Bulk reconstruction in AdS/CFT

arXiv:1405.6394, 1505.03755 with Gilad Lifschytz

previous and related works with Lifschytz, Lowe, Hamilton, Roy, Sarkar and by other authors

CRM workshop - 7/6/15

Motivation

Suppose the CFT is exactly soluble.

Formulate bulk reconstruction as a precise mathematical problem, soluble by a definite procedure in the 1/N expansion.

Finite N? Black holes?

For example AdS_2 / CFT_1 with $ds^2 = \frac{1}{Z^2}\left(-dT^2 + dZ^2\right)$

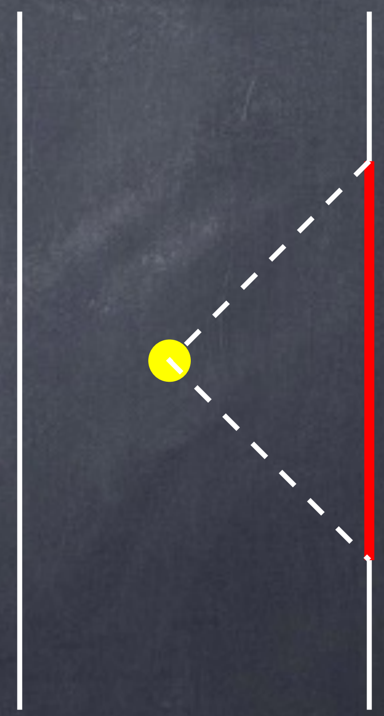
$\phi(T,Z)$ massless bulk scalar

$\mathcal{O}(T)$ single-trace dimension-1 primary

At infinite N the CFT only has 2-point functions.

Express free bulk field in terms of $\mathcal{O}$.

Pretty easy:



$$\Box\phi = 0 \qquad \phi(T,Z) \sim Z\mathcal{O}(T) \quad \text{as } Z \to 0$$

$$\Rightarrow \phi(T,Z) = \frac{1}{2}\int_{T-Z}^{T+Z} dT'\mathcal{O}(T')$$

Generalizes to higher dimensions, fields with spin, other coordinates: $\phi = \int K\mathcal{O}$

HKLL, Morrison

What about interactions?

Various approaches have been developed.

KLL

HMPS

I'll focus on bulk locality (microcausality).

$$\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \rangle$$

$\downarrow$ apply $\int K(x, z|x_1)$

$$\langle \phi(x,z) \mathcal{O}_2 \mathcal{O}_3 \rangle$$

Something's wrong, K obeys free wave equation!

Depends on $\chi = \frac{\left((x-x_2)^2+z^2\right)\left((x-x_3)^2+z^2\right)}{z^2(x_2-x_3)^2}$ > 0 if spacelike

Find $\langle \phi \mathcal{O} \mathcal{O} \rangle \sim F(\cdot, \cdot, \cdot, 1/\chi)$

Branch point at $\chi = 1$ => violates locality.

What to do? Set $\phi = \int K\mathcal{O} + \sum_n a_n \int K_n \mathcal{O}_n$

Math problem: choose a_n's to restore locality.

Claim: soluble in 1/N expansion. For example in AdS_2 / CFT_1 with $\Delta = 1$ operators

$\mathcal{O}$ single trace primary

$\mathcal{O}_n$ double trace, $\mathcal{O}\partial^{2n}\mathcal{O}$

$$\langle\, [\, \textstyle\int K\mathcal{O}, \mathcal{O}\,]\; \mathcal{O}\,\rangle \sim Q_0\big(\sqrt{1-\chi}\,\big)$$

$$\langle [\textstyle\int K_n \mathcal{O}_n,\, \mathcal{O}]\, \mathcal{O}\rangle \sim P_{2n+1}(\sqrt{1-\chi})$$

Fortunately $Q_0(x) = \sum_{n=0}^{\infty} \frac{4n+3}{(2n+1)(2n+2)} P_{2n+1}(x)$

Recovering bulk e.o.m.?

$$\phi = \int K\mathcal{O} + \sum_n a_n \int K_n \mathcal{O}_n$$

Act with $\Box^2$.

$$\Box^2 \phi = \sum_n a_n M_n^2 \int K_n \mathcal{O}_n$$

Stick r.h.s. in a 3-point function.

$$\langle \text{r.h.s.}\, \mathcal{O}\, \mathcal{O} \rangle \sim \frac{z}{(x-x_2)^2 + z^2} \, \frac{z}{(x-x_3)^2 + z^2}$$

Product of two bulk - boundary correlators!

r.h.s. $\sim \big(\phi(x,z)\big)^2$ so $\Box^2 \phi \sim \phi^2$

Generalizes to

scalars in higher dimensions, arbitrary masses

bulk scalars coupled to gauge fields

recover $\Box\phi \sim A_\mu \partial^\mu \phi$

bulk scalars coupled to gravity

recover $\Box\phi \sim h_{\mu\nu} \partial^\mu \partial^\nu \phi$

Ambiguities in reconstruction? Banks

Bulk side: free to make local field redefinitions

CFT side: free to add operators which preserve desired analyticity at spacelike separation

These ambiguities are in 1:1 correspondence.

For example consider the field redefinition

$$(\Box^2 - m^2)\phi = \lambda\phi^2$$

$$\phi \to \phi + \epsilon\phi^2$$

$$\left(\Box^2 - m^2\right)\phi = \left(\lambda - \epsilon m^2\right)\phi^2 - 2\epsilon\partial\phi\partial\phi$$

From the bulk point of view this is a field redefinition which is invisible at the boundary. How does this look from the CFT?

$$(\Box^2 - m^2)\phi = \lambda\phi^2$$

suppose we've reconstructed this e.o.m. from the CFT

$$\phi \to \phi + \epsilon\phi^2$$

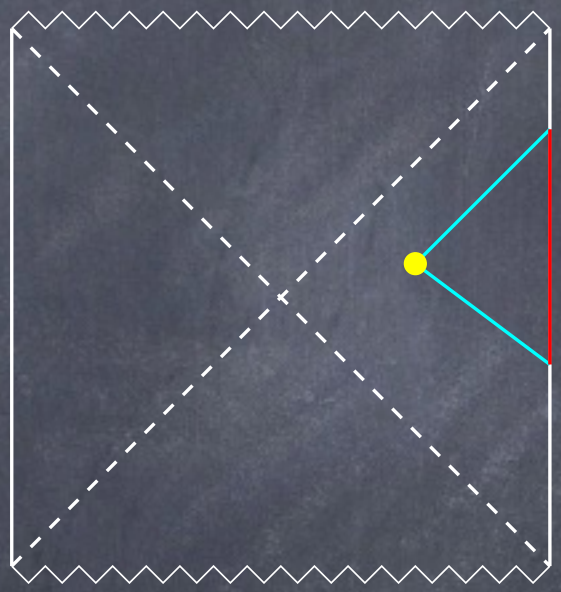
a redefinition of the CFT operator which preserves analyticity...

$$\left(\Box^2 - m^2\right)\phi = \left(\lambda - \epsilon m^2\right)\phi^2 - 2\epsilon\partial\phi\partial\phi$$

...but reconstructs a different e.o.m.

Finite N and black holes

At finite N microcausality should break down: the CFT doesn't have enough d.o.f. to build local bulk fields. A simple context where we can see this happening in a 2-point function is an AdS-Schwarzschild black hole.



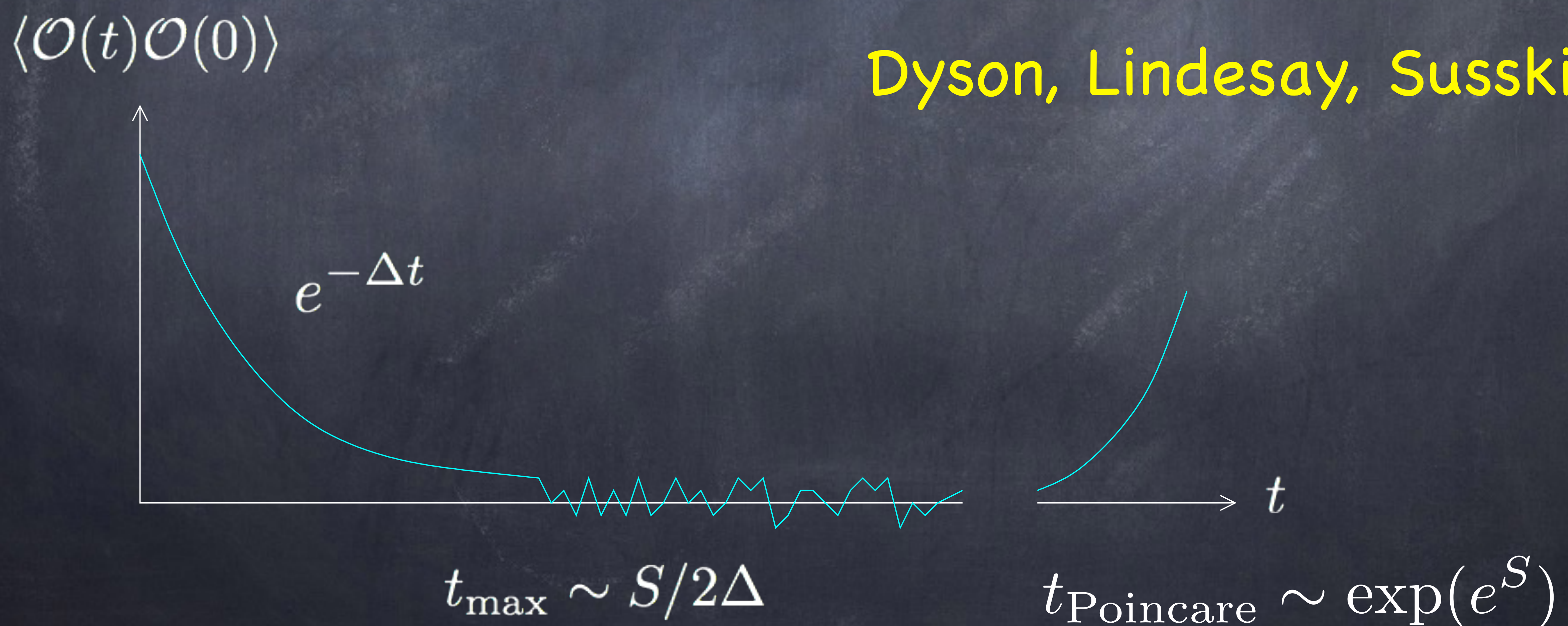
As the bulk point approaches the future horizon the smearing extends to $t = +\infty$ and becomes sensitive to the late-time behavior of CFT correlators.

This is problematic. At finite entropy a correlator can't go below the generic inner product of two normalized vectors.

$$\left|\langle\psi_1|\psi_2\rangle\right| \sim \frac{1}{\sqrt{\dim\mathcal{H}}} = e^{-S/2}$$

So a finite-N correlator should look like

Dyson, Lindesay, Susskind



A simple cure is to cut off the smearing function at $t \sim t_{\mathrm{max}}$. Seems ad hoc, but one consequence is finite energy resolution.

HKLL

$$\Delta E \sim 1/t_{\mathrm{max}} \sim 1/\beta S$$

This lets us recover a smooth bulk geometry, perhaps as an average over microstates.

The price to be paid is a failure of bulk locality, with commutators at spacelike separation $\sim e^{-N^2}$

(no firewalls?)

Kabat & Lifschytz,
Papadodimas & Raju

Open problems

What happens at $O(1/N^2)$?

Locality in 4-point functions => constraints on spectrum?

Heemskerk et al.

Bulk loops and renormalization?

Precise statements at finite N?

The cutoff procedure is a bit ad hoc.

Is there a guiding principle we can use?