Low temperature dynamics of nonlinear Luttinger liquids

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[Christoph Karrasch, Rodrigo Pereira, JS, arXiv: 1410.2226 (2014)]

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MATERIALS IN MAINZ SCIENCE

Longtime asymptotics of finite-T correlation functions

$$H = J\sum_i \left(s_i^x s_{i+1}^x + s_i^y s_{i+1}^y + \Delta s_i^z s_{i+1}^z\right) + H_{\mathrm{pert}}$$

$$G(r,t) \equiv \langle S^z_{j+r}(t) S^z_j(0)\rangle = \frac{1}{Z}\mathrm{Tr}\left\{S^z_{j+r}(t)S^z_j(0)\mathrm{e}^{-\beta H}\right\}$$

Questions

- What is the asymptotic behavior for $0 \leq \Delta < 1$, $T/J \ll 1$, and $t \gg r/v$?
- Is there spin diffusion? $G(r,t) \sim \mathrm{e}^{-r^2/4Dt}/\sqrt{t}$

Aim

Develop a finite temperature nonlinear Luttinger liquid theory

Phenomenological spin diffusion theory

[N. Bloembergen, Physica **15**, 386 (49); P.G. de Gennes, J. Phys. Chem. Solids **4**, 223 (58)]

Diffusion equation: $\partial_t S^z_q(t) = -Dq^2\, S^z_q(t)$

This implies: $\langle S^z_q(t) S^z_{-q}(0)\rangle \sim \mathrm{e}^{-Dq^2 t}$

$G(r,t) = \langle S^z_{j+r}(t) S^z_j(0)\rangle \sim \mathrm{e}^{-r^2/4Dt}/\sqrt{t}\,; \quad G(0,t) \sim 1/\sqrt{t}$

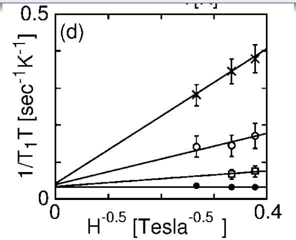
Problems

- Assumes independent Gaussian modes for 3D FM at high T
- Implies diffusive spin transport, but transport is ballistic in the integrable case: $\lim_{t\to\infty} \lim_{L\to\infty} \langle J(t) J(0)\rangle_T \neq 0$

[Prosen, PRL **106**, 217206 (11), NPB **886**, 1177 (14); Pereira, Pasquier, JS, Affleck, JSTAT P09037 (14)]

Experiments and numerical studies

NMR, if diffusive: $$\frac{1}{T_1} = \int \frac{dq}{2\pi} \underbrace{|A(q)|^2}_{\sim\text{const}} S^{zz}(q,\omega) \sim \frac{1}{\sqrt{\omega}}$$



- Experiment on Sr_2CuO_3

  [Thurber *et al*, PRL **87**, 247202 (01)]

- Other experiments, e.g.:

  [Pratt *et al*, PRL **96**, 247203 (2006); Xiao *et al*, PRB **91**, 1444417 (15)]

Numerical studies and short time expansions inconclusive

- Quantum model: [Böhm *et al*, PRB **49**, 417: PRB **49**, 15669 (94); Fabricius, Löw, Stolze, PRB **55**, 5833 (97); Fabricius, McCoy, PRB **57**, 8340 (98); Starykh, Sandvik, Singh, PRB **55**, 14953 (97)]
- Classical model:

  [Müller, PRL **60**, 2785 (88); Gerling, Landau, PRL **63**, 812 (89); Bagchi, PRB **87**, 075133 (12)]

Outline

- Noninteracting case: Dominant contributions to the longtime asymptotics of the autocorrelation function
- Nonlinear Luttinger liquid theory
- Temperature dependent corrections and consequences of integrability/integrability breaking
- Comparison with tDMRG data

Jordan-Wigner: Fermionic representation

$$H = \sum_i \left[-\frac{1}{2}\left(c_i^\dagger c_{i+1} + h.c.\right) + \Delta \left(c_i^\dagger c_i - \frac{1}{2}\right)\left(c_{i+1}^\dagger c_{i+1} - \frac{1}{2}\right)\right]$$

$$G^{(0)}(0,t) = \langle c_i(t) c_i^\dagger(0)\rangle \langle c_i^\dagger(t) c_i(0)\rangle = \left(\int_{-\pi}^{\pi} \frac{dk}{2\pi} \frac{e^{i\varepsilon_k t}}{e^{\beta\varepsilon_k}+1}\right)^2$$

- $\varepsilon_k = -\cos k$
- $G^{(0)}(0,t)$ oscillates: Saddle points, $d\varepsilon_k/dk = 0$, occur at $k=0$ and $k=\pi$
- Saddle points dominate longtime asymptotics, and not the Fermi points, even for $T=0$!
- Regular Luttinger liquid theory cannot desribe longtime behavior

Mode expansion

$T \ll 1$: keep states near the (smeared) Fermi surface and near the saddle points

$$c_{j=x} \sim \Psi_R(x) e^{i\pi x/2} + \Psi_L e^{-i\pi x/2} + \bar{d}^\dagger(x) + e^{i\pi x} d(x)$$

Low energy modes: linear dispersion, CFT result

$$\langle \Psi_{R,L}(x,t) \Psi^\dagger_{R,L}(x,0) \rangle \sim \pi T / \sinh(\pi T t)$$

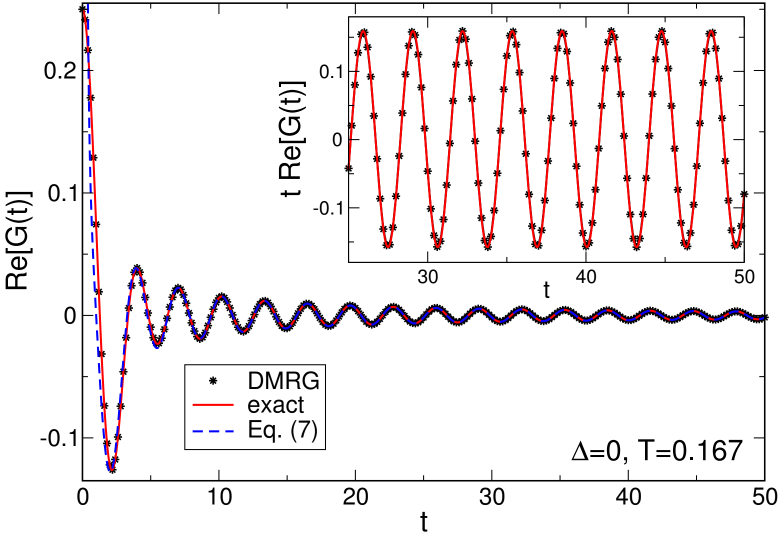
Band edges, $\varepsilon_k \approx 1 - k^2/2$: $\langle d(x,t) d^\dagger(x,0) \rangle \sim e^{-it}/\sqrt{t}$

$$G^{(0)}(0,t) \approx \frac{i e^{-2it}}{2\pi t} + \frac{\sqrt{2}\, T e^{-i(t+\pi/4)}}{\sqrt{\pi t}\, \sinh(\pi T t)} - \frac{T^2}{\sinh^2(\pi T t)}$$

For $t \gg 1/T$ the first term (high energy particle and hole) dominates.

[JS, PRB **73**, 224424 (06)]

Exact result, longtime approximation, and tDMRG



Nonlinear Luttinger liquid theory

Using the mode expansion and bosonizing the low-energy modes:

$$\begin{aligned}\mathcal{H} &= d^\dagger\left(\varepsilon+\frac{\partial_x^2}{2m}\right)d+\bar{d}^\dagger\left(\varepsilon+\frac{\partial_x^2}{2m}\right)\bar{d}+Vd^\dagger d\,\bar{d}^\dagger\bar{d}\\ &+\frac{v}{2}[(\partial_x\theta)^2+(\partial_x\phi)^2]+\frac{v\alpha}{\sqrt{\pi K}}\partial_x\phi(d^\dagger d-\bar{d}^\dagger\bar{d})\end{aligned}$$

- Dual fields obeying $[\phi(x),\partial_{x'}\theta(x')]=i\delta(x-x')$
- Velocity and bandwidth: $v=\varepsilon=\frac{1}{m}=\frac{\pi\sqrt{1-\Delta^2}}{2\arccos\Delta}$
- Luttinger parameter: $K=1-\frac{\alpha}{2\pi}=\frac{\pi/2}{\pi-\arccos\Delta}$
- Impurity-impurity interaction: $V\approx-4\Delta$ for $\Delta\ll 1$

Decoupling of impurities and bosonic modes by a unitary transformation: $d,\bar{d}(x)\to d,\bar{d}(x)e^{\pm i\alpha\theta(x)/\sqrt{\pi K}}$

Autocorrelation in simplest approximation

Impurity-Impurity contrib. for $t \gg 1/V^2$ (from ladder diagrams):

$$G_2(t) \approx \frac{B(T)}{t^2}\, e^{-2i\varepsilon t}$$

Impurity-boson contribution: $\eta = K, \quad \varphi = \pi[\eta - 1/2]/2$

$$G_1(t) \approx \frac{A(T)}{\sqrt{t}} \left[\frac{\pi T}{\sinh(\pi T t)}\right]^{\eta} e^{-i[\varepsilon t + \varphi]}$$

Luttinger liquid contribution:

$$G_{\mathrm{LL}}(t) \approx A' \left[\frac{\pi T}{\sinh(\pi T t)}\right]^{2} + B' \left[\frac{\pi T}{\sinh(\pi T t)}\right]^{2K}$$

Generalization of Pereira, White, Affleck, PRL **100**, 027206 (08)

But: Parameters significantly renormalized at finite T by irrel. ops.

T-dependent renormalization from dimension three operators

Kinematically allowed dimension three operators:

$$\begin{aligned}\delta\mathcal{H} &= g[(\partial_x\theta)^2+(\partial_x\phi)^2](d^\dagger d+\bar{d}^\dagger\bar{d}) \\ &+g'[(\partial_x\theta)^2-(\partial_x\phi)^2](d^\dagger d+\bar{d}^\dagger\bar{d})-\mu_+\partial_x^2\theta\left(d^\dagger d-\bar{d}^\dagger\bar{d}\right) \\ &+\mu_-\partial_x\theta\left(-id^\dagger\partial_x d+i\bar{d}^\dagger\partial_x\bar{d}+h.c.\right)\end{aligned}$$

To lowest order: $g=-\Delta, \quad \mu_-=-\Delta/\sqrt{\pi}, \quad g'=\mu_+=0$

Renormalization of parameters in the autocorrelation

- Effective bandwidth decreases: $\varepsilon\to\varepsilon(T)=\varepsilon+cgT^2$
- $G_1(t)$ decays slower: $\eta\to\eta(T)=\eta+\tilde{c}gT$

T dependence holds for arbitrary $0<\Delta<1$ as long as $T\ll 1$!

Integrability and integrability breaking

Integrable systems have L local conserved charges: $[H, Q_n] = 0$
$\rightarrow$ Field theory: Not all kinematically allowed terms are present

- Energy current density: $\partial_x j(x) = i[\mathcal{H}(x), H]$
- Energy operator conserved: $[J_E, H] = [\int dx\, j(x), H] = 0$

Consider $g'[(\partial_x\theta)^2 - (\partial_x\phi)^2](d^\dagger d + \bar{d}^\dagger \bar{d})$

- Conservation of $J_E \rightarrow g' \equiv 0$
- g' introduces a nonzero impurity decay rate for $T > 0$!

[Castro Neto, Fisher, PRB **53**, 9713 (96); Karzig, Glazman, von Oppen, PRL **105**, 226407 (10)]

System with integrability breaking perturbation

$$G_2(t) \approx \frac{B(T)}{t^2}\, e^{-2i\varepsilon t} e^{-t/\tau} \text{ with } \tau^{-1} \sim (g')^2 (T/\varepsilon)^2$$

Umklapp scattering and diffusive decay

Umklapp scattering at half-filling: $\delta H_U = \lambda \cos(4\sqrt{\pi K}\phi)$

Boson propagator: [Sirker, Pereira, Affleck, PRL **103**, 216602 (09); PRB **83**, 035115 (11)]

$$\langle\phi\phi\rangle^{\mathrm{ret}}(q,\omega) = \frac{v}{\omega^2 - v^2q^2 + 2i\gamma\omega}; \quad \gamma \propto \lambda^2 T^{8K-3}$$

$$G(r,t) = -\frac{2K}{\pi}\int\frac{d\omega}{2\pi}\int\frac{dq}{2\pi}\, q^2 e^{i(qr-\omega t)}\frac{\mathrm{Im}\,\langle\phi\phi\rangle^{\mathrm{ret}}(q,\omega)}{1-e^{-\omega/T}}$$

- Pole and branch cut contributions
- Umklapp dangerously irrel.; changes asymptotics completely

Analytical result for $t \gg 1/\gamma \gg 1/T$

$$G_{\mathrm{diff}} = \frac{\Gamma}{\sqrt{t}}, \qquad \Gamma = \frac{KT}{\pi v^2}\sqrt{\frac{\gamma}{2\pi}} \qquad (\Gamma \text{ known for XXZ})$$

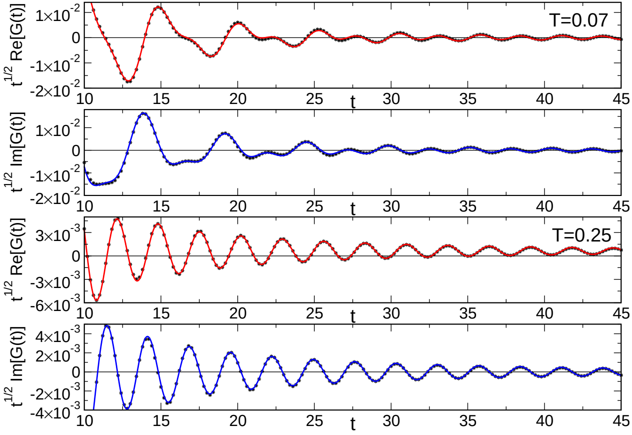
Final result for the autocorrelation function

$$
\begin{aligned}
G(t) \;\sim\; & \frac{\Gamma}{\sqrt{t}} + \frac{B(T)}{t^2}\, e^{-2i\varepsilon(T)t} \\
+ \; & \frac{A(T)}{\sqrt{t}} \left[\frac{\pi T}{\sinh(\pi T t)}\right]^{\eta(T)} e^{-i[\varepsilon(T)t+\varphi(T)]} \\
+ \; & A' \left[\frac{\pi T}{\sinh(\pi T t)}\right]^{2} + B' \left[\frac{\pi T}{\sinh(\pi T t)}\right]^{2K}
\end{aligned}
$$

Integrable case

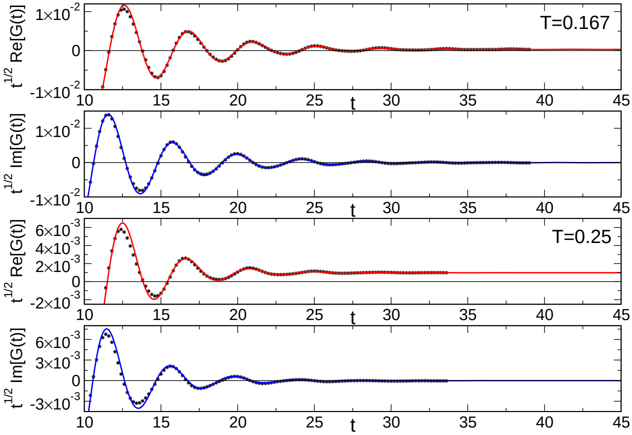
- Γ, A', and B' known exactly
- $\eta(0) = K$, $\varepsilon(0) = v$, and $\varphi(0)$ known exactly; finite T corrections known to first order in Δ
- $A(T)$ and $B(T)$ are the only completely free fitting parameters

$\sqrt{t}G(t)$ at $\Delta = 0.3$



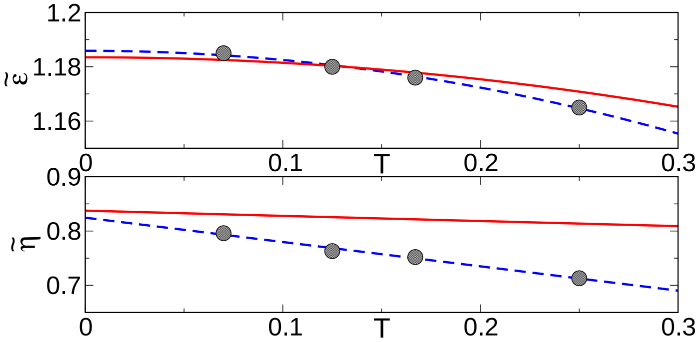
- $\Gamma_{\text{fit}}(T = 0.07) \approx 0$ and $\Gamma_{\text{th}}(T = 0.25) = 2.1 \cdot 10^{-5}$
- $\Gamma_{\text{fit}}(T = 0.25) \approx 6.3 \cdot 10^{-4}$ and $\Gamma_{\text{th}}(T = 0.25) = 7.9 \cdot 10^{-4}$

$\sqrt{t}G(t)$ at $\Delta = 0.8$



- $\Gamma_{\rm fit}(T = 0.167) \approx 3.3 \cdot 10^{-4}$ and $\Gamma_{\rm th}(T = 0.25) = 5.6 \cdot 10^{-4}$
- $\Gamma_{\rm fit}(T = 0.25) \approx 9.9 \cdot 10^{-4}$ and $\Gamma_{\rm th}(T = 0.25) = 1.3 \cdot 10^{-3}$

$\varepsilon(T)$ and $\eta(T)$ for $\Delta = 0.3$: fit versus $\mathcal{O}(\Delta)$ theory



- $\varepsilon(T) = \varepsilon(0) - cT^2$ (effective bandwidth)
- $\eta(T) = \eta(0) - \tilde{c}T$ (exponent for single imp. contribution)

Finite temperature nonlinear Luttinger liquid theory

Longtime asymptotics of $\langle S_j^z(t) S_j^z(0)\rangle$ for the XXZ chain:

- Mode expansion including Fermi points and band edges
- $T > 0$: CFT for linear modes; edge modes not affected for $T \ll 1$
- Bandwidth ($\sim -T^2$) and 'exponents' ($\sim -T$) renormalized
- Umklapp leads to diffusive contribution $\sim 1/\sqrt{t}$
- Predictions confirmed by unbiased tDMRG calculations

The integrable XXZ chain shows spin diffusion—in the sense of a $1/\sqrt{t}$ longtime decay of $G(r,t)$—coexisting with ballistic spin transport; diffusion is generic ballistic transport is not.

NLL approach can be generalized to other models, e.g. Bose gas