Finite size spectra of the OSp(3|2) superspin chain

Critical properties of an intersecting loop model

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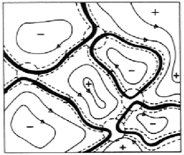
Beyond integrability: The mathematics and physics of integrability and its breaking
Centre de recherches mathématiques, Montréal 13-17 July 2015

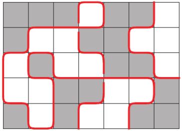
HF & MJ Martins: Nucl. Phys. B **894** (2015) 665-684 [arXiv:1502.05305]

Spin chains based on superalgebras

Disorder problems

- network models for disorder problems (e.g. QH plateau transitions)
- lattice gas with random scatterers / (intersecting) loops





≻ replica trick or <u>supersymmetry techniques</u>

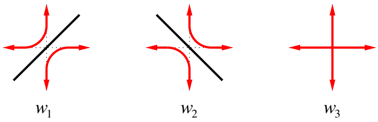
Complications

- superalgebras: finite/infinite irreps, indecomposable reps
- non-Hermitian hamiltonian / non-unitary CFT: non-normalizable states, continuous spectrum, ...

Challenges

- lattice regularization with local interactions and finite dimensional state space (possibly after truncation) [Zirnbauer ...]
- strong subleading finite size corrections [Essler,HF,Saleur (2005), ...]
- characterization of CFT with non-compact target [Ikhlef,Jacobsen,Saleur (2012),...]
- corrections to scaling in presence of continuous spectrum [HF&Seel (2014)]

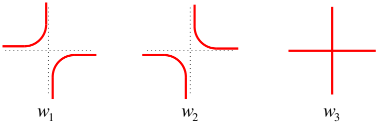
Lattice gas with random obstacles



- mirrors with two orientations placed randomly on square lattice
- probabilities w_i

$\succ$ consider deterministic motion of particles on the lattice

Intersecting loops – Goldstone phase



- Boltzmann weights w_k of local configurations
- fugacity z for closed loops (nonlocal)

partition function ($\mathcal{N}$: no. of closed loops)

$$Z = \sum_{\text{loop configurations}} w_1^{m_1}\, w_2^{m_2}\, w_3^{m_3}\, z^{\mathcal{N}} \,,$$

continuum limit $\sim O(N)$ scalar field theory $\sim OSp(m|2n)$ susy model with loop fugacity $z = N = m - 2n$ (e.g. $z \to 1$ for SAW/polymers)

low T phase for $N < 2$ in $2d$ [Jacobsen,Read,Saleur (2003); Martins *et al.* (1998)]:

- massless Goldstone modes of broken $O(N)$ or supersymmetry ($\neq$ 'dense loops' phase of *non*-intersecting loops for $|N| < 2$)
- marginally irrelevant coupling constant $g \sim 1/\log \Lambda$
- operators have integer conformal weights (including 0)

Intersecting loops – integrable models

- integrable realization of local loop configurations in terms of generators of braid-monoid algebra for Boltzmann weights

$$w_1 \sim 1\,, \quad w_2 \sim \frac{\lambda}{1-\frac{1}{2}z-\lambda}\,, \quad w_3 \sim \lambda$$

  $\succ$ R-matrix in terms of braid-monoid generators satisfying a Yang-Baxter equation

- for integer fugacity z: (local) vertex model based on finite-dimensional representation of $OSP(m|2n)$ for $z = m - 2n$ [Martins,Nienhuis,Rietmann (1998)]
- finite size scaling for integrable superspin model:
  - central charge $c = z - 1$
  - indication for vanishing critical exponent

  $\sim$ fractal dimension of loops $d_f = 2$ ('superdiffusion' in lattice gas picture).

$\succ$ **integrable realization of Goldstone phase in a lattice model**
(finite weight for crossings)

The $OSp(3|2)$ superspin chain

Irreducible representations

- irreps are labelled by two integer or half-integer numbers $(p \geq 0; q \geq \frac{1}{2})$
- quadratic Casimir: $I_2 = (p(p+1) + 2q(1-2q))$
- $OSp(3|2) \supset SU(2) \oplus SU(2)$: decompose into $\ell \otimes s$ irreps of $SU(2) \oplus SU(2)$:
  - trivial representation $(0;0) = (0 \otimes 0)$:
    ground state of even L superspin chain
  - 5d fundamental representation $(0;\frac{1}{2}) = (1 \otimes 0) \oplus (0 \otimes \frac{1}{2})$
    local spins, ground state of odd L superspin chain

Partition function

exact largest eigenvalue of transfer matrix

$$\Lambda_{\max} = (w_1 + w_2 + w_3)^L$$

$\succ$ ground state energy $E_0 = -\left.\partial_\lambda \Lambda_{\max}\right|_{\lambda=0} = -3L$ without finite size corrections: low energy effective field theory with central charge $c = 0$ (non-unitary)

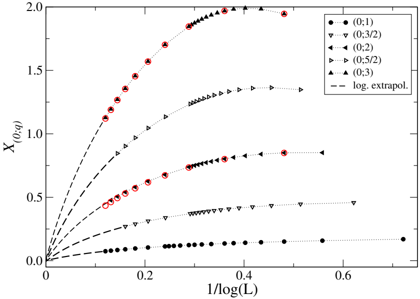
Ground state and lowest excitations

Ground state

- $OSp(3|2)$-singlet $(0;0)$ for even L, quintet $(0;\frac{1}{2})$ for odd L
- degenerate Bethe root configuration for even L

Excitations

- exact diagonalization: lowest states are $(0;q\geq 1)$ multiplets, $2q\sim L \mod 2$
- strong subleading corrections to finite size estimates of scaling dimensions [Martins, Nienhuis, Rietman (1998)]



perturbative RG for symmetry broken low-T (Goldstone) phase of $O(N=1)$ model with long distance cutoff L gives coupling constant

$$g\sim 1/\log L$$

vanishing dimensions $X_{(0;q)}\to 0$ for all q: log. fine-structure in finite size spectrum

Fine structure of finite size spectrum

Conjecture for log. amplitudes based on finite size data:

$$X_{(0;q)} = 0 + \frac{q(2q-1)}{\log(L/L_0)}\,, \quad q = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots$$

(cf. quadratic Casimir?)

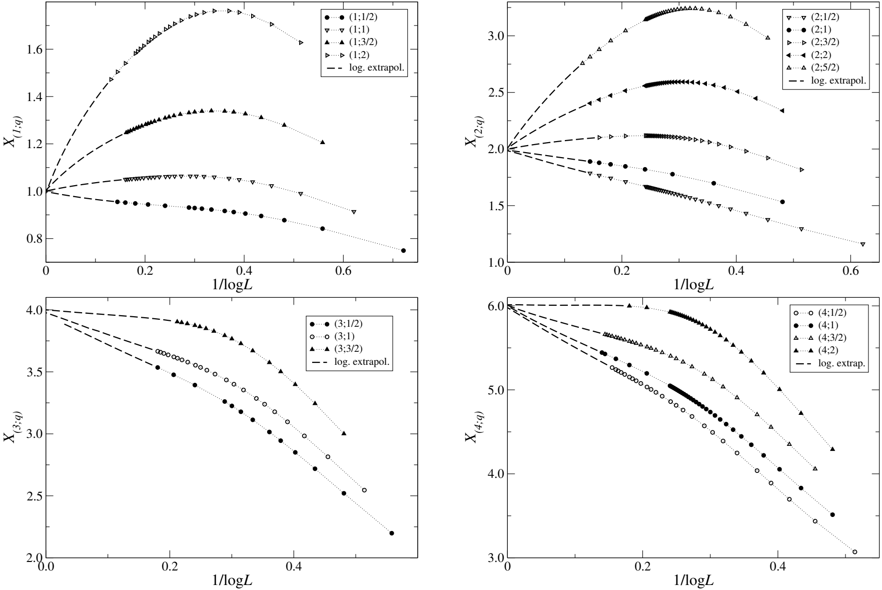
Long distance asymptotics of *watermelon* correlators $G_k(r)$
$\equiv$ probability of k loop segments connecting two points at distance r

$$G_k(r) \sim 1/(\log r)^{\alpha_k}\,, \quad \alpha_k \in \{2q(2q-1) : q = 1, \frac{3}{2}, 2, \ldots\}$$

$\succ$ identification of $(0;q)$ primary with the $k = 2q$-leg operator: agrees with perturbative RG and numerical results for G_2 and G_4 [Nahum *et al.* (2013)]

NB: The $(0;L/2)$ multiplet of the L-site superspin chain is (one of) the ferromagnetic reference states used in the Bethe ansatz: therefore the spectrum of the $(0;q)$ states extends from $E_0 = -3L$ to $E_{\text{fm}} = -L$
$\succ$ **continuous spectrum of scaling dimensions!**

Other symmetry sectors



Discrete part of conformal spectrum

$\succ$ continua starting at integer X! The lowest levels in the $(p;q)$ sector of the spectrum of the superspin chain correspond to operators with conformal weight

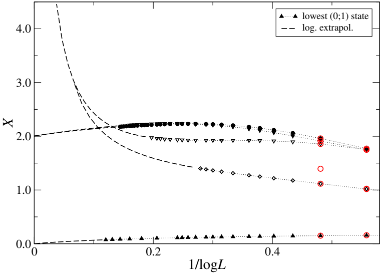
$$(h,\bar{h}) = \begin{cases} (h_{p/2}, h_{p/2}) & \text{for } p \text{ even}, \\ (h_{(p\pm1)/2}, h_{(p\mp1)/2}) & \text{for } p \text{ odd}. \end{cases}$$

where $h_k = \frac{1}{2}k(k+1)$, $k = 0, 1, 2, \ldots$

Continua of critical exponents with lower edges

sector $(p;q)$	$X = h + \bar{h}$	$s = h - \bar{h}$	$(h,\bar{h})$
$(0;q)$	0	0	$(0,0)$
$(1;q)$	1	±1	$(1,0)$, $(0,1)$
$(2;q)$	2	0	$(1,1)$
$(3;q)$	4	±2	$(3,1)$, $(1,3)$
$(4;q)$	6	0	$(3,3)$
...			

Continuum limit of a (compact) lattice model



Consider low E states in given symmetry sector $(p;q) = (0;1)$ for $L = 6, 8$ sites:

- primary with scaling dimension $X = 0$
- two doublets with conformal spin ± 1
- three excitations with conformal spin 0

Bethe ansatz for larger system sizes:

- two spin zero states extrapolate to $X = 2$: descendants of the primary
- one doublet and third spin 0 state: finite size extrapolation does not converge: disappear from the low E spectrum
- the 2^{nd} spin ± 1 level has not been identified in the BA (level-1 descendant?)

consequence of the vanishing coupling constant
$\succ$ generic property of lattice regularizations (with compact quantum space) field theories with continuous spectrum of critical exponents!

Summary

integrable lattice realization for the low-T Goldstone phase of intersecting loops

finite size study of $OSp(3|2)$ superspin chain:
continuum limit characterized by compact **and non-compact** degree of freedom:

- discrete part of spectrum is characterized by integer conformal weights (label p of $OSp(3|2)$ irrep $(p;q)$)
- emergence of continua of scaling dimensions (label q of $OSp(3|2)$ irrep)
- identification of log. amplitudes in lowest levels $\succ$ asymptotics of "watermelon" correlators
- compact spin chain $\rightarrow$ non-compact continuum limit:
  massive reordering of finite size levels

Some open problems

- amplitudes of subleading log. corrections in sectors $(p \geq 1; q)$:

$$\sim q(2q-1) + f(p)$$

  relation to quadratic $OSp(3|2)$ Casimir?
  $\succ$ NLIE formulation to deal with larger systems ($L \gg 10^4$)

- other models:
  - $OSp(2|2) \Leftrightarrow$ self-dual/isotropic limit of staggered $U_q[sl(2|1)]$ susy chain: at the border (critical level) of a phase described by the $SL(2,\mathbb{R})/U(1)$ sigma model at level $k = \pi/\gamma$ (2d Euclidean black hole) [HF&Martins (2011,2012), HF&Seel (2014)]
  - $OSp(1|2)$: rank 1 super algebra, central charge $c = -2$ but simple spectrum without room for continuous spectrum [Martins (1995)]
    reason: k-leg operators $\sim$ contraction of susy tensors vanishing for small m
    $\succ$ need to consider $OSp(m|2n)$ with $m - 2n = -1$ but m sufficiently large

- low energy effective theory for the Goldstone phase?
  - characterization of density of states in continua
    $\succ$ non-standard conserved charges?

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