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Linear difference equations, frieze patterns, and projective polygons: Three sides of the same coin

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Motivated by the recent progress in the study of the so-called pentagram map (a discrete completely integrable system defined by a projectively natural construction on polygons), I shall describe three closely interrelated spaces: the space of linear difference equations with periodic coefficients and (anti)periodic solutions, the space of frieze patterns, and the moduli space of polygons in projective space.

Of these three, frieze patterns, beautiful combinatorial objects, introduced and studied by Coxeter and Conway in the 1970s, are lesser known. Recently they have attracted much attention due to their relation to the theory of cluster algebras. In my talk, I shall introduce frieze patterns and their higher generalizations.

I shall describe the combinatorial Gale transform, a somewhat surprising duality between difference equations of different orders with (anti)periodic solutions. For example, the solutions of a third-order difference equation $V_i = a_i V_{i-1} - b_i V_{i-2} + V_{i-3}$ with 5-periodic coefficients are 5-periodic $V_{i+5} = V_i$ if and only if the solutions of the second-order difference equation $W_i = a_i W_{i-1} - W_{i-2}$ are 5-antiperiodic $W_{i+5} = -W_i$.

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