Asymptotics of correlation functions of the Heisenberg-Ising chain in the easy-axis regime

Frank Göhmann

Bergische Universität Wuppertal
Fachgruppe Physik

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Outline

- More than a decade of steady progress in our understanding of correlation functions of integrable models, chief example XXZ chain

$$H = J \sum_{j=1}^{L} \left(\sigma_{j-1}^x \sigma_j^x + \sigma_{j-1}^y \sigma_j^y + \Delta \left(\sigma_{j-1}^z \sigma_j^z - 1 \right) \right) - \frac{h}{2} \sum_{j=1}^{L} \sigma_j^z$$

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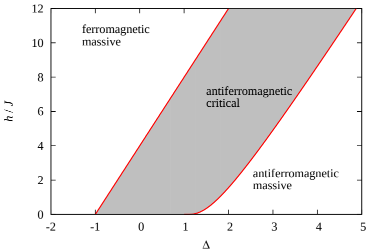
- 1d variant of the fundamental model of antiferromagnetism in solids
- Can also be simulated by cold atoms in optical traps
- in which time- and space resolved two-point functions can be measured
- We have calculated their long-time large-distance asymptotics in the massive antiferromagnetic regime



Joint project with Maxime Dugave, Karol Kozlowski and Junji Suzuki

Form factor series

- The longitudinal ground-state two-point functions of the Heisenberg-Ising chain for $\Delta > 1$ have the form-factor expansion (see e.g. DGKS 15)

$$\langle \sigma_1^z \sigma_{m+1}^z(t) \rangle$$
$$= \frac{(q^2;q^2)^4}{(-q^2;q^2)^4}(-1)^m + \sum_{\substack{\iota=0,1\\ n\in 2\mathbb{N}}} \frac{(-1)^{\iota m}}{n!} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\mathrm{d}^n \nu}{(2\pi)^n} \mathcal{F}_{\iota,n}^{(z)} \prod_{j=1}^n \mathrm{e}^{\mathrm{i}[p(\nu_j)m - \varepsilon(\nu_j)t]}$$

Here we used the standard notation for q-multi factorials

$$(a;q_1,\dots,q_p) = \prod_{n_1,\dots,n_p=0}^{\infty} (1 - aq_1^{n_1}\dots q_p^{n_p})$$

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- Sum over pairs of spinons with single particle momentum and energy p and ε parameterized in terms of a rapidity variable ν,

$$p(\nu) = \frac{\pi}{2} + \nu - \mathrm{i}\ln\left(\frac{\vartheta_4(\nu + \mathrm{i}\gamma/2, q^2)}{\vartheta_4(\nu - \mathrm{i}\gamma/2, q^2)}\right), \quad \varepsilon(\nu) = -\frac{4JK\,\mathrm{sh}(\gamma)}{\pi}\,\mathrm{dn}\left(\frac{2K\nu}{\pi}\middle| k\right)$$

ϑ_4 is a Jacobi theta function and dn a Jacobi-elliptic function, $k = k(q)$ is the elliptic modulus, and $K = K(k)$ the complete elliptic integral of the first kind

Form factor series

- Spinon energy and momentum are related by

$$p'(\nu) = -\varepsilon(\nu)/2J\,\mathrm{sh}(\gamma)\,, \quad \varepsilon(p) = -\sqrt{v_{c_1} v_{c_2}} \cdot \sqrt{1/k^2 - \cos^2(p)}$$

The dispersion relation reveals the massive nature of the excitations. Here we have introduced two combinations of parameters

$$v_{c_1} = \frac{4JKk^2\,\mathrm{sh}(\gamma)}{\pi(1+k')}\,, \quad v_{c_2} = \frac{4JKk^2\,\mathrm{sh}(\gamma)}{\pi(1-k')}$$

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- Form factor densities $\mathcal{F}^{(z)}_{\iota,n}$ depend on an even number n of rapidities $\nu_1, \dots, \nu_n$. For general n they were expressed by multiple integrals by JIMBO and MIWA 95 and by Fredholm determinants by DGKS 15. A more explicit expression is only known for the two-spinon case $n = 2$ and follows from LASHKEVICH'S 2002 result for XYZ

Two-spinon contribution

- LASHKEVICH's formula allows us to write the two-spinon contribution to the longitudinal correlation function as

$$\left\langle \sigma_1^z \sigma_{m+1}^z(t) \right\rangle_2 = \frac{(q^2;q^2)^4}{(-q^2;q^2)^4}(-1)^m + \frac{1}{2} I_2(m,t)$$

where

$$I_2(m,t) = \left[\prod_{j=1}^{2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\mathrm{d}\nu_j}{2\pi} \mathrm{e}^{\mathrm{i}[p(\nu_j)m - \varepsilon(\nu_j)t]} \right] f(\nu_1, \nu_2)$$

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The function f can be expressed as

$$f(\nu_1,\nu_2) = A(\nu_1,\nu_2)(-1)^m + A(\nu_1 + \pi, \nu_2)$$

where

$$A(\nu_1,\nu_2) = \frac{32\cos^2\big((p(\nu_1)+p(\nu_2))/2\big)\sin^2(\nu_{12})}{\sin((\nu_{12}+i\gamma)/2)\sin((\nu_{12}-i\gamma)/2)}$$

$$\times q(q^2;q^2)^2 \vartheta_4^2(\nu_{12}/2,q) \prod_{\sigma=\pm} \frac{(q^4;q^4,q^4)^2}{(q^2;q^4,q^4)^2} \frac{(q^4 e^{2i\sigma\nu_{12}};q^4,q^4)^2}{(q^2 e^{2i\sigma\nu_{12}};q^4,q^4)^2} \frac{(q^2 e^{2i\sigma\nu_{12}};q^4)}{(q^4 e^{2i\sigma\nu_{12}};q^4)}$$

and $\nu_{12} = \nu_1 - \nu_2$

Saddle-point equation

- We have calculated the asymptotics $m, t \to \infty$ for fixed ratio $v = m/t \geq 0$
- In this limit the integral I_2 can be estimated by the method of steepest descent
- The calculation shows that the higher-spinon contributions contribute only higher corrections to the steepest descent result

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- Let us define the function

$$g(\lambda) = \mathrm{i}[p(\lambda)v - \varepsilon(\lambda)]$$

Then the phase in the integrand is $g(\lambda)t$. The asymptotics of the integral I_2 is determined by the roots of the saddle point equation $g'(\lambda) = 0$ on steepest descent contours joining $-\pi/2$ and $\pi/2$

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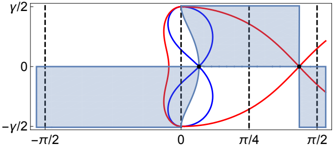
- Define $k_1 = v_{c_1}/v_{c_2} = (1-k')/(1+k')$, $K_1 = K(k_1)$. Then the saddle point equation can be written as

$$\mathrm{sn}\big(4K_1\lambda/\pi \big| k_1\big) = v/v_{c_1}$$

  The solutions divide the '*m*-*t* world plane' into three different asymptotic regimes R1, R2 and R3 .

Saddle-point equation

R1. 'Time-like regime' $0 < v < v_{c_1}$: SPE has two real solutions $\lambda_1^- < \lambda_1^+$ in $[-\pi/2, \pi/2]$, both located in $[0, \pi/2]$ such that $\lambda_1^+ = \pi/2 - \lambda_1^-$



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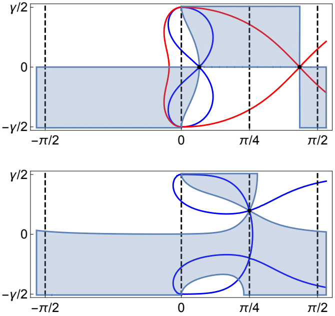
R2. 'Precursor regime' $v_{c_1} < v < v_{c_2}$: SPE has no real solutions. Let $\lambda_2 = \pi/4 + \mathrm{i}y$. Then

$$\mathrm{dn}\left(4K_1 y/\pi \middle| k_1'\right) = v_{c_1}/v$$

which has real solutions $\pm y \in [-\gamma/2, \gamma/2]$ as long as $v_{c_1} < v < v_{c_2}$



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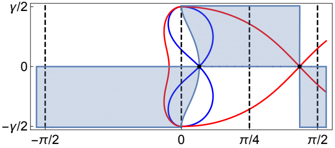

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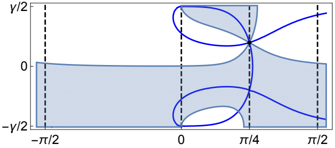

R3. 'Space-like regime' $v_{c_2} < v$: Let $\lambda_3 = \mathrm{i}\gamma/2 + x$. Then

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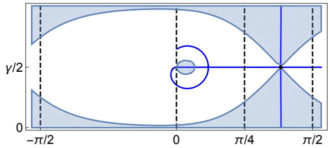

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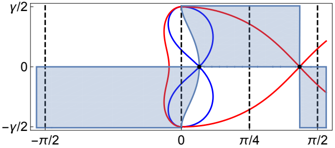

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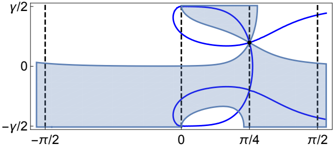

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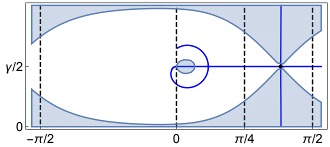

The above equations can be inverted as incomplete elliptic integrals to give $\lambda_1^\pm$, x and y as functions of v. Using these values we obtain the leading large-t asymptotics of I_2

Leading asymptotics

- In R1

$$I_2(m,t) \sim \frac{f(\lambda_1^+, \lambda_1^-)}{\pi t} \prod_{\sigma=\pm} \frac{e^{tg(\lambda_1^\sigma)}}{\sqrt{g''(\lambda_1^\sigma)}}$$

$g(\lambda_1^\pm)$ is purely imaginary and I_2 shows oscillations and algebraic decay. Note that we have obtained a factor of $1/\sqrt{t}$ per integration

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- For $v \to 0$: $\lambda_1^- = 0$ and $\lambda_1^+ = \pi/2$, yielding an explicit result for the leading large-t asymptotics of the dynamical part of the auto-correlation function

$$I_2(0,t) \sim \frac{e^{i v_{c_2} t}}{J\pi t} \frac{8(q^2;q^2)^4(-q^4;q^4)^2(q^8;q^8,q^8)^4}{(q^{-2}-q^2)(q^4;q^4)^6(q^4;q^8,q^8)^4}$$

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- In R2 and R3, only one solution of the saddle point equation is relevant. Since $f(\nu,\nu)=0$ this changes the algebraic contribution to the asymptotics:

$$I_2(m,t) \sim \frac{[\nabla^2 f](\lambda_j,\lambda_j)}{4\pi t^2} \cdot \frac{e^{2tg(\lambda_j)}}{g''(\lambda_j)^2}$$

for $j=2,3$. $\lambda_2 = \pi/4 + iy$, where $y \in [0,\gamma/2]$, and $\lambda_3 = i\gamma/2 + x$, where $x \in [\pi/4,\pi/2]$. In R2 $g(\lambda_2)$ has a negative real part and a non-vanishing imaginary part. In R3 $g(\lambda_3)$ is real negative

Large-distance asymptotics in static case

- Specialyzing to pure space direction we obtain an explicit formula for the next-to-leading term in the asymptotics of the static longitudinal two-point function

$$\langle \sigma_1^z \sigma_{m+1}^z \rangle = \frac{(q^2;q^2)_\infty^4}{(-q^2;q^2)_\infty^4} (-1)^m$$
$$+ A \cdot \frac{k(q^2)^m}{m^2} \left((-1)^m - \mathrm{th}^2(\eta/2) \frac{(q;q^2)_\infty^4}{(-q;q^2)_\infty^4} \right) (1 + \mathcal{O}(m^{-1}))$$

where

$$k(q^2) = \frac{\vartheta_2^2(0|q^2)}{\vartheta_3^2(0|q^2)}, \quad A = \frac{1}{\pi \, \mathrm{sh}^2(\eta/2)} \frac{(-q;q^2)_\infty^4}{(q^2;q^2)_\infty^2} \frac{(q^4;q^4,q^4)_\infty^8}{(q^2;q^4,q^4)_\infty^8}$$

generalizing the result of the correlation length of [Johnson, Krinsky and McCoy 73]

Explicit evaluation of functions at the saddle points

- Amazingly, it is possible to obtain the saddle-point values $g(\lambda)$, $g''(\lambda)$, $\lambda = \lambda_1^\pm, \lambda_2, \lambda_3$ and $[\nabla^2 f](\lambda_j, \lambda_j)$, $j = 2, 3$ as explicit algebraic functions of v. We can rewrite the saddle-point equation as

$$v\varepsilon(p) - v_{c_1} v_{c_2} \cos(p)\sin(p) = 0 \quad (*)$$

This can be solved for $z = \cos^2(p)$ at the saddle points. Introducing the rescaled velocity parameter $r = v/\sqrt{v_{c_1} v_{c_2}}$ we obtain two solutions

$$2z_\pm = 1 + r^2 \pm \begin{cases} \sqrt{(r_1^2 - r^2)(r_2^2 - r^2)} & \text{in R1, R3,} \\ \mathrm{i}\sqrt{(r^2 - r_1^2)(r_2^2 - r^2)} & \text{in R2} \end{cases}$$

Here $r_1^2 = v_{c_1}/v_{c_2}$ and $r_2^2 = v_{c_2}/v_{c_1}$. Hence, $0 < r < r_1$ in R1, $r_1 < r < r_2$ in R2 and $r_2 < r$ in R3. From here we obtain $\cos(p)$ and $\sin(p)$ and therefore $\mathrm{e}^{\mathrm{i}p}$ at the saddle points. $\varepsilon(p)$ then follows from $(*)$

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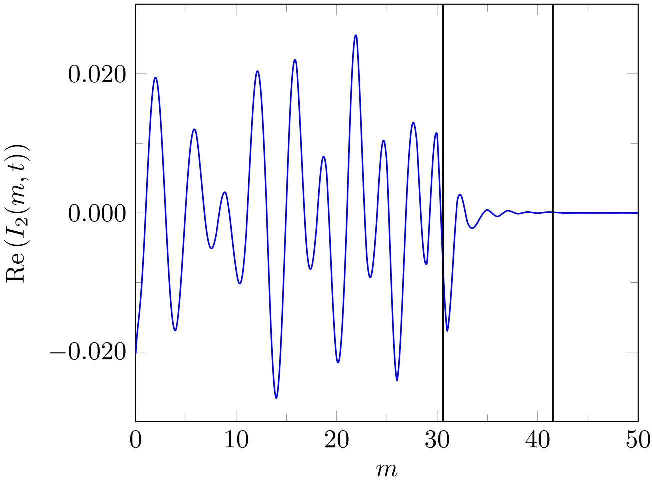
- Using the explicit form of the asymptotics we can easily plot the full space and time dependent two-point functions for large times and far separated points

Quantum signal at a fixed instance of time



Real part of $I_2(m,t)$ as a function of m for fixed $t = 4$ and $\Delta = 2.375$. Data points calculated for $m \in \mathbb{Z}$ and connected by means of splines. Vertical lines separate different asymptotic regimes, first line $m = v_{c_1} t$, second line $m = v_{c_2} t$.

Quantum signal at a fixed instance of time

- The wave excited at $m = 1$ and $t = 0$ contains all frequency components and hence spreads out with the maximal possible group velocity v_{c_1}. The dispersion relation implies

$$\max_{p \in [-\pi/2, \pi/2]} |\varepsilon'(p)| = v_{c_1}$$

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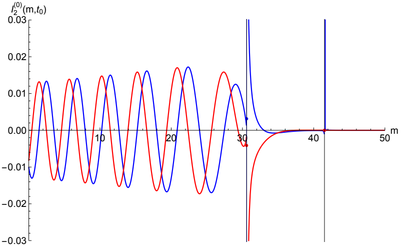
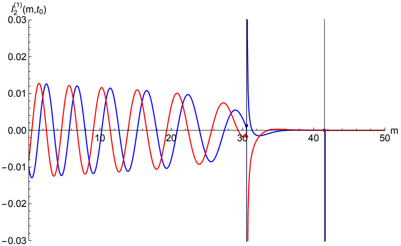
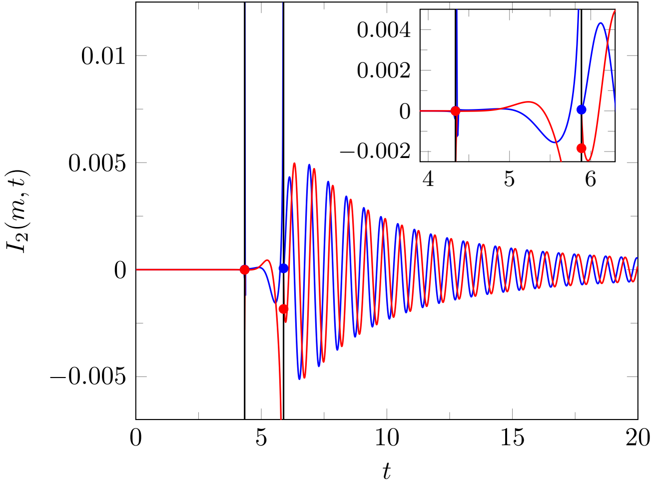
- The irregular appearance of the wave train is due to the interference of commensurate and incommensurate components. In fact,

$$I_2(m,t) = I_2^{(0)}(m,t)(-1)^m + I_2^{(1)}(m,t)$$





Quantum signal at a fixed site



$I_2(m,t)$ as a function of t for fixed $m = 45$ and $\Delta = 2.375$ (real part blue, imaginary part red). Vertical lines separate different asymptotic regimes, first line $t = m/v_{c_2}$, second line $t = m/v_{c_1}$. Dots denote asymptotics values exactly at the boundaries between the different regimes

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- As the analytic behaviour of the correlation functions changes across the Stokes lines, one sector cannot be obtained from the other by analytic continuation, and different effective field theories are needed
- The correlation functions describe the time evolution of ‘a signal’ whose front is traveling with maximal group velocity v_{c_1} (equal to the ‘Lieb-Robinson bound’) and is preceded by a precursor decaying in forward direction and spreading out with velocity v_{c_2}

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- We have obtained particularly simple expressions for pure time and space directions. In particular the auto-correlation function decays algebraically like $A(q)\mathrm{e}^{\mathrm{i}v_{c_2}t}/t$, which can be interpreted as spin diffusion of two spinons (factor of $1/\sqrt{t}$ per spinon)