The Ising model and a sum involving Toeplitz determinants

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We describe two results in joint work with Craig A. Tracy.

In the theory of the 2D Ising model there is a quantity called the “diagonal magnetic susceptibility” which equals an infinite sum involving Toeplitz determinants. In terms of a certain parameter it is analytic inside the unit circle, and it was conjectured that the unit circle is a natural boundary. We prove this, and generalizations to a larger class of Toeplitz determinants.

The magnetic susceptibility itself is an analytic function of the same parameter and for it there is an as yet unproved natural boundary conjecture. The susceptibility equals a sum of multiple integrals and recent attention has focussed on the singularities of the summands. We show that the summands have no other singularities on the unit circle than those identified by B. G. Nickel.

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