Quantum quenches in 2D with chain array matrix product states

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arXiv:1504.00237

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Outline

- MPS for many body systems
  - Entanglement and role of dimension
- MPS for arrays of chains
  - DMRG and 2D quantum Ising phase transition
- Time evolution for arrays of coupled chains
  - TEBD and iTEBD
  - Quenches of 2D quantum Ising

Many-Body States

$$|\psi\rangle = \sum_{\{\sigma\}} c_{\sigma_1 \cdots \sigma_N} |\sigma_1 \cdots \sigma_N\rangle$$

General many-body state requires exponentially many numbers

$$d_\sigma^N = \dim(\sigma)^N$$

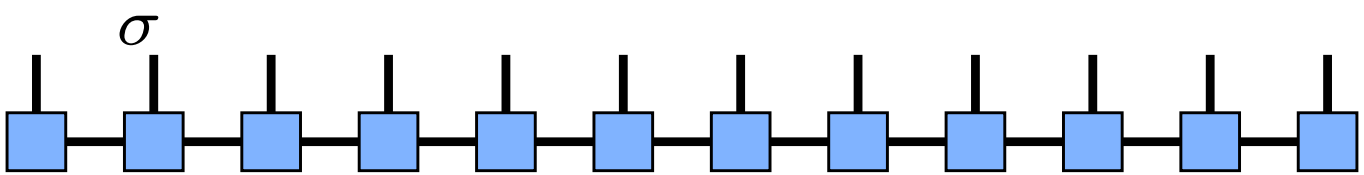
But for ‘typical’ states many are zero, e.g. product state

$$|\psi\rangle = |\sigma_1 \cdots \sigma_N\rangle$$

Look for efficient representations between these extremes

Matrix Product States

$$|\psi\rangle = \sum_{\sigma} A^{\sigma_1}_{1a_1} A^{\sigma_2}_{a_1 a_2} \cdots A^{\sigma_N}_{a_{N-1},1} |\sigma_1 \cdots \sigma_N\rangle$$



- Matrix Product States provide a particularly efficient representation in 1D (very successful algorithms for eigenstates, time evolution…)
- Form a variational basis for states with restricted entanglement, related to 'bond' (matrix) dimension

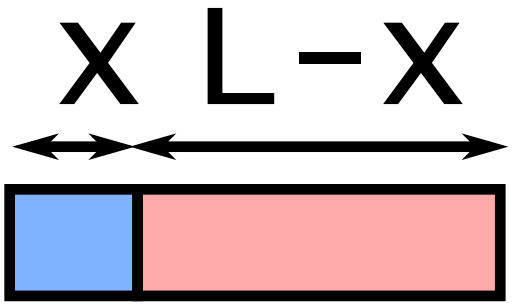
$$(d_\sigma - 1) N D^2 \quad \text{vs.} \quad d_\sigma^N$$

- Max entanglement at a partition $S_E \leq \log D$

Why do MPS algorithms work so well in 1D?

Success of MPS methods in 1D relies on, at worst, log growth of entanglement (bond dimension doesn't grow too fast)

Bipartite 1D system



$$S_A = \frac{c}{6} \log L_{eff} + \cdots$$

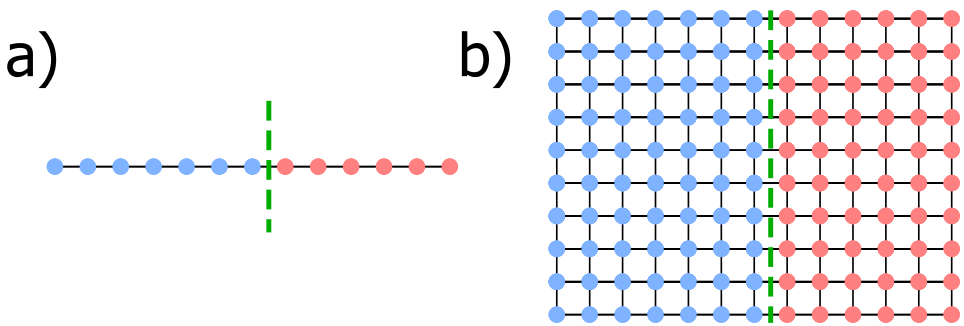
$$L_{eff} = \begin{cases} \frac{L}{\pi} \sin \frac{\pi x}{L} & \text{critical} \\ \xi / a & \text{gapped} \end{cases}$$

Holzhey, Larsen & Wilczek 1994, Calabrese & Cardy 2004

How does entanglement entropy behave above 1D?

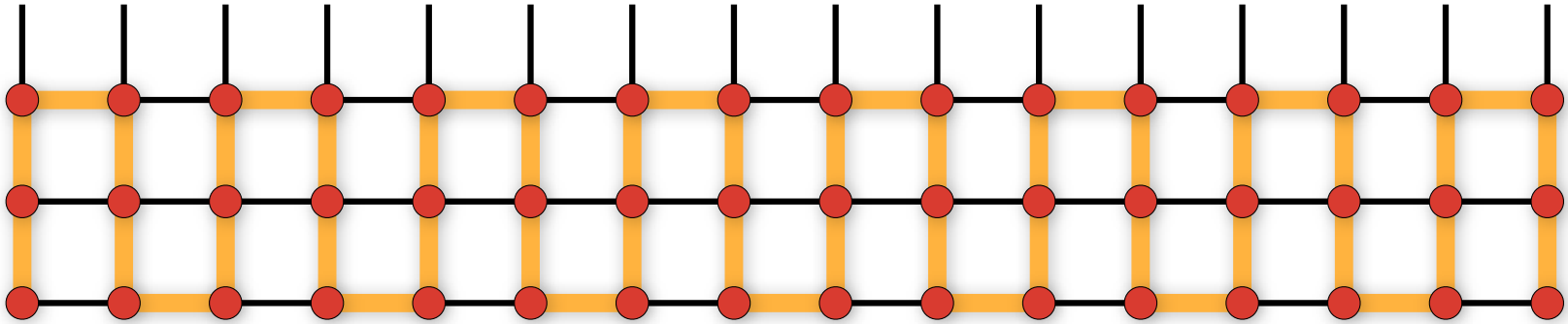
Generally an ‘area law’ is expected

$$S_E \sim \mathcal{A}/a^{D-1}$$



MPS in 2D

Growth of S_E with area bond dimension grows with system size: general problem for tensor methods



MPS manifestation: mapping the 2D system to 1D generates long ranged interactions

Stoudenmire and White, Ann. Rev. Condens. Matter Phys. (2012)

Long ranged interactions require more sophisticated time evolution algorithms

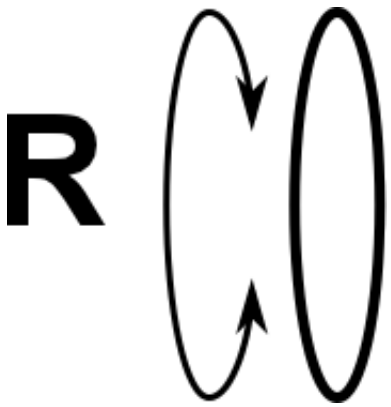
Zalatel et al. PRB 91 165112 (2015) (MPO method)

Haegeman et al. arXiv:1408.5056 (TDVP integration)

MPS for arrays of chains

Bending the area law

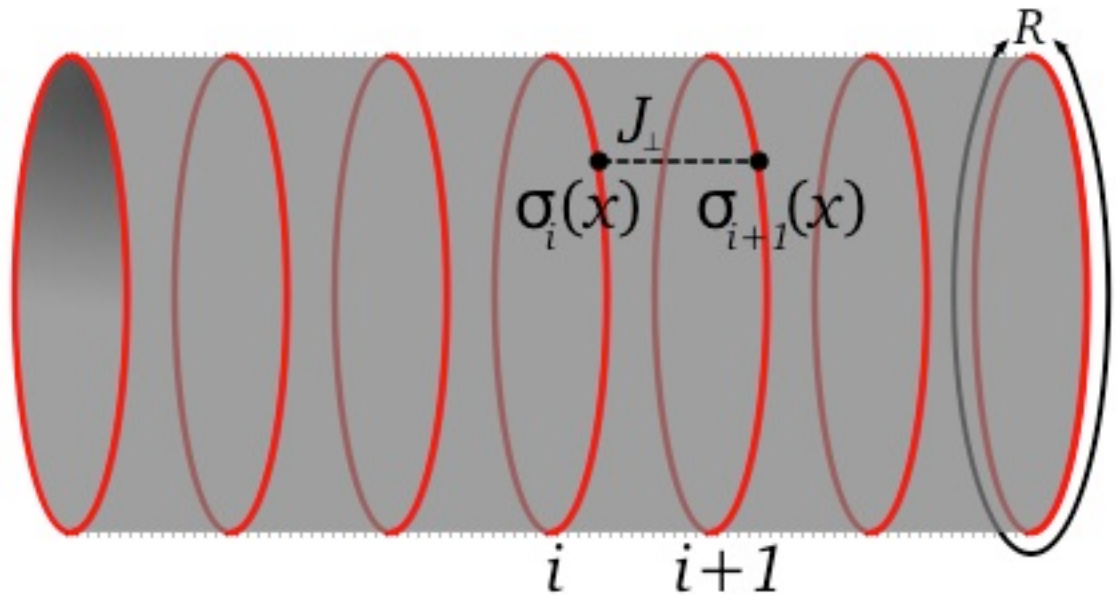
Continuum chain, length R, periodic b.c.s, with Hamiltonian that is integrable



- Continuum limit: finite size corrections exponential, keep R small $\sim e^{-\Delta R}$
- Integrable: exact spectrum and matrix elements known
- Use chains as 'sites' in MPS?

$$H = \sum_i \left\{ H_{1D,i} + J_\perp \int_0^R dx \hat{O}_i(x) \hat{O}_{i+1}(x) \right\}$$

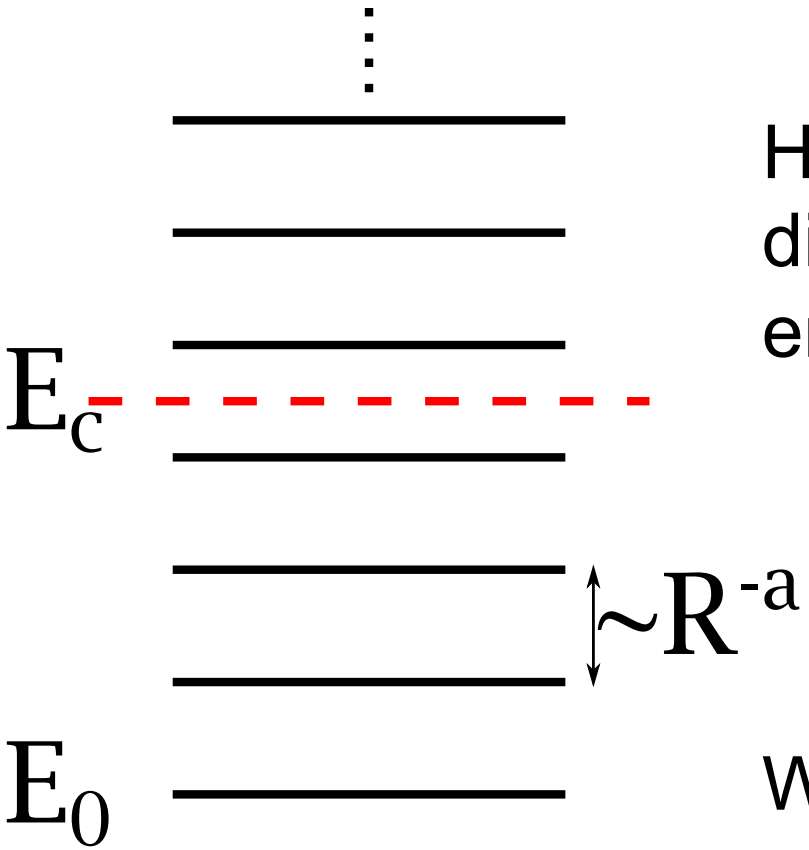
Array of coupled chains



$$H = \sum_i \left\{ H_{1D,i} + J_\perp \int_0^R dx \hat{O}_i(x) \hat{O}_{i+1}(x) \right\}$$

Chain energy cutoff

The spectrum of each chain is infinite…



However for finite R it is discrete, so we can order by energy and truncate

When can we hope to get away with this?

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Truncated Conformal Space Approach

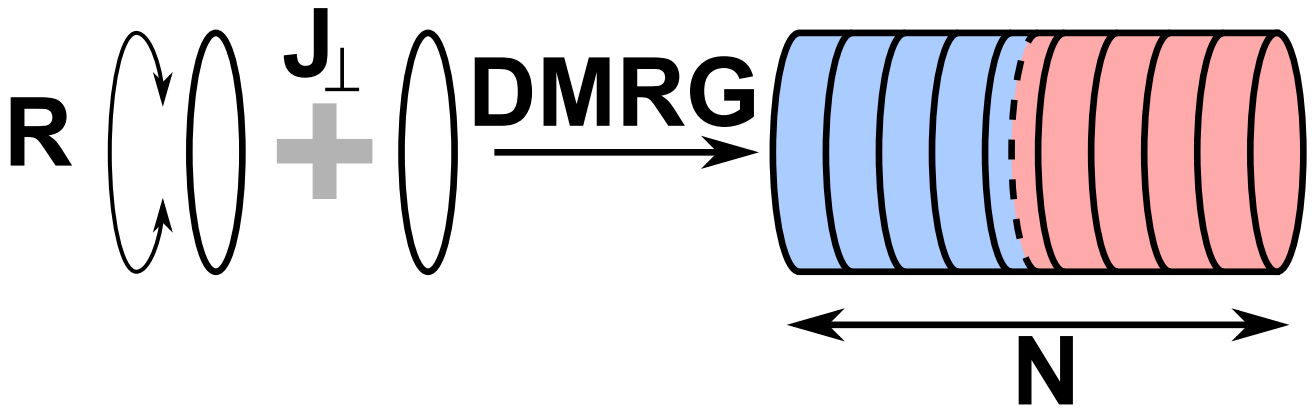
V. P. Yurov and Al. B. Zamolodchikov Int. J. Mod. Phys. A 6, 4557 (1991)

$$H = H_{CFT} + \lambda_0 \int d^2x \, \phi_0(x)$$

Relevant perturbing operator, most important effect is mixing of low energy states

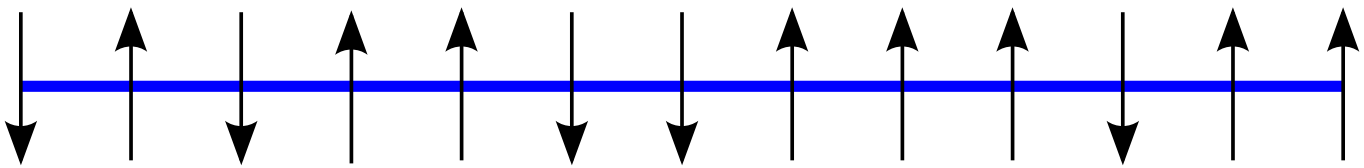
Critical Ising chain in magnetic field well described (error ~ 1%) by keeping only 39 states

DMRG with coupled chain system



- Control area law with small R
- Consider relevant interchain coupling
- Truncate in energy after each DMRG step to keep consistent cutoff
- Bond dimension ~30 in gapped phase to 100's nearer criticality for 10^{-5} truncation error

Example: Quantum Ising chain



- Continuum limit of lattice Ising chain in transverse field
- Fermionic field theory: negative mass, Δ, (disordered) or positive Δ (ordered) chains

$$H^{1D}_{\text{lattice}} = -J \sum_m \left(\sigma^z_m \sigma^z_{m+1} + (1+g)\sigma^x_m \right)$$

$$H^{1D} = \int dx \left[\frac{v}{2} \left(\bar{\Psi} \frac{\partial \bar{\Psi}}{\partial x} - \Psi \frac{\partial \Psi}{\partial x} \right) + \Delta \bar{\Psi} \Psi \right]$$

$$\Delta = -2gJ, v = 2Ja$$

Quantum Ising chain spectrum

- Chain eigenstates organised into two 'sectors' Ramond and Neveu-Schwarz with integer and half-integer momenta

$$\text{NS: } |p_1, p_2, \cdots, p_{N_f}\rangle = \alpha^\dagger_{p_1}\alpha^\dagger_{p_2}\cdots\alpha^\dagger_{p_{N_f}}|0\rangle_{NS}, \quad p_i \in (\mathbb{Z}+1/2)\frac{2\pi}{R}$$

$$\text{R: } |k_1, k_2, \cdots, k_{N_f}\rangle = \alpha^\dagger_{k_1}\alpha^\dagger_{k_2}\cdots\alpha^\dagger_{k_{N_f}}|0\rangle_{R}, \quad k_i \in \mathbb{Z}\frac{2\pi}{R}$$

- Permissible states depend on sign of mass

$\Delta > 0$, NS states when N_f even, R states when N_f even.

$\Delta < 0$, NS states when N_f even, R states when N_f odd.

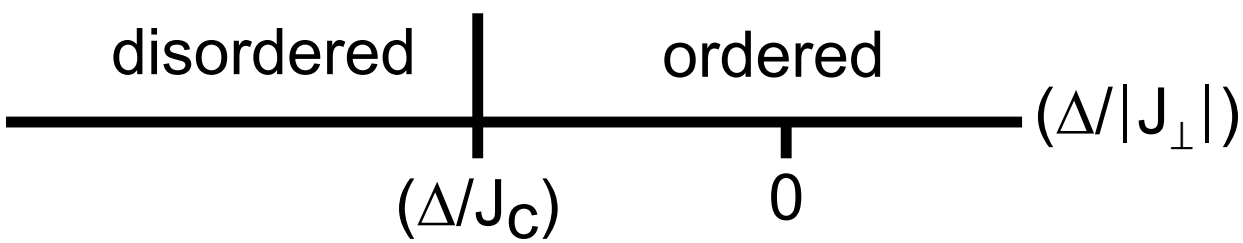
- Fermion energies

$$E_{k_i} = \sqrt{\Delta^2 + k_i^2}$$

2D Quantum Ising model (2+1)

$$H = \sum_i \left\{ H_{1D,i} + J_\perp \int_0^R \mathrm{d}x \; \sigma_i(x)\sigma_{i+1}(x) \right\}$$

- Should be an order/disorder transition (disordered chains will order with enough coupling)



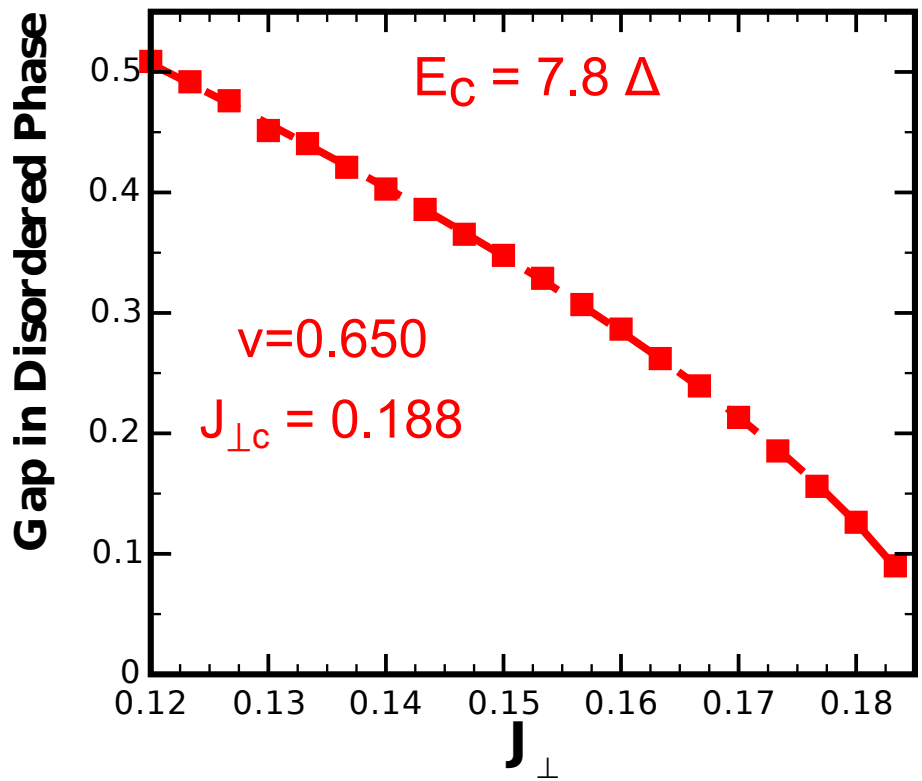
- Start with disordered (negative mass) chains and sweep chain coupling, study the energy gap
- Test that the correct many-body behaviour is captured

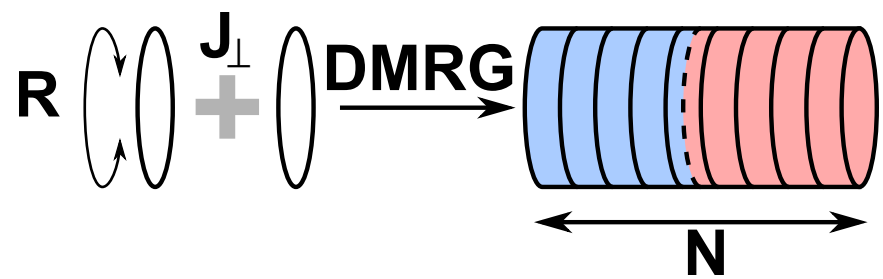
2D Quantum Ising model (2+1)

$$\Delta_{2DQI} \sim |J_c - J_\perp|^\nu$$

$$N = 60,\ R\Delta = 10,\ E_c = 7.8\Delta$$

R. M. Konik and Y. Adamov, PRL 102, 097203 (2009)





In 3D classical Ising universality class,

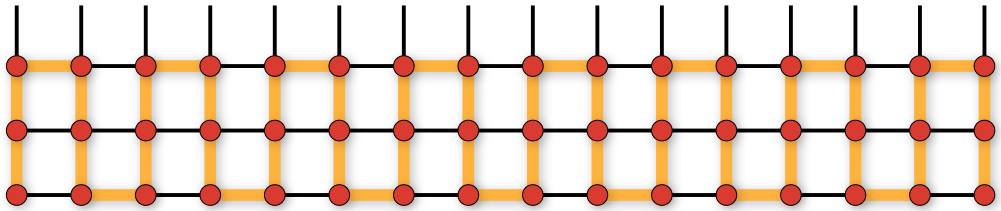
$$\nu = 0.630$$

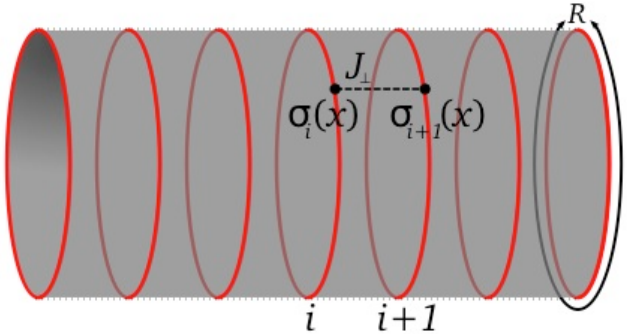
Time evolution and quenches

Time evolution for coupled chains

- Isotropic lattice case requires time evolution with a long ranged Hamiltonian

  Zalatel et al. PRB 91 165112 (2015) (MPO method)
  Haegeman et al. arXiv:1408.5056 (TDVP integration)

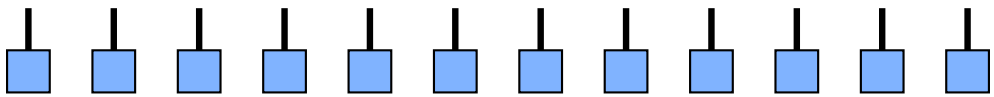




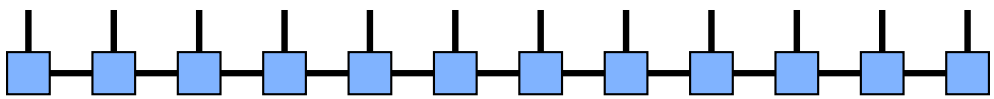
- For an array of chains we can adapt existing 1D MPS algorithms for time evolution (TEBD and iTEBD), particularly easy to work with infinite cylinders

Quantum quenches for coupled quantum Ising chains

Start with uncoupled chains in their ground states (pure state, zero entanglement, integrable)



Quench by turning on finite chain coupling

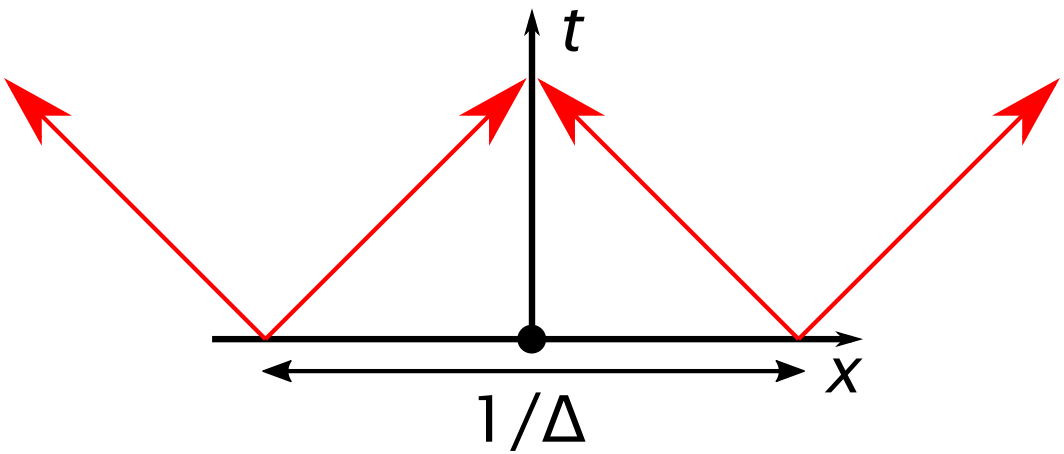


Consider fermion mode occupation on chain i,

$$n_{i,k}$$

Zero in prequench state (chain vacuum states) but not conserved by interchain spin-spin coupling

Quench produces quasiparticles



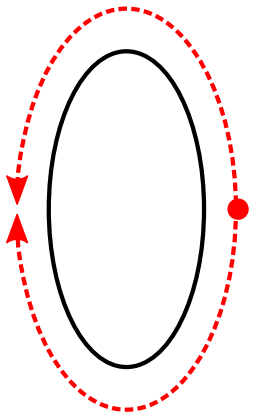
Time scales

Quench produces quasiparticles

- Short time: quasiparticles entangled on scale of initial correlation length

$$t_\Delta \sim \frac{1}{\Delta}$$

- Longer time: quasiparticles created at same point circuit chain and meet again, because of finite length R



$$t_R \sim R/2$$

Very weak quench

For very weak coupling, compare with perturbation theory

Use unitary perturbation theory to avoid secular terms

Moeckel and Kehrein, PRL 100, 175702 (2008)

Kollar, Wolf and Eckstein, PRB 84, 054304 (2011)

Very weak quench

For very weak coupling, compare with perturbation theory

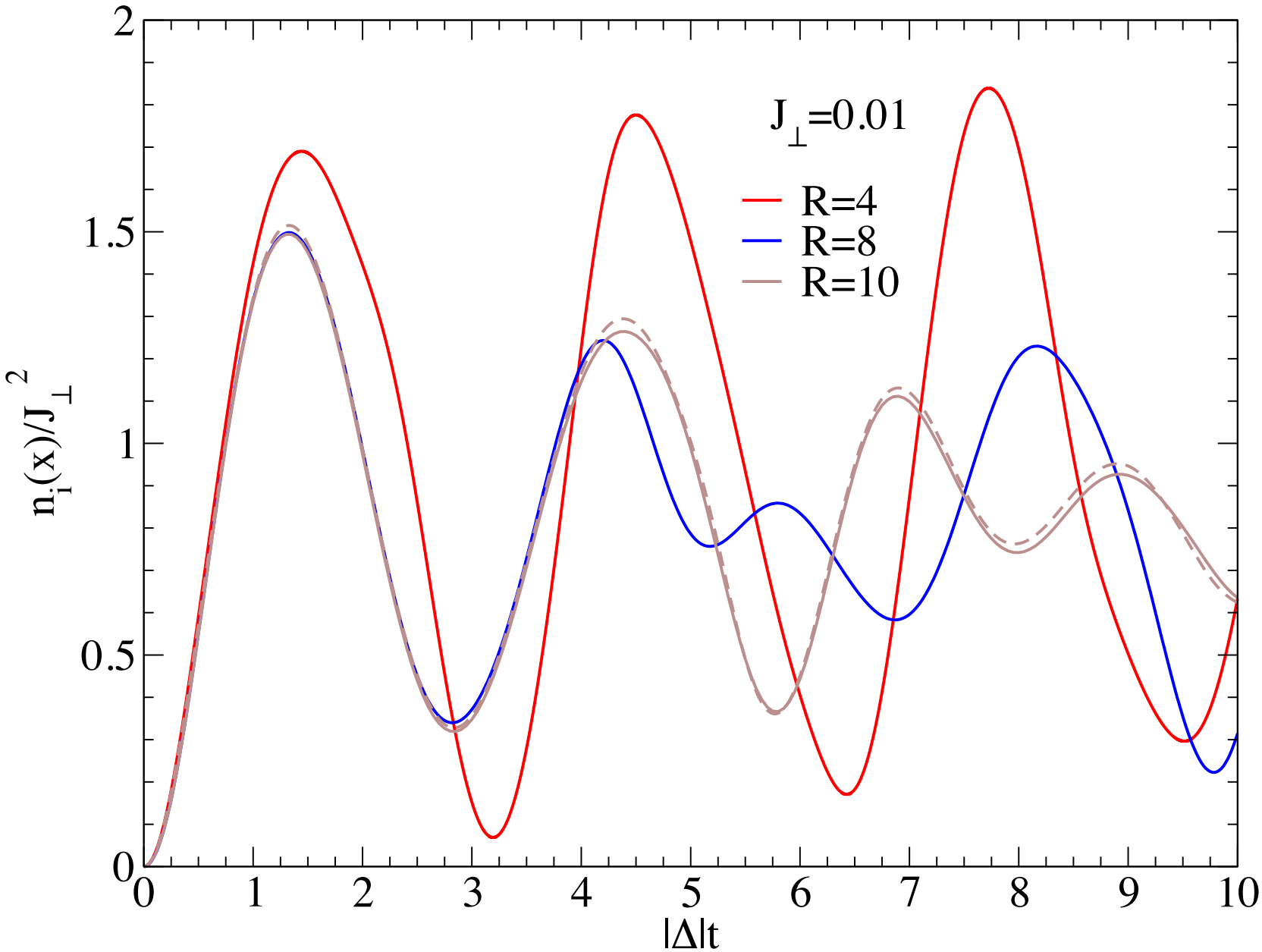
Expectation of operator at time *t* on chain *i*

$$\langle \mathcal{O}_i \rangle_t = \langle \mathcal{O}_i \rangle_0 + 8 J_\perp^2 R^2 \sum_{n_i, n_{i+1}} \Big(\frac{\sin^2(t[E_i + E_{i+1}]/2)}{(E_i + E_{i+1})^2} \times \delta_{k_i, -k_{i+1}} |\langle 0|\sigma_i|n_i\rangle \langle 0|\sigma_{i+1}|n_{i+1}\rangle|^2 \langle n_i|\mathcal{O}_i|n_i\rangle \Big)$$

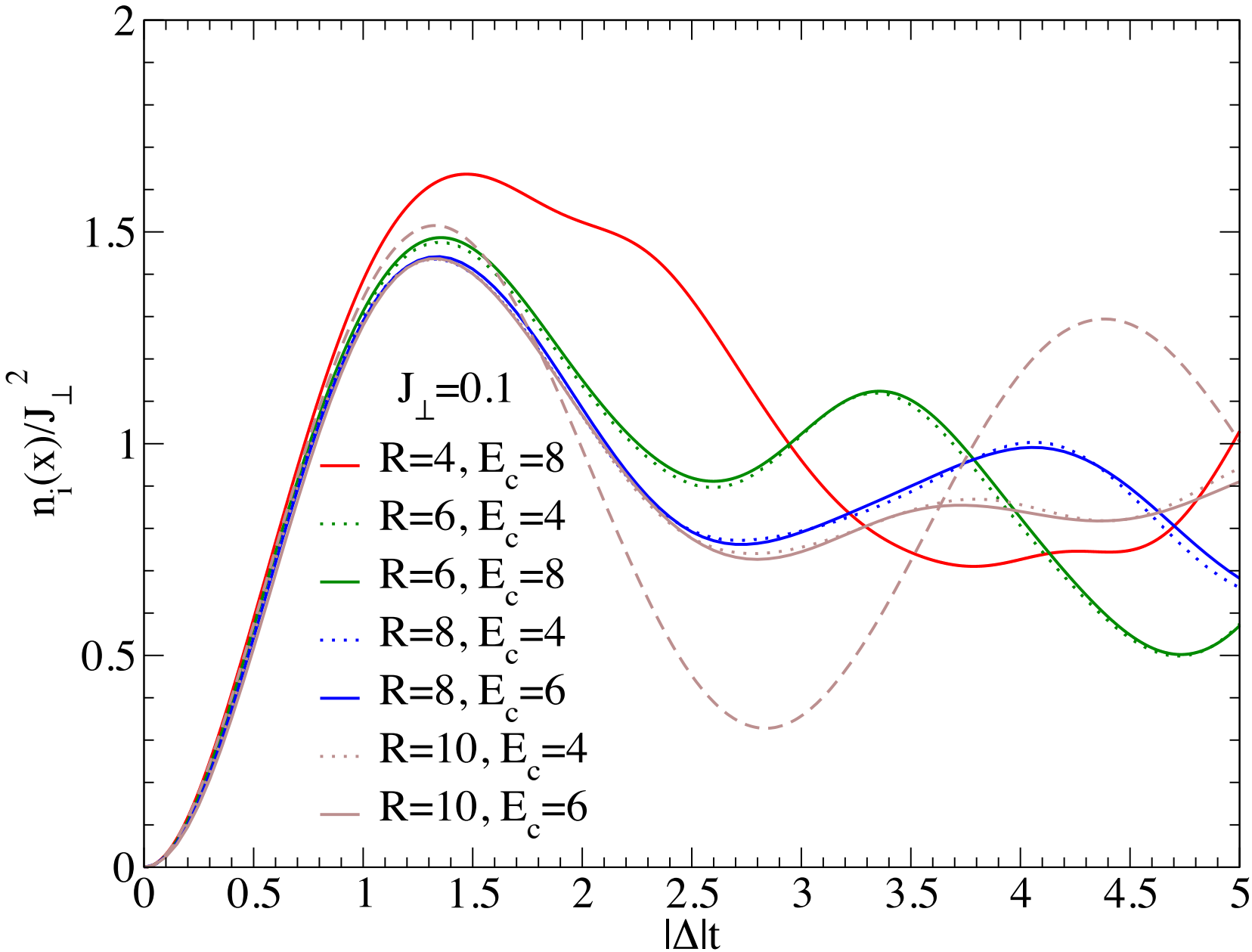
Spin matrix elements off diagonal in sector

GS is NS, this means only Ramond states contribute at this order

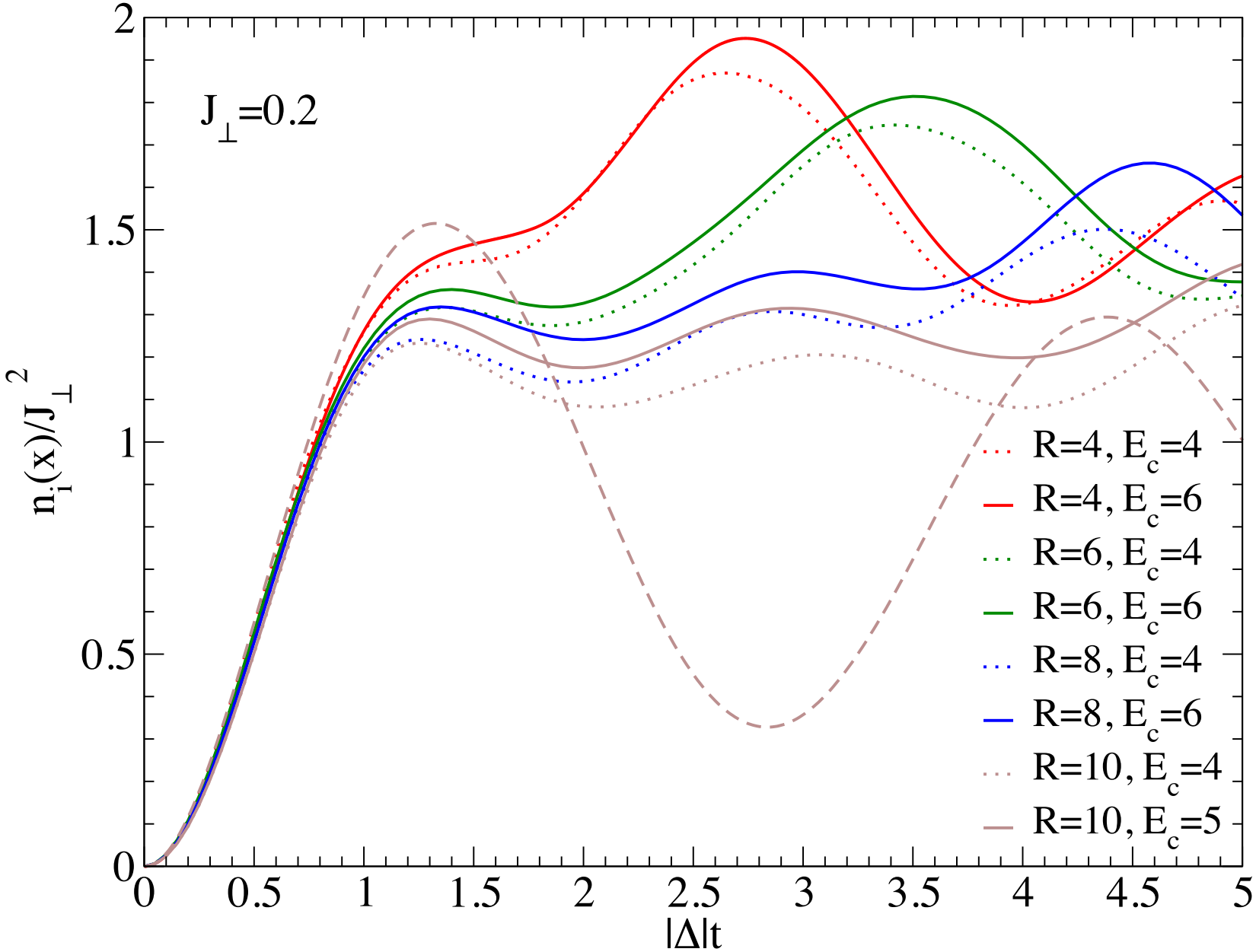
Very weak quench $\frac{J_\perp}{J_c} \approx 0.053$



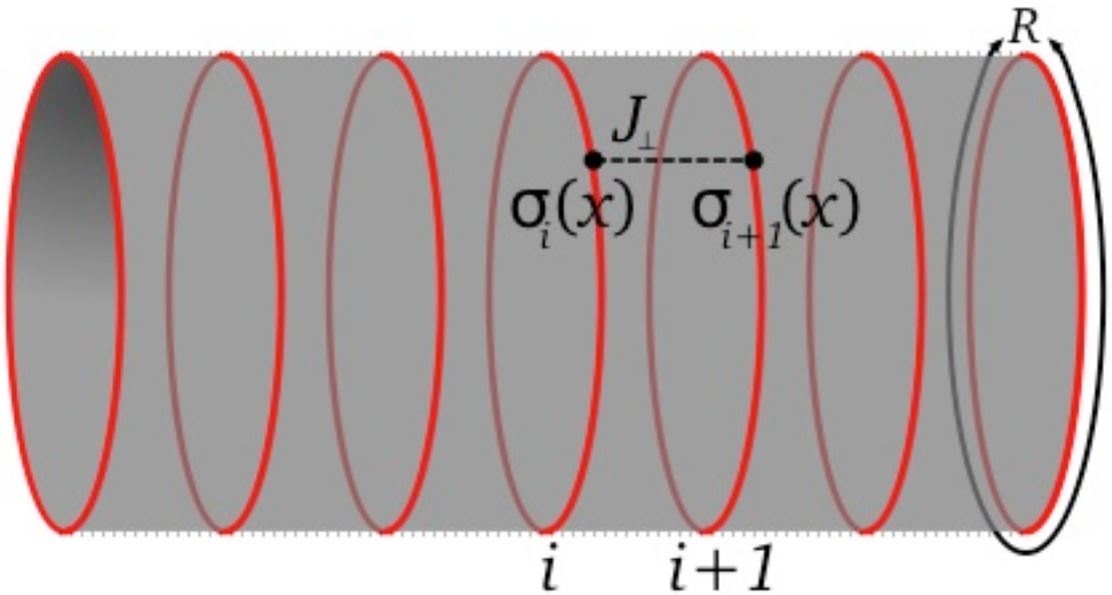
Stronger quench $\frac{J_\perp}{J_c} \approx 0.53$



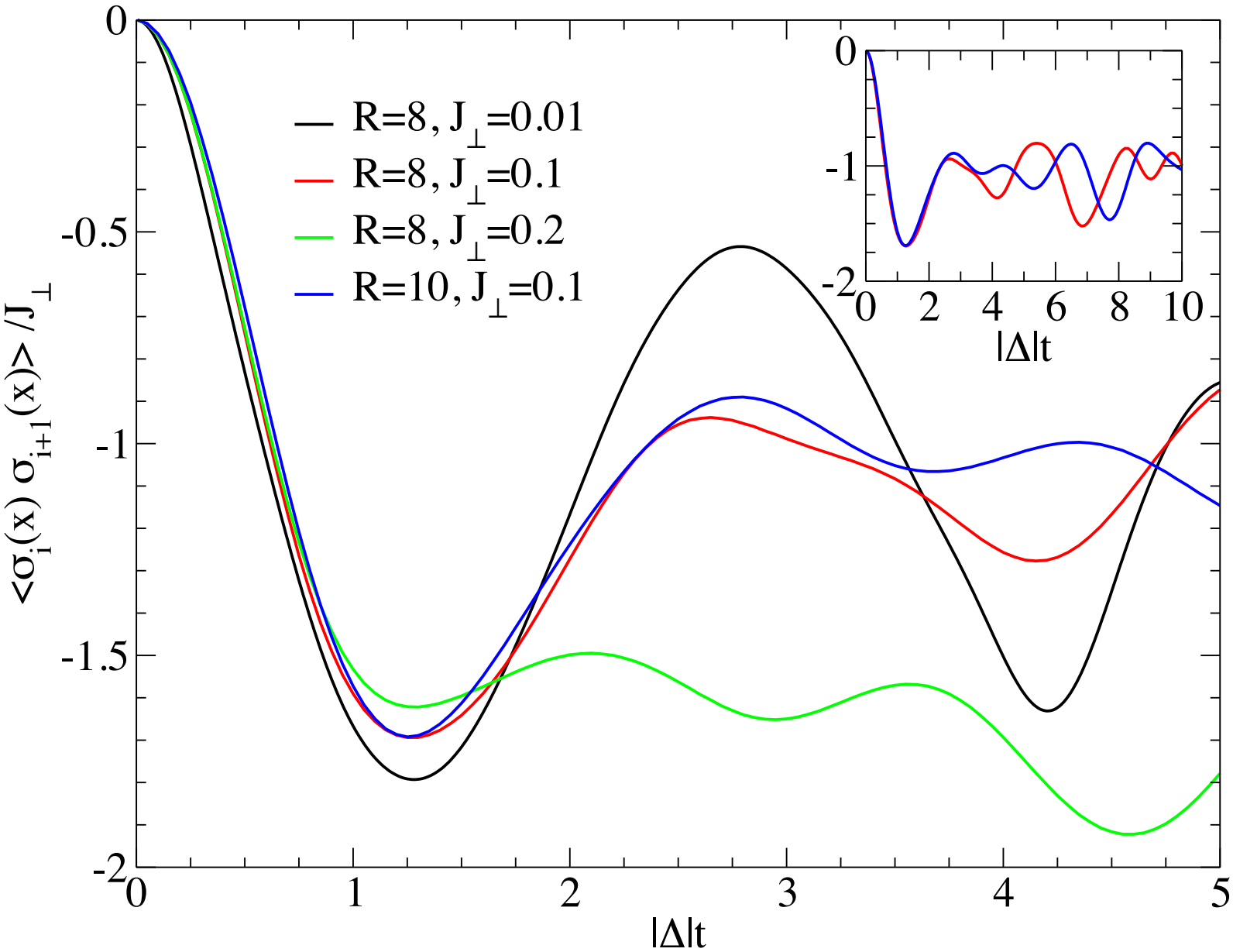
Quench through critical coupling $\frac{J_\perp}{J_c} \approx 1.1$



Correlations



Correlations



Return probability/Loschmidt echo

Global quantity, overlap with initial state

$$G(t) = |\langle\psi(0)|e^{-iHt}|\psi(0)\rangle|^2$$

Define a rate function

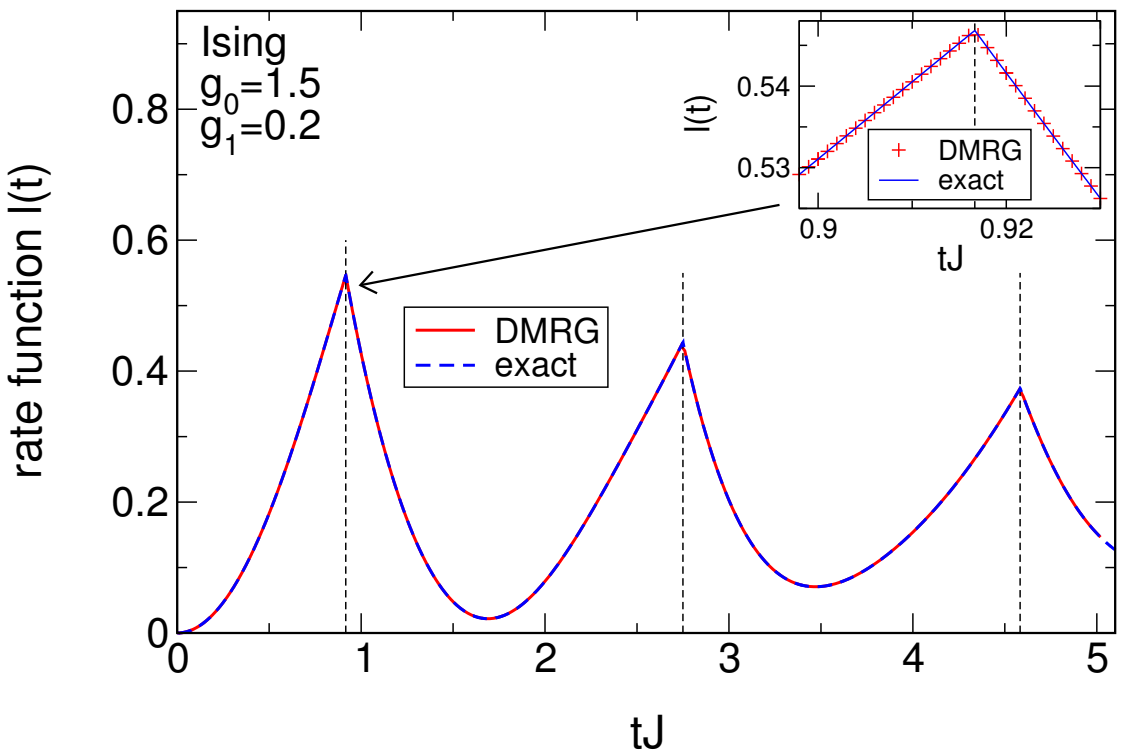
$$l(t) = -\lim_{N\to\infty}\frac{1}{N}\log|G(t)|^2$$

Non analytic rate in 1D

Heyl, Polkovnikov and Kehrein, PRL 110, 135704 (2013)

For TFIM, nonanalytic rate if quench is through critical point

$$l(t) = -\lim_{N\to\infty} \frac{1}{N} \log|G(t)|^2$$



Karrasch and Schuricht
PRB 87, 195104 (2013)

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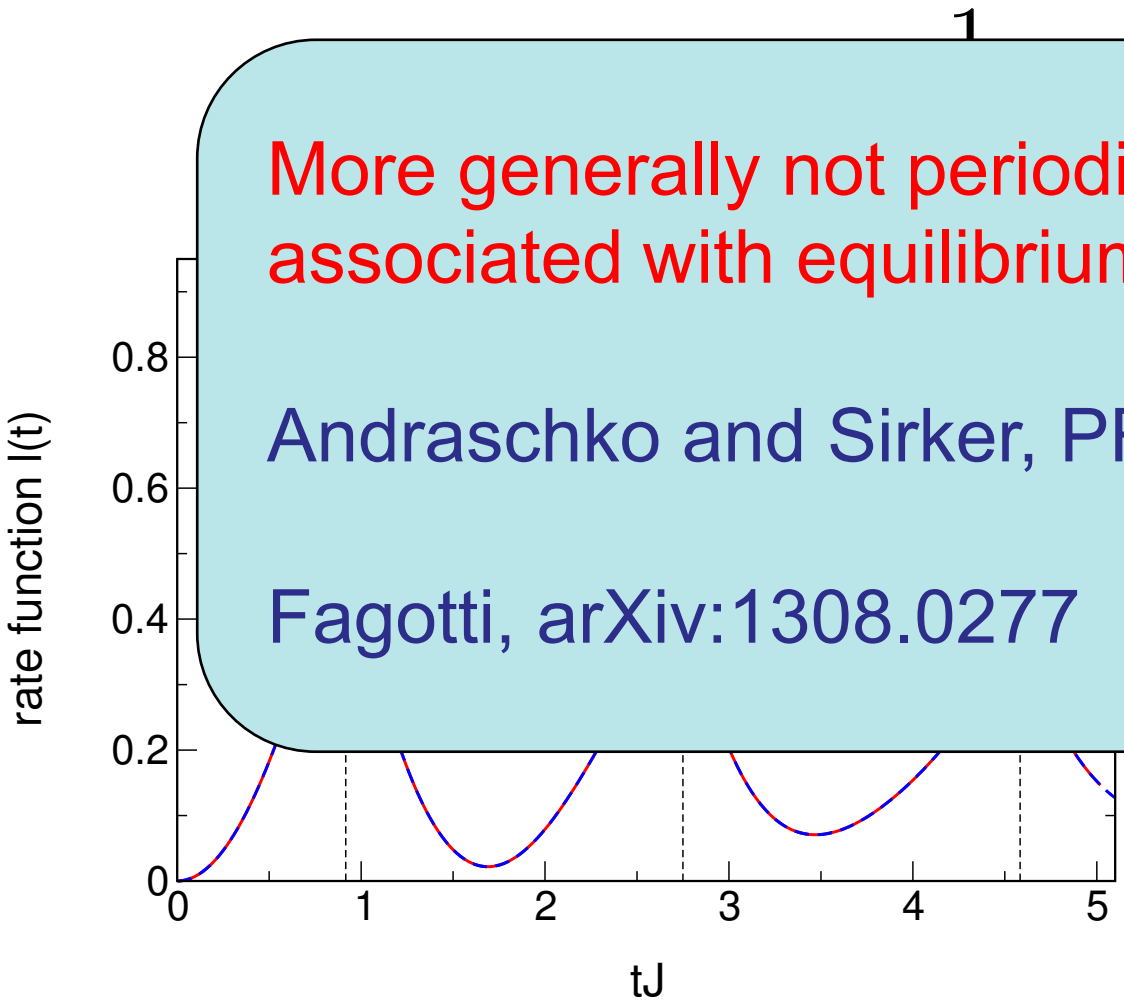

More generally not periodic, nor uniquely associated with equilibrium critical points

Andraschko and Sirker, PRB 89 (2014)

Fagotti, arXiv:1308.0277

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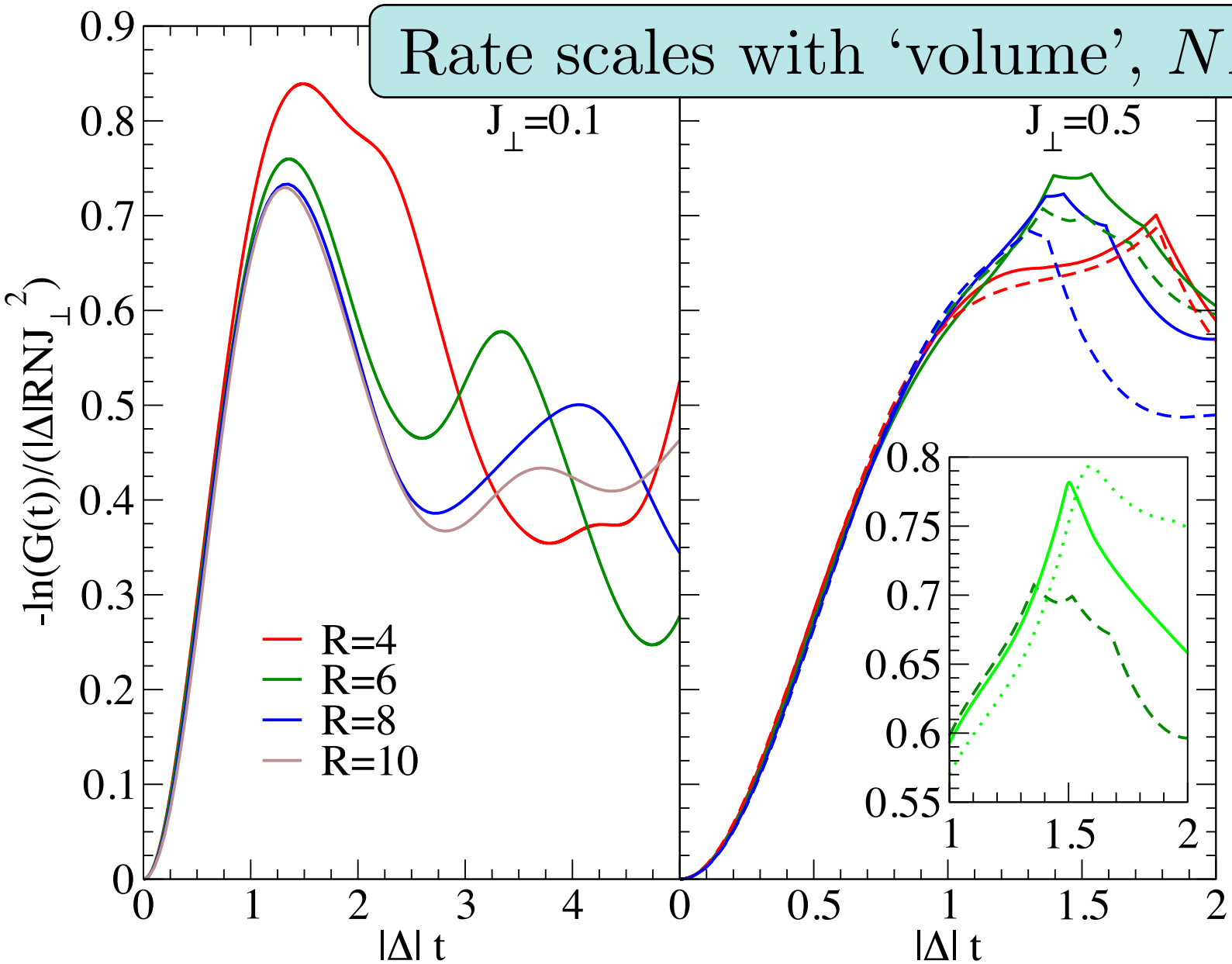
Return probability/Loschmidt echo



Conclusions

- Coupling integrable chains allows straightforward extension of 1D MPS methods to 2D systems
- Captures 2D quantum Ising phase transition
- Can study time evolution of infinite cylinders
  - Especially effective for weak quenches
  - Applicable to quenches through critical points
- Future work
  - Coupled Luttinger liquids
  - Coupled XXZ chains (with Caux group)