Cosmological Perturbations & AdS/CFT

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CRM, July 7 2015

1. Motivation - Cosmology

std. paradigm inflation

time line

s-t diagram

matter bounce

$a(t)$

s-t diagram

Ekpyrotic

Q. source of bounce

$$ds^2 = dt^2 - a(t)^2 d\underline{x}^2$$

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho = H^2$$

$$\dot H = -4\pi G(\rho + p)$$

} bounce: $\rho = 0$, $\rho + p < 0$

violation of NEC ghost condensation

modified gravity HL gravity with spatial curv.

Problem: not UV complete

Challenge: obtain bounce in UV complete theory of gravity

$\rightarrow$ AdS/CFT

2. Fluctuations

$$ds^2 = (1+2\Phi)d\eta^2 - [(1-2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j$$

$\Phi(\underline{x}, \eta)$ cosm. pert.

$h_{ij}(\underline{x}, \eta)$ grav. waves

$\varphi(\underline{x}, \eta)$ test scalar

$$S = \int d^4x \left(\frac{1}{16\pi G} R + \mathcal{L}_m\right)$$

$$\mathcal{L}_m = \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - V(\varphi)$$

$\varphi = \varphi_0 + \delta\varphi$

$S^{(2)} = \frac{1}{2}\int d^4x \left(v'^2 - v_{,i}v^{,i} + \frac{z''}{z} v^2 \right)$ curv. pert.

$v = a(\delta\varphi + \frac{z}{a}\Phi)$ $\qquad \zeta = z^{-1} v$

$z = a \frac{\varphi_0'}{\mathcal{H}}$

$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0$

oscillations sub-Hubble

squeezing super-Hubble

$v_k(\eta_i) = \frac{1}{\sqrt{2k}}$ vacuum IC

$S^{(2)} = \frac{1}{2}\int d^4x \left(u'^2 - u_{,i}u^{,i} + \frac{a''}{a} u^2 \right)$ grav. waves

$u = ah$

$u = a\,\delta\varphi$ test scalar

$P_\zeta(k) = k^3 |\zeta(k)|^2$

$P_h(k) = k^3 |h(k)|^2$ } power spectra

$P_\zeta(k) \sim k^{n_s - 1}$ $\quad n_s$ spectral index

require: $n_s \simeq 1$ scale-invariant $\quad n_s = 3$ for vacuum

Ex. Inflation (sketch)

$P_\zeta(k,t) = P_\zeta(k, t_H(k)) \left(\frac{a(t)}{a(t_H(k))}\right)^2$

$= \underbrace{P_\zeta(k, t_i(k))}_{k^2} \underbrace{\left(\frac{a(t)}{a(t_H(k))}\right)^2}_{k^{-2}}$ $\qquad a(t_H(k))\,k^{-1} = H^{-1}$

$\Rightarrow n_s = 1$

Ex Matter Bounce D. Wands, (1998)

F. Finelli & RB, (2002)

sketch

$$v_k(\eta) = c_1 \eta^2 + c_2 \eta^{-1}$$

$$P_\varphi(k,\eta) \sim k^3 |v_k(\eta)|^2 \sim k^3 |v_k(\eta_H(k))|^2 \left(\frac{\eta_H(k)}{\eta}\right)^2 \sim k^0$$

$n_s = 1$

but: decaying mode has a different spectrum like for inflation

Inflation: matching at t_R trivial
no singularities

Bounce: matching at $t=0$ non-trivial
- singularities
  or new physics
- gauge ambiguities in matching surface
  $t = \text{const}$
  is not diffeo. invariant

R. Durrer, F. Vernizzi (2002)

Challenge: map bounce problem to AdS/CFT

- bounce in weak coupling
- bounce in gauge theory
- no diffeom. ambiguities

key question: change in spectral index across bounce

3. Setup : deformed $AdS_5 \times S_5$ — S. Das et al 2006 –
time-dep. deformation — C. Chu & P. Ho 2006, 2007

$$ds^2 = \frac{R^2}{z^2}(dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu)$$

$$\downarrow$$

$$ds^2 = \frac{R^2}{z^2}(dz^2 + \tilde{g}_{\mu\nu} dx^\mu dx^\nu)$$

$$R^4 = 16\pi N l_s^4 g_s^{-1}$$

$$g_s = e^{-\phi}$$

deformation: $\phi = \phi(t) \Leftrightarrow \tilde{g}_{\mu\nu} = \tilde{g}_{\mu\nu}(t)$ — $|t| < t_c$

$\phi(t) = \text{const}$ — $|t| > t_c$

$a(t) \sim t^p$

$\phi = \frac{\alpha}{p} \ln a(t) \qquad \alpha^2 = 6p(1-p)$

$t = 0$ — curvature singularity
strong string coupling
weak gauge coupling

$\pm t_b$: $g_s = 1$

Fig. 1

<u>Goal</u>: study evolution of test scalar field bulk fluct.
$t < -t_b$ to $t > t_b$

test scalar field : dilaton

<u>Note</u>: expect this to tell us how gravit. wave
spectrum will evolve from $-t_b$ to $+t_b$

4. Calculation (Fig. 2)

Input: spectrum of dilaton flucts (bulk) at $-t_c$

step 1: evolution in bulk from $-t_c$ to $-t_b$

step 2: map flucts. to bdry at $-t_b$

$\Rightarrow \delta A_\mu(k)$

step 3: evolve bdry flucts. in gauge theory to $+t_b$

step 4: reconstruct dilaton flucts. in bulk at t_b

step 5: evolution in bulk from t_b to t_c

Output: spectrum of dilaton flucts. (bulk) at t_c

step 1: bulk evolution

$$S_\varphi = \int d^5x \sqrt{-g}\left(g^{MN}\partial_M\varphi\,\partial_N\varphi\right)$$

$$\ddot\varphi + 3H\dot\varphi - a^{-2}\varphi_{,ii} - \varphi_{,zz} + \frac{3}{z}\varphi_{,z} = 0$$

$$\varphi(t,z,x^i) = T(t)\,Z(z)\,X(x^i)$$

$$D^2X + k^2X = 0 \qquad \text{plane waves}$$

$$\tilde T(\eta) = a(\eta)\,T(\eta)$$

$$\tilde T'' + \left(w^2a^2 + k^2 - \frac{a''}{a}\right)\tilde T = 0$$

$$Z_{,zz} - \frac{3}{z}Z_{,z} + w^2 Z = 0$$

$$Z(z) = C\,z^2 J_2(wz)$$

IR modes: $\tilde T'' - \frac{a''}{a}\tilde T = 0$

5. Dual Boundary Theory

$$S_{YM} = -\frac{1}{4}\int d^4x\, e^{\phi(t)}\, \mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu}$$

bulk to boundary correspondence

$$\phi(z,t,x) \rightarrow O(t,x) = \mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$$

field redef. to get std. kinetic term

$$\tilde{A}_\mu \equiv e^{-\frac{\phi}{2}}A_\mu$$

$$\partial_\mu \partial^\mu \tilde{A}_\nu + M_{YM}^2(t)\tilde{A}_\nu = 0$$

$$M_{YM}^2 = -\frac{d^2}{4t^2} - \frac{d}{2t^2}$$

$$\ddot{\tilde{A}}_k + \underbrace{(k^2}_{\text{negligible}} + M_{YM}^2)\tilde{A}_k = 0 \qquad \text{gauge } A_0 = 0$$

$$\tilde{A}_k(t) = |t|^{1/2}\left(D_J(k)\, J_{\nu_J}(|kt|) + D_Y(k)\, Y_{\nu_Y}(|kt|)\right)$$

$$\nu_J = \frac{1 \pm d}{2}$$

N.B. $\tilde{A}_k$ diverges at $t=0$ → matching criterion needed

But: Matching in CFT → well defined

step 2: IC for $A_\mu(k)$ at $-t_0$

$$m_{pl}^3\phi(k) \sim \frac{1}{4}\int d^3k'\, \dot{A}_i\left(\frac{k+k'}{2}\right)\dot{A}_i\left(\frac{k-k'}{2}\right)V^{1/2}$$

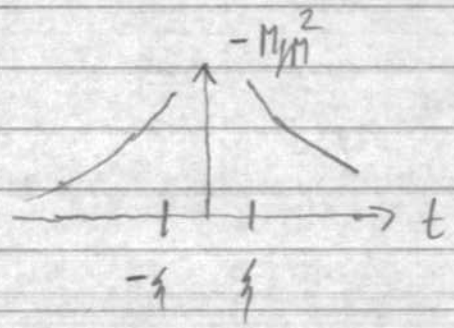
R1: $k' < k$

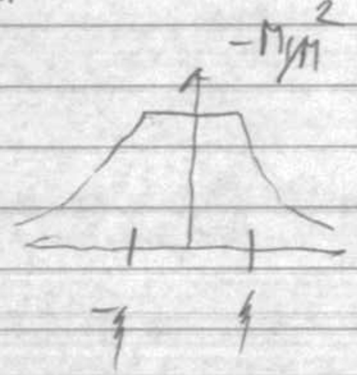
R2: $k' > k$

Result : $A_i(k) \sim k^{-9/4}$ not vacuum!

step 3 bdry evolution : need cutoff

$t = \pm \xi \qquad \xi = \sqrt{\alpha'}$





Match $A_j, \dot{A}_j$ from $-\xi$ to ξ

double matching well defined

<u>Result</u> : $\tilde{A}_k(t_B) \simeq \left(\frac{t_B}{\xi}\right)^{2\nu_j} \tilde{A}_k(-t_B)$

- non-vanishing particle production
- non-trivial mode matching
- k independent mode matching

6. Bulk Reconstruction (Fig. 3)

$$\phi(x, t, z) = \int dt' d^3x' \, K(x', t'; x, t, z) \, O(x', t')$$

use KKLL propagator

$$\phi(k, t, z) = V^{-1/2} \int d^3x \, \phi(x, t, z) e^{i\underline{k}\cdot\underline{x}}$$

$$= V^{-1/2} \int ds \, d^3y \, K(y, s; z) \int d^3x \, e^{i k \cdot x} \, O(x+y, t+s$$

$x' = x + y$
$t' = t + s$

$$= \int ds' d^3y' K(y', s, z)\, \mathcal{O}(\underset{\sim}{k}, t')$$

only k dependence

Result: spectral index unchanged

Conclusions:

Prescription for evolving fluct. through cosmological bounce
using AdS/CFT
Spectral index unchanged
Amplitude enhanced
$\Rightarrow$ stringy realization of matter bounce alternative
to inflation