Asymptotics of correlation functions of the Heisenberg-Ising chain in the easy-axis regime

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More than a decade of steady progress in our understanding of correlation functions of integrable models, chief example XXZ chain

\[ H = J \sum_{j=1}^{L} \left( \sigma_{j-1}^{x} \sigma_{j}^{x} + \sigma_{j-1}^{y} \sigma_{j}^{y} + \Delta (\sigma_{j-1}^{z} \sigma_{j}^{z} - 1) \right) - \frac{h}{2} \sum_{j=1}^{L} \sigma_{j}^{z} \]

\[ \Delta = \frac{q + q^{-1}}{2} = \text{ch}(\gamma), \; L \text{ even} \]
Outline

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- 1d variant of the fundamental model of antiferromagnetism in solids
- Can also be simulated by cold atoms in optical traps
  - in which time- and space resolved two-point functions can be measured
- We have calculated their long-time large-distance asymptotics in the massive antiferromagnetic regime

Joint project with Maxime Dugave, Karol Kozlowski and Junji Suzuki
The longitudinal ground-state two-point functions of the Heisenberg-Ising chain for $\Delta > 1$ have the form-factor expansion (see e.g. DGKS 15)

$$\langle \sigma_1^z \sigma_{m+1}^z (t) \rangle$$

$$= \frac{(q^2; q^2)^4}{(-q^2; q^2)^4} (-1)^m + \sum_{t=0,1} \sum_{n \in 2\mathbb{N}} \frac{(-1)^m}{n!} \int_{-\pi/2}^{\pi/2} \frac{d^n \nu}{(2\pi)^n} f_t(z) \prod_{j=1}^n e^{i[p(\nu_j)m-\epsilon(\nu_j)t]}$$

Here we used the standard notation for $q$-multi factorials

$$(a; q_1, \ldots, q_p) = \prod_{n_1, \ldots, n_p=0}^{\infty} (1 - aq_1^{n_1} \ldots q_p^{n_p})$$
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Sum over pairs of spinons with single particle momentum and energy $p$ and $\varepsilon$ parameterized in terms of a rapidity variable $\nu$,

$$p(\nu) = \frac{\pi}{2} + \nu - i \ln \left( \frac{\vartheta_4(\nu + i\gamma/2, q^2)}{\vartheta_4(\nu - i\gamma/2, q^2)} \right), \quad \varepsilon(\nu) = -\frac{4JK \text{sh}(\gamma)}{\pi} \text{dn} \left( \frac{2K\nu}{\pi} \Big| k \right)$$

$\vartheta_4$ is a Jacobi theta function and $\text{dn}$ a Jacobi-elliptic function, $k = k(q)$ is the elliptic modulus, and $K = K(k)$ the complete elliptic integral of the first kind.
Spinon energy and momentum are related by

\[ p'(ν) = -\varepsilon(ν)/2J \text{sh}(γ), \quad \varepsilon(p) = -\frac{v_{c_1} v_{c_2}}{\pi} \cdot \sqrt{1/k^2 - \cos^2(p)} \]

The dispersion relation reveals the massive nature of the excitations. Here we have introduced two combinations of parameters

\[ v_{c_1} = \frac{4JKk^2 \text{sh}(γ)}{\pi(1 + k')}, \quad v_{c_2} = \frac{4JKk^2 \text{sh}(γ)}{\pi(1 - k')} \]

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Form factor densities \( \mathcal{F}_{l,n}(z) \) depend on an even number \( n \) of rapidities \( \nu_1, \ldots, \nu_n \). For general \( n \) they were expressed by multiple integrals by Jimbo and Miwa 95 and by Fredholm determinants by DGKS 15. A more explicit expression is only known for the two-spinon case \( n = 2 \) and follows from Lashkevich’s 2002 result for XYZ.
LASHKEVICH’s formula allows us to write the two-spinon contribution to the longitudinal correlation function as

\[ \langle \sigma_1^z \sigma_{m+1}^z (t) \rangle_2 = \frac{(q^2; q^2)^4}{(-q^2; q^2)^4} (-1)^m + \frac{1}{2} l_2(m, t) \]

where

\[ l_2(m, t) = \prod_{j=1}^{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{d\nu_j}{2\pi} e^{i[\nu_j(m-\epsilon_j)t]} \] \[ \times f(\nu_1, \nu_2) \]
**Two-spinon contribution**

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l_2(m, t) = \left[ \prod_{j=1}^{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{d\nu_j}{2\pi} e^{i[p(\nu_j) m - \epsilon(\nu_j)t]} \right] f(\nu_1, \nu_2)
\]

The function \( f \) can be expressed as

\[
f(\nu_1, \nu_2) = A(\nu_1, \nu_2)(-1)^m + A(\nu_1 + \pi, \nu_2)
\]

where

\[
A(\nu_1, \nu_2) = \frac{32 \cos^2((p(\nu_1) + p(\nu_2))/2) \sin^2(\nu_{12})}{\sin((\nu_{12} + i\gamma)/2) \sin((\nu_{12} - i\gamma)/2)}
\]

\[
\times q(q^2; q^2)^2 \delta^2(\nu_{12}/2, \nu) \prod_{\sigma = \pm} \frac{(q^4; q^4, q^4)^2 (q^4 e^{2i\nu_{12}}; q^4, q^4)^2 (q^2 e^{2i\nu_{12}}; q^4)^2}{(q^2; q^4, q^4)^2 (q^2 e^{2i\nu_{12}}; q^4, q^4)^2 (q^4 e^{2i\nu_{12}}; q^4)^2}
\]

and \( \nu_{12} = \nu_1 - \nu_2 \)
Saddle-point analysis

We have calculated the asymptotics $m, t \to \infty$ for fixed ratio $\nu = m/t \geq 0$

In this limit the integral $I_2$ can be estimated by the method of steepest descent

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Let us define the function

$$g(\lambda) = i [\rho(\lambda) \nu - \epsilon(\lambda)]$$

Then the phase in the integrand is $g(\lambda)t$. The asymptotics of the integral $I_2$ is determined by the roots of the saddle point equation $g'(\lambda) = 0$ on steepest descent contours joining $-\pi/2$ and $\pi/2$. 

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Define \( k_1 = \nu_{c_1}/\nu_{c_2} = (1-k')/(1+k') \), \( K_1 = K(k_1) \). Then the saddle point equation can be written as

\[
\text{sn}(4K_1 \lambda/\pi | k_1) = \nu/\nu_{c_1}
\]

The solutions divide the ‘\( m-t \) world plane’ into three different asymptotic regimes R1, R2 and R3.
R1. ‘Time-like regime’ $0 < v < v_{c_1}$: SPE has two real solutions $\lambda_1^- < \lambda_1^+$ in $[-\pi/2, \pi/2]$, both located in $[0, \pi/2]$ such that $\lambda_1^+ = \pi/2 - \lambda_1^-$.
Saddle-point equation

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R2. ‘Precursor regime’ $v_{c1} < v < v_{c2}$: SPE has no real solutions. Let $\lambda_2 = \pi/4 + iy$. Then

$$dn\left(4K_1 y/\pi | k'\right) = v_{c1}/v$$

which has real solutions $\pm y \in [-\gamma/2, \gamma/2]$ as long as $v_{c1} < v < v_{c2}$. 
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R3. ‘Space-like regime’ $v_{c_2} < v$: Let $\lambda_3 = i\gamma/2 + x$. Then

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having two real solutions $x_- < x_+$ in $[-\pi/2, \pi/2]$, located in $[0, \pi/2]$, such that $x_+ = \pi/2 - x_-$
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The above equations can be inverted as incomplete elliptic integrals to give $\lambda^\pm_1$, $x$ and $y$ as functions of $v$. Using these values we obtain the leading large-$t$ asymptotics of $I_2$
In R1

\[ I_2(m, t) \sim \frac{f(\lambda_1^+, \lambda_1^-)}{\pi t} \prod_{\sigma=\pm} \frac{e^{tg(\lambda_1^\sigma)}}{\sqrt{g''(\lambda_1^\sigma)}} \]

\( g(\lambda_1^\pm) \) is purely imaginary and \( I_2 \) shows oscillations and algebraic decay. Note that we have obtained a factor of \( 1/\sqrt{t} \) per integration.
Leading asymptotics

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- For \( \nu \to 0: \lambda_1^- = 0 \) and \( \lambda_1^+ = \pi/2 \), yielding an explicit result for the leading large-\( t \) asymptotics of the dynamical part of the auto-correlation function

\[ l_2(0, t) \sim \frac{e^{i\nu_2 t}}{J\pi t} \frac{8(q^2; q^2)^4(-q^4; q^4)^2(q^8; q^8)^4}{(q^{-2} - q^2)(q^4; q^4)^6(q^4; q^8, q^8)^4} \]
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- In R2 and R3, only one solution of the saddle point equation is relevant. Since \( f(\nu, \nu) = 0 \) this changes the algebraic contribution to the asymptotics:

\[ l_2(m, t) \sim \frac{[\nabla^2 f](\lambda_j, \lambda_j)}{4\pi t^2} \cdot \frac{e^{2tg(\lambda_j)}}{g''(\lambda_j)^2} \]

for \( j = 2, 3 \). \( \lambda_2 = \pi/4 + iy \), where \( y \in [0, \gamma/2] \), and \( \lambda_3 = iy/2 + x \), where \( x \in [\pi/4, \pi/2] \). In R2 \( g(\lambda_2) \) has a negative real part and a non-vanishing imaginary part. In R3 \( g(\lambda_3) \) is real negative.
Specializing to pure space direction we obtain an explicit formula for the next-to-leading term in the asymptotics of the static longitudinal two-point function

$$\langle \sigma_z^1 \sigma_z^m \rangle = \frac{(q^2; q^2)_\infty^4}{(-q^2; q^2)_\infty^4} (-1)^m$$

$$+ A \cdot \frac{k(q^2)^m}{m^2} \left( (-1)^m - \text{th}^2(\eta/2) \frac{(q; q^2)_\infty^4}{(-q; q^2)_\infty^4} \right) (1 + O(m^{-1}))$$

where

$$k(q^2) = \frac{\vartheta_2^2(0|q^2)}{\vartheta_3^2(0|q^2)}, \quad A = \frac{1}{\pi \text{sh}^2(\eta/2)} \frac{(-q; q^2)_\infty^4}{(q^2; q^2)_\infty^2} \frac{(q^4; q^4, q^4)_\infty^8}{(q^2; q^4, q^4)_\infty^8}$$

generalizing the result of the correlation length of [Johnson, Krinsky and McCoy 73]
Amazingly, it is possible to obtain the saddle-point values \( g(\lambda), g''(\lambda), \lambda = \lambda_1^\pm, \lambda_2, \lambda_3 \) and \([\nabla^2 f](\lambda_j, \lambda_j), j = 2, 3\) as explicit algebraic functions of \( v\). We can rewrite the saddle-point equation as

\[ v\varepsilon(p) - v_{c_1} v_{c_2} \cos(p) \sin(p) = 0 \quad (\ast) \]

This can be solved for \( z = \cos^2(p) \) at the saddle points. Introducing the rescaled velocity parameter \( r = v / \sqrt{v_{c_1} v_{c_2}} \) we obtain two solutions

\[
2z_\pm = 1 + r^2 \pm \begin{cases} 
\sqrt{(r_1^2 - r^2)(r_2^2 - r^2)} & \text{in } R1, R3, \\
\pm i\sqrt{(r^2 - r_1^2)(r_2^2 - r^2)} & \text{in } R2
\end{cases}
\]

Here \( r_1^2 = v_{c_1} / v_{c_2} \) and \( r_2^2 = v_{c_2} / v_{c_1} \). Hence, \( 0 < r < r_1 \) in R1, \( r_1 < r < r_2 \) in R2 and \( r_2 < r \) in R3. From here we obtain \( \cos(p) \) and \( \sin(p) \) and therefore \( e^{ip} \) at the saddle points. \( \varepsilon(p) \) then follows from (\ast)
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abla^2 f(\lambda, \lambda_j)$, $j = 2, 3$ as explicit algebraic functions of $v$. We can rewrite the saddle-point equation as

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Using the explicit form of the asymptotics we can easily plot the full space and time dependent two-point functions for large times and far separated points.
Real part of $I_2(m, t)$ as a function of $m$ for fixed $t = 4$ and $\Delta = 2.375$. Data points calculated for $m \in \mathbb{Z}$ and connected by means of splines. Vertical lines separate different asymptotic regimes, first line $m = v_{c_1} t$, second line $m = v_{c_2} t$. 
The wave excited at \( m = 1 \) and \( t = 0 \) contains all frequency components and hence spreads out with the maximal possible group velocity \( v_{c_1} \). The dispersion relation implies

\[
\max_{p \in [-\pi/2, \pi/2]} |\epsilon'(p)| = v_{c_1}
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Such a maximal group velocity in spin systems is called a Lieb-Robinson bound.
Quantum signal at a fixed instance of time

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The irregular appearance of the wave train is due to the interference of commensurate and incommensurate components. In fact,

$$I_2(m, t) = I_2^{(0)}(m, t)(-1)^m + I_2^{(1)}(m, t)$$

![Graphs showing the behavior of $I_2^{(0)}(m, t)$ and $I_2^{(1)}(m, t)$]
Quantum signal at a fixed site

$I_2(m, t)$ as a function of $t$ for fixed $m = 45$ and $\Delta = 2.375$ (real part blue, imaginary part red). Vertical lines separate different asymptotic regimes, first line $t = m/v_{c_2}$, second line $t = m/v_{c_1}$. Dots denote asymptotics values exactly at the boundaries between the different regimes.
Summary and implications

We have analysed the longitudinal two-point functions of the XXZ chain for long times $t$ and large distances $m$ for fixed ratio $v = m/t$.

- Asymptotics of correlation functions for massive XXZ
We have analysed the longitudinal two-point functions of the XXZ chain for long times $t$ and large distances $m$ for fixed ratio $\nu = m/t$.

We identified three different asymptotic regimes, separated by two Stokes lines corresponding to two critical velocities $\nu_{c_1}$ and $\nu_{c_2}$. As the analytic behaviour of the correlation functions changes across the Stokes lines, one sector cannot be obtained from the other by analytic continuation, and different effective field theories are needed.

The correlation functions describe the time evolution of ‘a signal’ whose front is traveling with maximal group velocity $\nu_{c_1}$ (equal to the ‘Lieb-Robinson bound’) and is preceded by a precursor decaying in forward direction and spreading out with velocity $\nu_{c_2}$. The signal is a superposition of commensurate and incommensurate components.

We have obtained particularly simple expressions for pure time and space directions. In particular the auto-correlation function decays algebraically like $A(q) e^{i\nu_{c_2}t/t}$, which can be interpreted as spin diffusion of two spinons (factor of $1/\sqrt{t}$ per spinon).
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