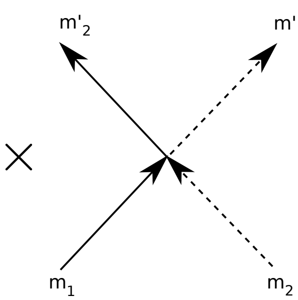
Semiclassical approach $\rightarrow$ extend the results

Semiclassical approach



- Low density: the kinks follow classical trajectories (straight lines)
- when they collide, need QM: use the low energy universal S-matrix

$$S^{m_1,m_2}_{m'_1,m'_2} = (-1)\,\delta_{m_1,m'_2}\delta_{m_2,m'_1} = (-1)\;\times$$

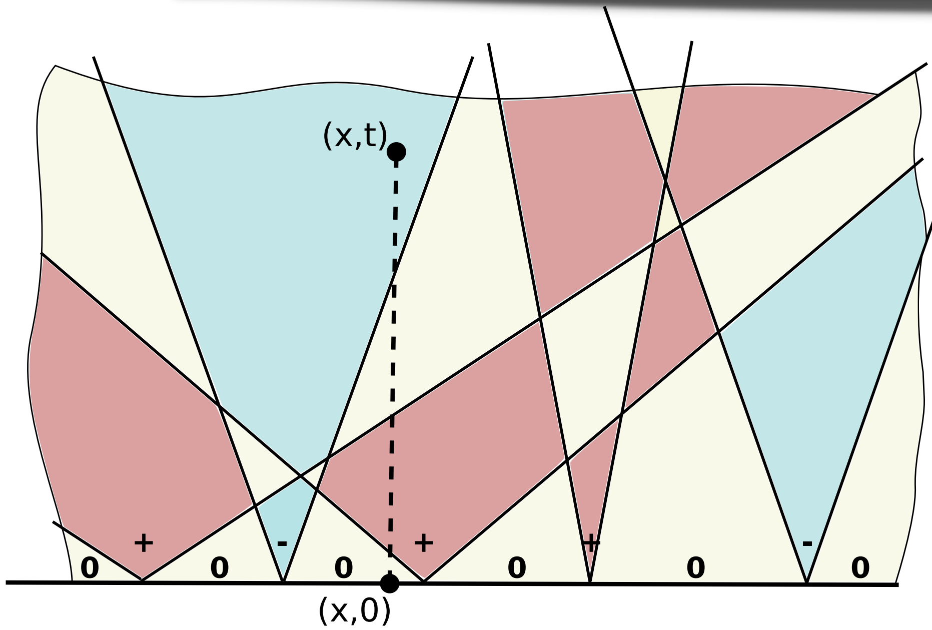


- expectation values are computed as averages over the semiclassical configurations given by $\{x_i, v_i, m_i\}$
  velocities are drawn from the distribution $f_{mm'}(v) \approx \dfrac{M}{2\pi\rho}|K_{mm'}(v/c)|^2$
- kinks are domain walls separating domains of constant $\Phi = n\dfrac{2\pi}{\beta}, \quad n \in \mathbb{Z}$
- the sequence of kinks and domains is unchanged in time

S. Sachdev, A. P. Young, Phys. Rev. Lett. 78, 2220 (1997), S. Sachdev, K. Damle, Phys. Rev. Lett. **78**, 943 (1997), Phys. Rev. Lett. **95**, 187201 (2005), A. Rapp, G. Zaránd, Physical Review B **74**, 014433 (2006).

F. Iglói, H. Rieger, Phys. Rev. Lett. **106**, 35701 (2011), B. Blass, H. Rieger, F. Iglói, EPL **99**, 30004 (2012), S. Evangelisti, J. Stat. Mech. P04003 (2013)

Expectation value $\langle e^{i\alpha\Phi(x,t)}\rangle$



Need to count how many domains we shift while traveling along the dashed line

$$s = n_+ - n_-$$

even:

$$\frac{\langle e^{i\alpha\Phi(x,t)}\rangle}{\mathcal{G}_\alpha} = 1$$

odd:

$$\frac{\langle e^{i\alpha\Phi(x,t)}\rangle}{\mathcal{G}_\alpha} = \frac{1}{2}\left(e^{i\alpha\frac{2\pi}{\beta}} + e^{-i\alpha\frac{2\pi}{\beta}}\right)$$

Probability that a given pair intersects the segment from the left (right):

$$p = \int_0^\infty \mathrm{d}v \frac{vt}{L} f(v)$$

Probability of a configuration with n_+ right and n_- left crossings:

$$p(n_+, n_-) = \frac{1}{n_+!}\frac{1}{n_-!} Q^{n_+ + n_-} e^{-2Q} \qquad Q = t\rho \int_0^\infty \mathrm{d}v\, v f(v) = \langle n_\pm \rangle$$

$$\boxed{\frac{\langle e^{i\alpha\Phi(x,t)}\rangle}{\mathcal{G}_\alpha} = \sum_{n_+,n_-=0}^{\infty} p(n_+,n_-)\left(\frac{1+(-1)^{n_+-n_-}}{2} + \frac{1-(-1)^{n_+-n_-}}{2}\cos(2\pi\alpha/\beta)\right) = \cos^2(\pi\alpha/\beta) + \sin^2(\pi\alpha/\beta)e^{-t/\tau}}$$

$$\tau^{-1} = 4\rho \int_0^\infty \mathrm{d}v v f(v)$$

Expectation value $\langle e^{i\alpha\Phi(x,t)}\rangle$

$$\langle e^{i\alpha\Phi(x,t)}\rangle/\mathcal{G}_\alpha = \cos^2(\pi\alpha/\beta) + \sin^2(\pi\alpha/\beta)e^{-t/\tau} \qquad \tau^{-1} = 4\rho\int_0^\infty \mathrm{d}v\, v f(v)$$

- exponential relaxation to an operator dependent **constant** follows from the purely reflective S-matrix
- time scale is set by the collision time
- agrees with the FF calculation:

$$\langle e^{i\alpha\Phi(x,t)}\rangle/\mathcal{G}_\alpha = 1 - \sin^2(\pi\alpha/\beta)\,\frac{t}{\tau} + \mathcal{O}(t^2)$$

- $\alpha = \beta/2$ gives the Ising case: $e^{i\frac{\beta}{2}n\frac{2\pi}{\beta}} = (-1)^n$

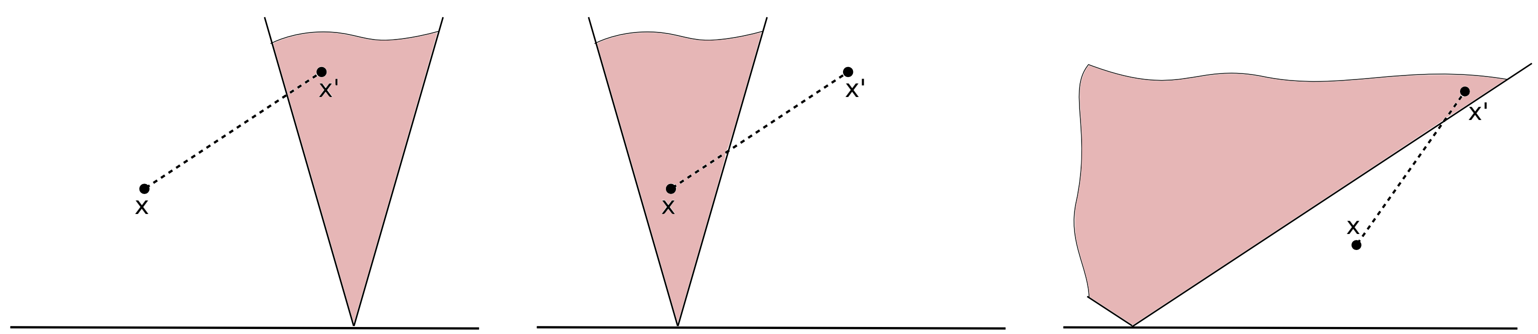
$$\langle e^{i\beta/2\,\Phi(x,t)}\rangle = \mathcal{G}_{\beta/2}e^{-t/\tau}$$

agrees exactly with the FF calculation!
suggests that the exact sG S-matrix, FF's, conserved charges are not really necessary (at leading order)

Dynamical correlation functions

$$\tilde{C}_\alpha(\Delta x; t, t') \equiv \frac{\left\langle e^{i\alpha\Phi(x,t)} e^{-i\alpha\Phi(x',t')} \right\rangle}{\left\langle e^{i\alpha\Phi(x,t)} e^{-i\alpha\Phi(x',t')} \right\rangle_{\mathrm{vac}}}$$

- $\alpha = \beta/2$ is equivalent to the Ising calculation:



(a) Right cut (R). (b) Left cut (L). (c) Left cut "from above" (A).

$$\boxed{\tilde{C}_{\beta/2}(\Delta x; t, t') = \exp\left\{-4\rho(t+t')\int_0^{\tilde{v}} \mathrm{d}v f(v) v\right\} \exp\left\{-4\rho(x'-x)\int_{\tilde{v}}^{v_s} \mathrm{d}v f(v)\right\} \exp\left\{-4\rho|t'-t|\int_{v_s}^{\infty} \mathrm{d}v f(v) v\right\}}$$

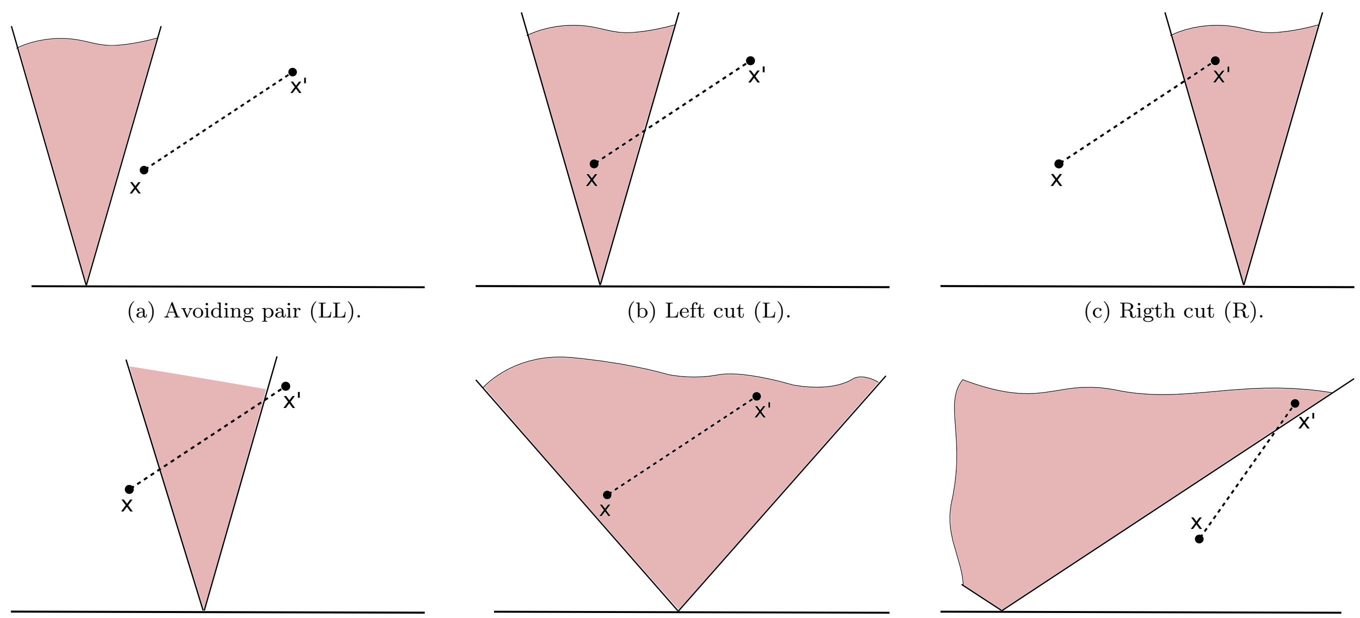
$$\tilde{v} = \frac{x'-x}{t'+t}\,, \qquad v_s = \frac{x'-x}{t'-t}$$

agrees with the scaling limit of the s.c. result of Evangelisti

scaling limit and semiclassical limit commute

Dynamical correlation functions

- **Generic** α



(a) Avoiding pair (LL). (b) Left cut (L). (c) Rigth cut (R).

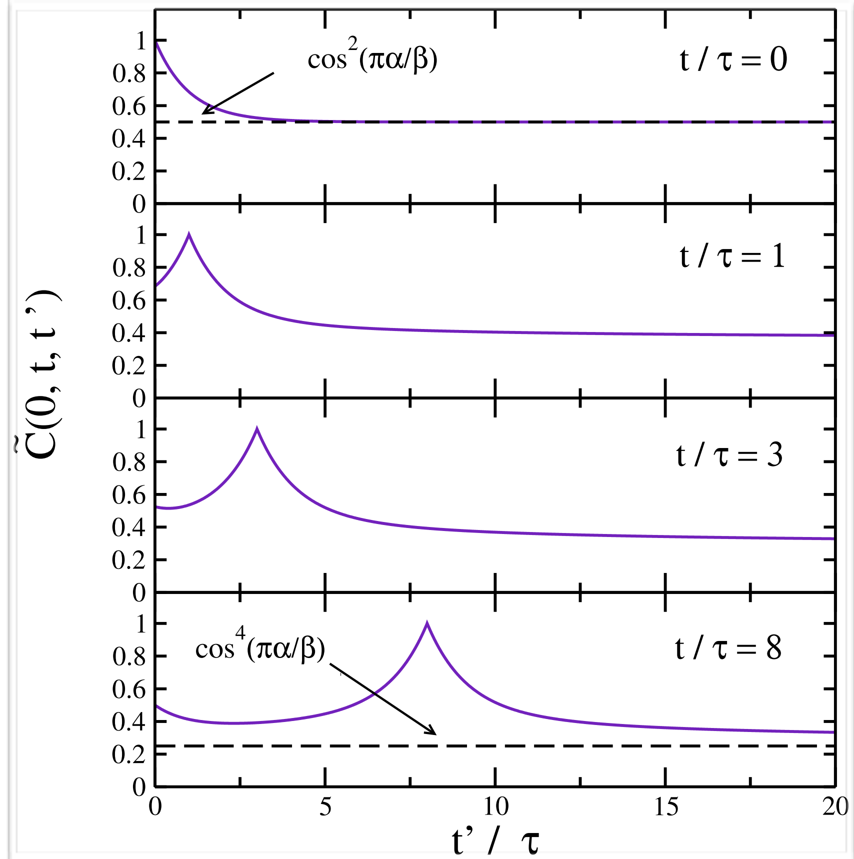
(d) Double cut (D) (e) Inclusion (I). (f) Left cut "from above" (A).

$$\tilde{C}_\alpha(x'-x;t,t') = \cos^4(\pi\alpha/\beta) + \sin^4(\pi\alpha/\beta)\, e^{-2Q_R-2Q_L-2Q_A} + \sin^2(\pi\alpha/\beta)\cos^2(\pi\alpha/\beta)\, e^{-2Q_I}(e^{-2Q_L}+e^{-2Q_R-2Q_A})$$
$$+\, 2\sin^2(\pi\alpha/\beta)\cos^2(\pi\alpha/\beta)\, e^{-(Q_L+Q_R+Q_D+Q_A)} \int_{-\pi}^{\pi} \frac{\mathrm{d}\phi}{2\pi} e^{Q_R e^{i\phi}+Q_D e^{2i\phi}+Q_A e^{-i\phi}} \left(e^{Q_L e^{i\phi}} - e^{-Q_L e^{i\phi}-2Q_I}\right)$$

with

$$Q_L = \rho\int_0^{\tilde{v}} \mathrm{d}v 2vtf(v) + \rho\int_{\tilde{v}}^{v_s} \mathrm{d}v[x'-x-v(t'-t)]f(v)$$
$$\vdots$$

Autocorrelation function



$$\tilde{C}_\alpha(0; t, t') = \cos^4(\pi\alpha/\beta) + \sin^4(\pi\alpha/\beta)\, e^{-\Delta t/\tau}$$
$$+ \sin^2(\pi\alpha/\beta)\cos^2(\pi\alpha/\beta)\left[e^{-t/\tau}(1 + e^{-\Delta t/\tau}) + 2e^{-\Delta t/(2\tau)}(1 - e^{-t/\tau}) I_0\left(\frac{\Delta t}{2\tau}\right)\right]$$

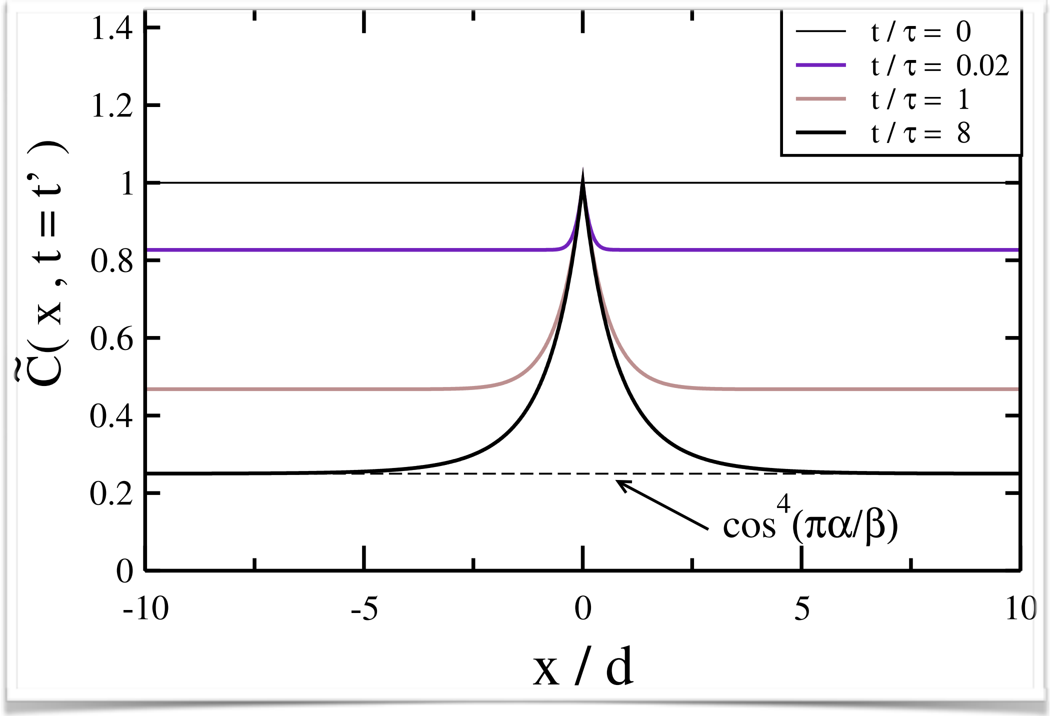
For large time separation shows **diffusive behavior**

$$\tilde{C}_\alpha(0; t, t' \to \infty) = \cos^4(\pi\alpha/\beta) + \sin^2(\pi\alpha/\beta)\cos^2(\pi\alpha/\beta)\left[e^{-t/\tau} + \frac{2(1 - e^{-t/\tau})}{\sqrt{\pi\Delta t/\tau}}\right]$$

For $\alpha = \beta/2$ **it is independent of** t **:** $\tilde{C}_{\beta/2}(0; t, t') = e^{-\Delta t/\tau}$

M. Kormos, M. Collura, P. Calabrese, Physical Review A **89**, 013609 (2014)

Equal time correlation function



$$\tilde{C}_\alpha(\Delta x; t, t) = \cos^4(\pi\alpha/\beta) + \sin^4(\pi\alpha/\beta)\, e^{-4\rho\Delta x + 4Q_D} + 2\sin^2(\pi\alpha/\beta)\cos^2(\pi\alpha/\beta)\left[e^{Q_D - 2\rho\Delta x} + e^{-t/\tau}(1 - e^{-Q_D}])\right]$$

$$Q_D(\Delta x, t) = \rho \int_0^{\tilde{v}} \mathrm{d}v(\Delta x - 2vt) f(v)$$

Cluster decomposition holds at all times

$$\tilde{C}_\alpha(\Delta x \to \infty; t, t) = \left[\cos^2(\pi\alpha/\beta) + \sin^2(\pi\alpha/\beta)\, e^{-t/\tau}\right]^2$$

Correlation length is set by the mean interparticle spacing

Asymptotic correlation functions

$t, t' \to \infty, \quad \Delta t$ fixed

$$\boxed{\tilde{C}^{as}_{\alpha}(\Delta x; \Delta t) = \cos^4(\pi\alpha/\beta) + \sin^4(\pi\alpha/\beta)\, e^{-4\rho\Delta x - 4Q_A} + 2\sin^2(\pi\alpha/\beta)\cos^2(\pi\alpha/\beta)\, e^{-2\rho\Delta x - 2Q_A} I_0\left(2\sqrt{Q^{\rm as}_A(Q^{\rm as}_A + 2\rho\Delta x)}\right)}$$

$$Q^{\rm as}_A(\Delta x, \Delta t) = \rho \int_{v_s}^{\infty} {\rm d}v (v\Delta t - \Delta x) f(v)$$

Assuming $f(v) = f_0 v^k + \mathcal{O}(v^{k+1})$

it is approached in a power law fashion $\sim 1/t^{(k+1)}$

Summary

arXiv:1507.02708

1. analytical approximations for 1-pt and dynamical 2-pt functions for small quenches through and intuitive method recovering and extending FF calculations
2. the method does **not** rely on integrability
3. results can be valid even in the attractive regime (breathers as spectators)
4. both the constant expectation values and the diffusive dynamical correlations are consequences of the special S-matrix after $T \sim \tau c^2/\bar{v}^2$ domains will change "color" $\longrightarrow$ pre-relaxation?

Thank you!