Numerical Holography and Heavy Ion Collisions

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based on work with Paul Chesler

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relativistic heavy ion collisions

Relevant dynamics:

Very early: partonic, marginally perturbative (?)

Plasma phase: strongly coupled

Evidence: screening lengths, viscosity, ...

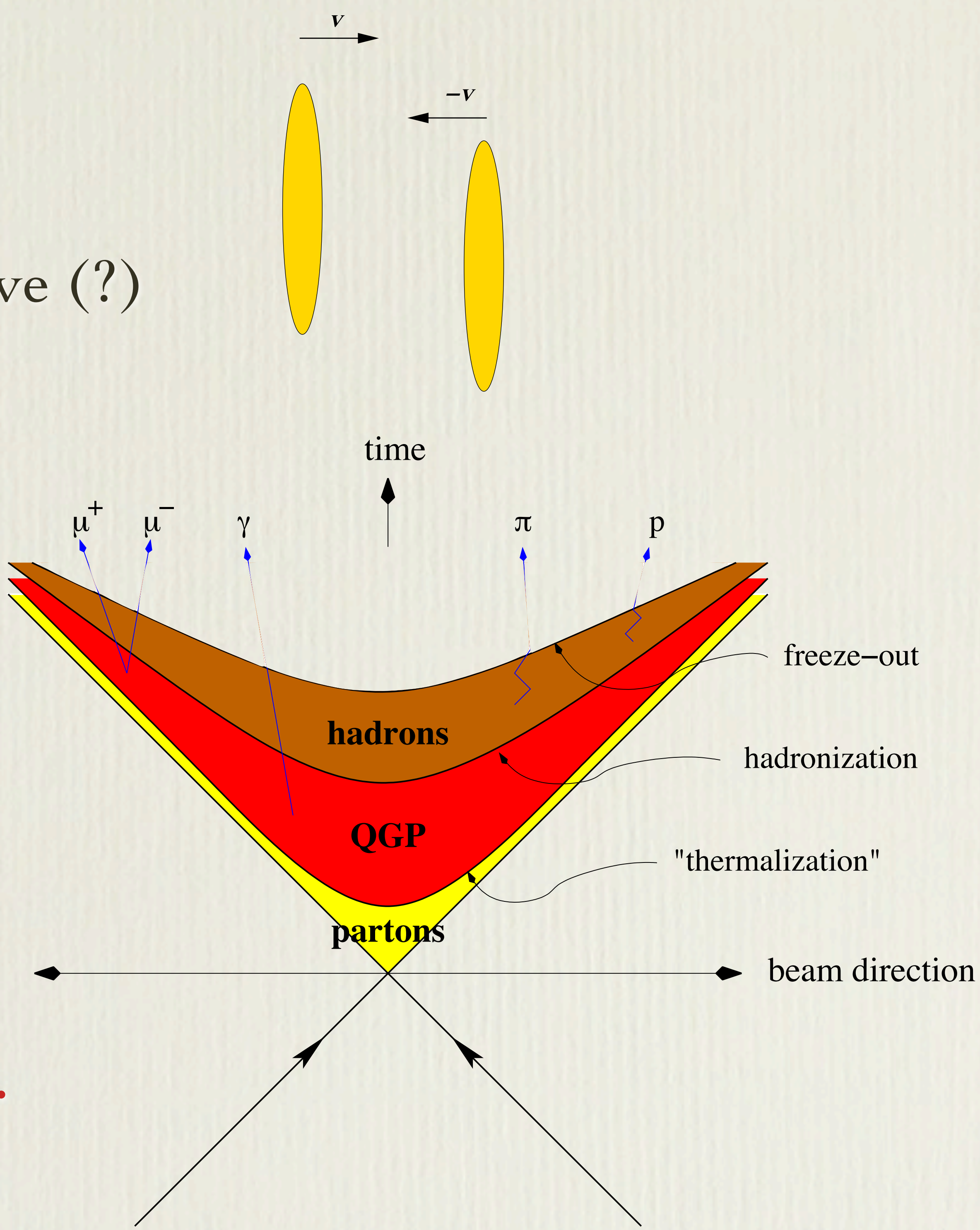
Many questions:

How fast do produced partons isotropize?

Initial conditions for hydrodynamics?

Signatures of strongly coupled dynamics?

No fully controlled theoretical methods.



idealize

QCD	➡	$\mathcal{N}$=4 SYM
# colors $N_c = 3$	➡	$N_c = \infty$
't Hooft coupling $\lambda \approx 10$	➡	$\lambda \gg 1$
highly boosted nuclei	➡	lightlike projectiles

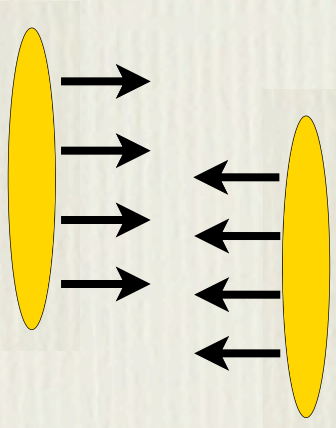
- ✓ non-Abelian plasma
- ✓ hydrodynamic response
- ✘ no hadronization
- ✓✘ conformal
- ✓ dual holographic description

holography

- strongly coupled, large N QFT = classical (super)gravity in higher dimension
  - valid description on all scales
  - gravitational fluctuations: $1/N^2$ suppressed
  - QFT state ↔ asymptotically AdS geometry
  - $O(N^2)$ entropy ↔ gravitational (black brane) horizon
  - thermalization ↔ gravitational infall, horizon formation & equilibration
  - non-equilibrium QFT dynamics ↔ classical gravitational initial value problem

holographic collisions

- scattering ➡ Poincaré patch AdS asymptotics
- warm-up steps:
  - homogeneous isotropization: 1+1D PDEs
    no spatial dynamics
  - boost invariant: 1+1D (no radial flow) or 2+1D PDEs
    unrealistic longitudinal dynamics
  - planar shocks: 2+1D PDEs
    no transverse dynamics
- recent work:
  - finite “nuclei”: 4+1D PDEs (off-center)
    sensible transverse and longitudinal dynamics



initial projectiles

- exact analytic solution for stable null projectile

Fefferman-Graham (FG) coordinates:

Gubser, ...; Romatschke,...

$$ds^2 = \frac{L^2}{s^2}\left[-dt^2 + d\boldsymbol{x}_\perp^2 + dz^2 + ds^2 + h_\pm(\boldsymbol{x}_\perp, z_\mp, s)\, dz_\mp^2\right]$$

$$\boldsymbol{x}_\perp \equiv \{x, y\} \qquad z_\mp \equiv z \mp t$$

- metric deformation function

$$\left(\partial_s^2 - \tfrac{3}{s}\,\partial_s + \nabla_\perp^2\right) h_\pm = 0$$

arbitrary

$$h_\pm(\boldsymbol{x}_\perp, z_\mp, s) = \int \frac{d^2\mathbf{k}_\perp}{(2\pi)^2}\, e^{i\mathbf{k}_\perp\cdot\mathbf{y}_\perp}\, \widetilde{H}_\pm(\mathbf{k}_\perp, z_\mp)\, 8(s^2/k_\perp^2)\, I_2(k_\perp s)$$

- stress-energy: $\langle T^{00}\rangle = \langle T^{zz}\rangle = \pm\langle T^{0z}\rangle = \kappa\, H_\pm(\boldsymbol{x}_\perp, z_\mp)$

- choose Gaussian profile for simplicity:

$$H_\pm(\boldsymbol{x}_\perp, z_\mp) = \frac{\mathcal{A}}{\sqrt{2\pi w^2}}\, \exp\left(-\tfrac{1}{2} z_\mp^2/w^2\right)\, \exp\left[-\tfrac{1}{2}(\boldsymbol{x}_\perp \mp \boldsymbol{b}/2)^2/R^2\right]$$

with: $\mathcal{A} = 1 \quad w = \tfrac{1}{2} \quad R = 4 \quad \boldsymbol{b} = \tfrac{3}{4} R\,\hat{\boldsymbol{x}}$

initial data

- Superpose left & right-moving shocks
- Transform to infalling Eddington-Finkelstein (EF)
  - ➡ must compute infalling radial null geodesic congruence

EF coordinates $X \equiv \{x^\mu, r\}$

boundary coordinates

affine parameter

FG coordinates $Y \equiv \{y^\mu, s\}$

geodesic congruence $Y(X)$

transformed metric $G_{MN}(X) = \dfrac{\partial Y^A}{\partial X^M} \dfrac{\partial Y^B}{\partial X^N} \widetilde{G}_{AB}(Y(X))$

numerical techniques

- characteristic formulation of Einstein equations
- spectral methods w. domain decomposition
- residual diffeomorphism freedom ➡ fix apparent horizon
- periodic spatial compactification
- Matlab implementation (shared memory, multicore)

characteristic formulation (1)

- null slicing of spacetime
- coordinates tied to infalling null geodesic congruence
- metric ansatz: $ds^2 = \frac{r^2}{L^2}\, g_{\mu\nu}(x,r)\, dx^\mu dx^\nu + 2\, dr\, dt$

  x^μ=const. is null geodesic, r = affine parameter

- residual diffeomorphism freedom: $r \rightarrow r + \lambda(x)$

  use to fix radial position of apparent horizon

characteristic formulation (2)

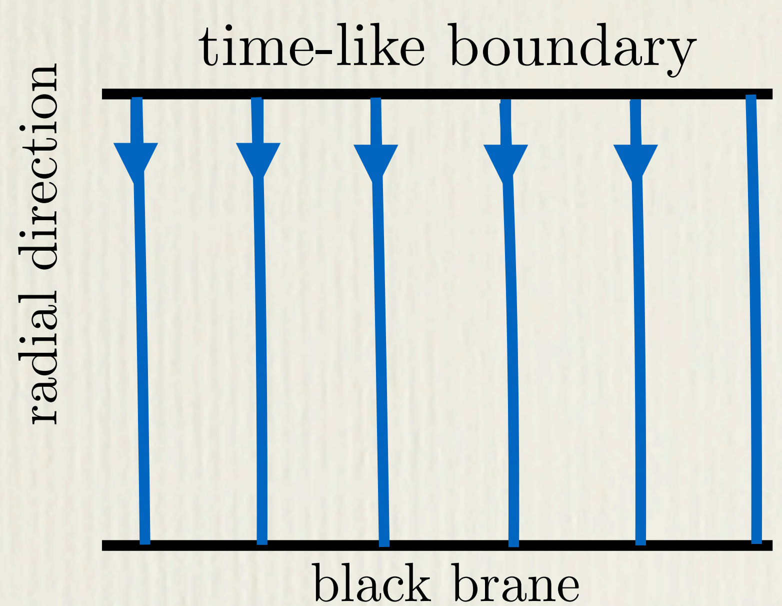
spatial scale factor

- rename: $\dfrac{r^2}{L^2}\, g_{00} = -2A\,, \quad \dfrac{r^2}{L^2}\, g_{0i} = -F_i\,, \quad \dfrac{r^2}{L^2}\, g_{ij} = G_{ij} \equiv \Sigma^2\, \hat{g}_{ij}$

unit determinant

- schematic form of resulting equations:

$$
\begin{aligned}
\left(\partial_r^2 + Q_\Sigma[\hat g]\right)\Sigma &= 0\,,\\
\left(\partial_r^2 + P_F[\hat g,\Sigma]\,\partial_r + Q_F[\hat g,\Sigma]\right) F &= S_F[\hat g,\Sigma]\,,\\
\left(\partial_r + Q_{\dot\Sigma}[\hat g,\Sigma]\right)\dot\Sigma &= S_{\dot\Sigma}[\hat g,\Sigma,F]\,,\\
\Big(\partial_r + Q_{\dot{\hat g}}[\hat g,\Sigma]\Big)\,\dot{\hat g} &= S_{\dot{\hat g}}[\hat g,\Sigma,F,\dot\Sigma]\,,\\
\partial_r^2\, A &= S_A[\hat g,\Sigma,F,\dot\Sigma,\dot{\hat g}]
\end{aligned}
$$



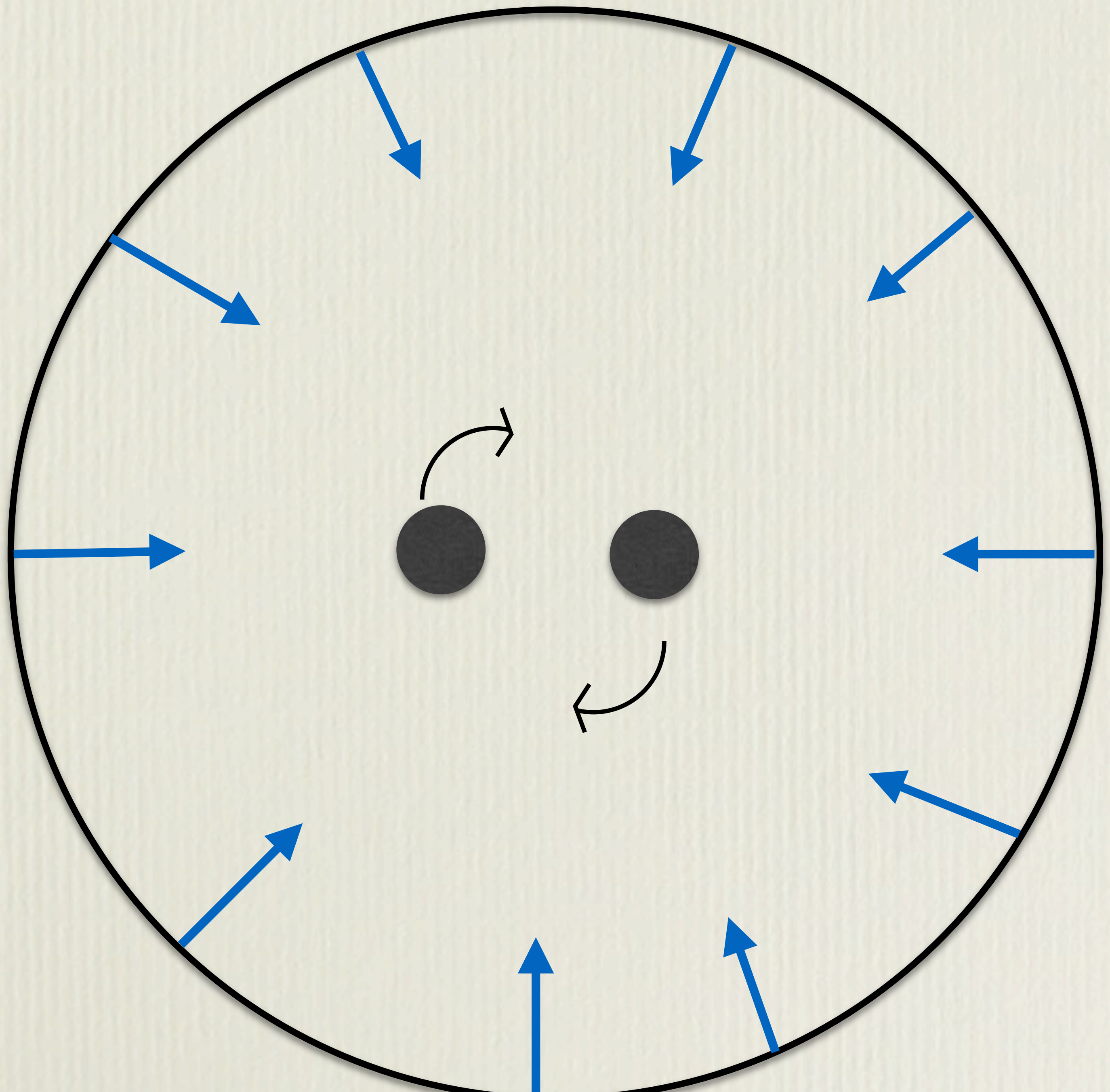
with $\dot h \equiv \partial_t h + \frac{1}{2} A\,\partial_r h$

➡ 5D PDEs $\longrightarrow$ nested linear radial ODEs !!!

laptop/desktop computable, no supercomputers

characteristic formulation (3)

- wonderful method, but not generally used in numerical relativity:



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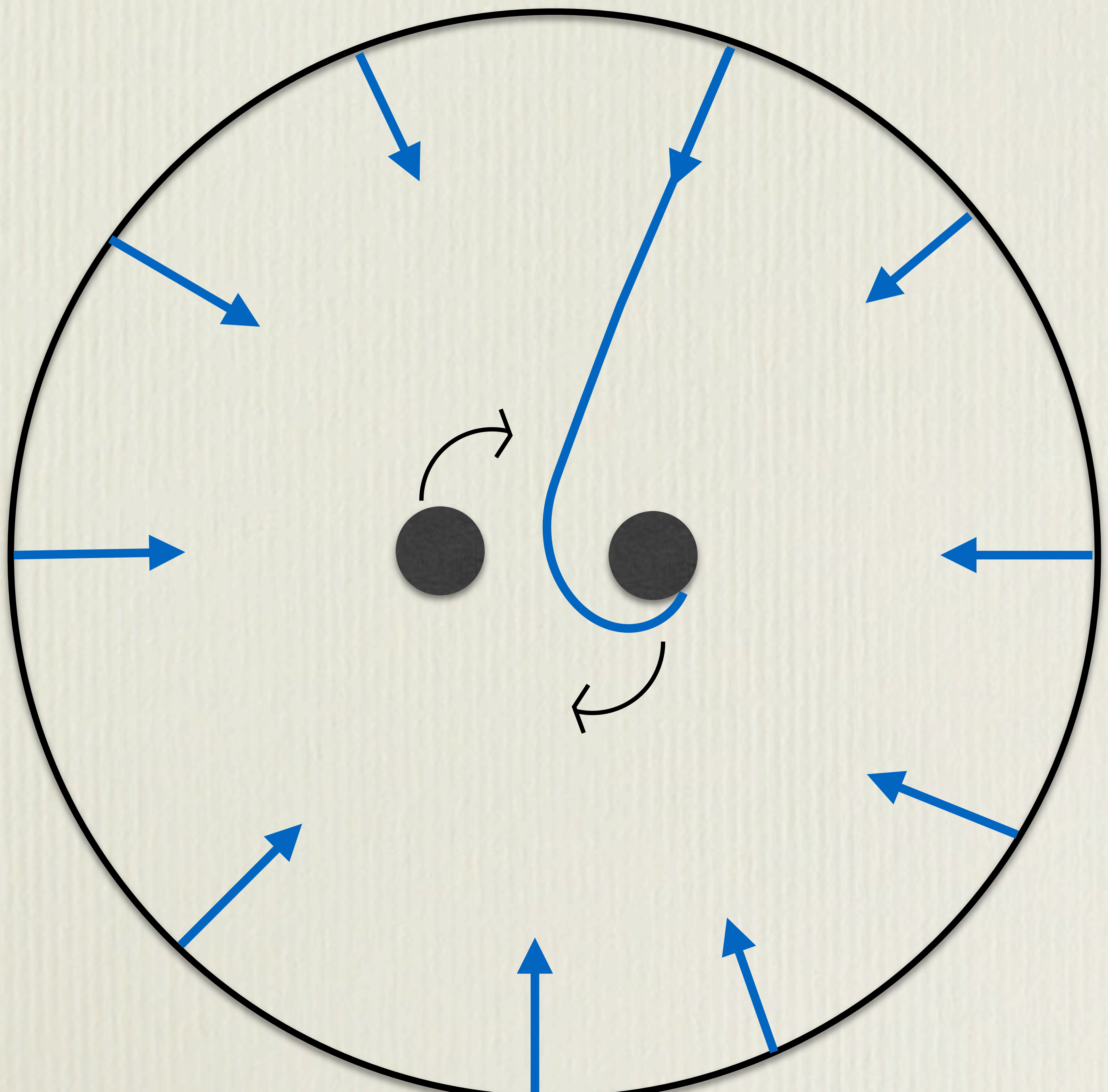

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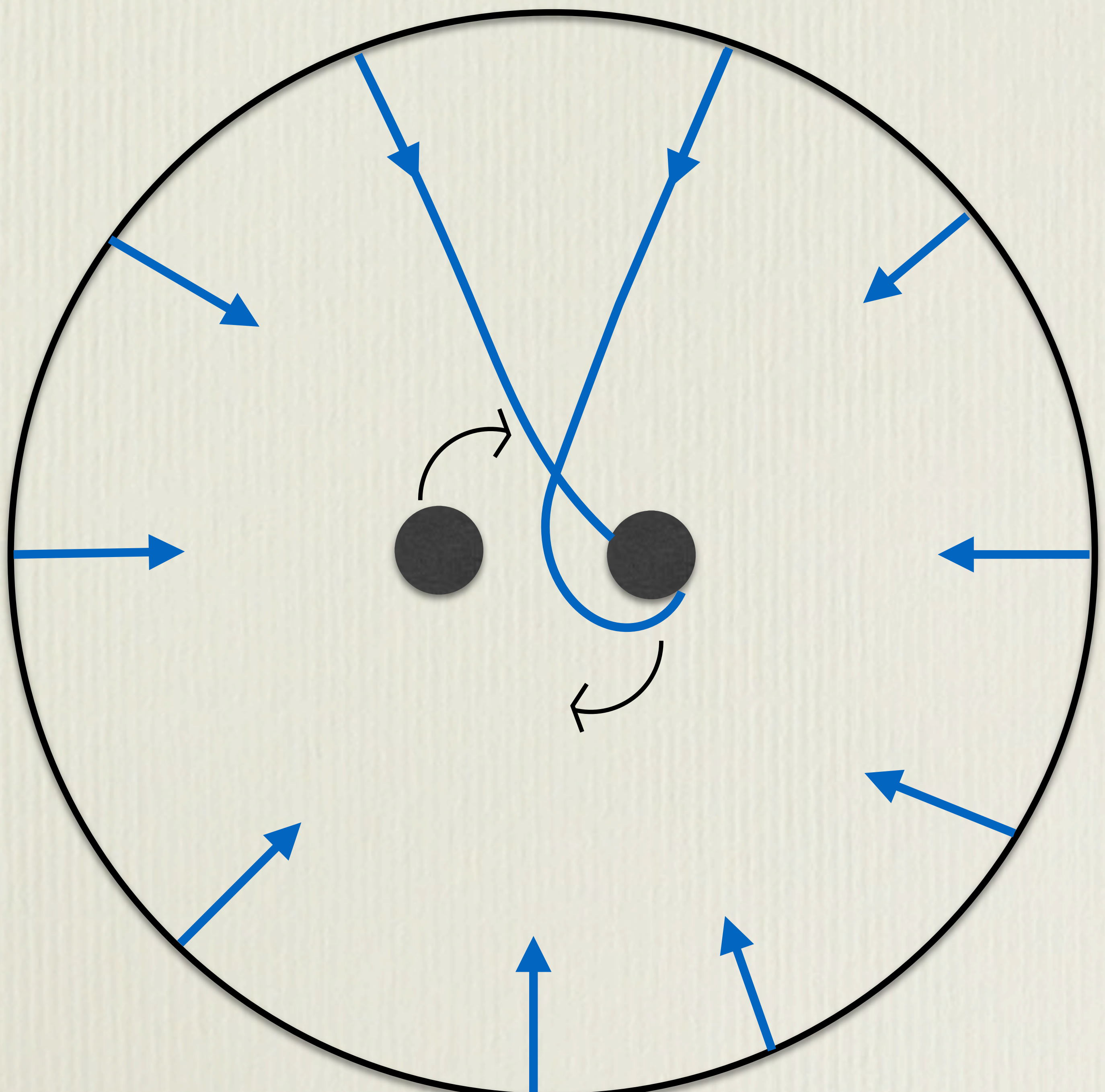

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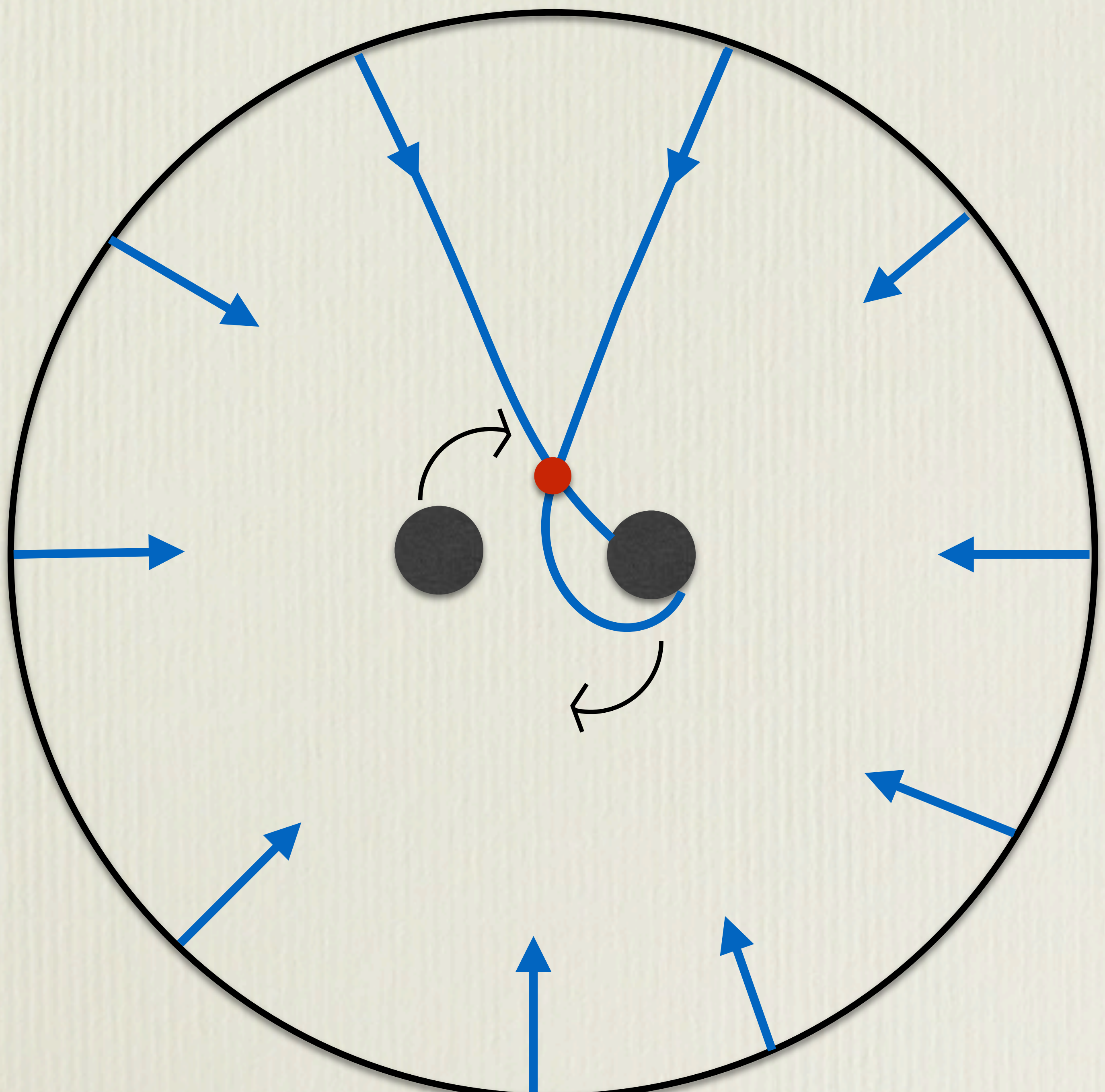

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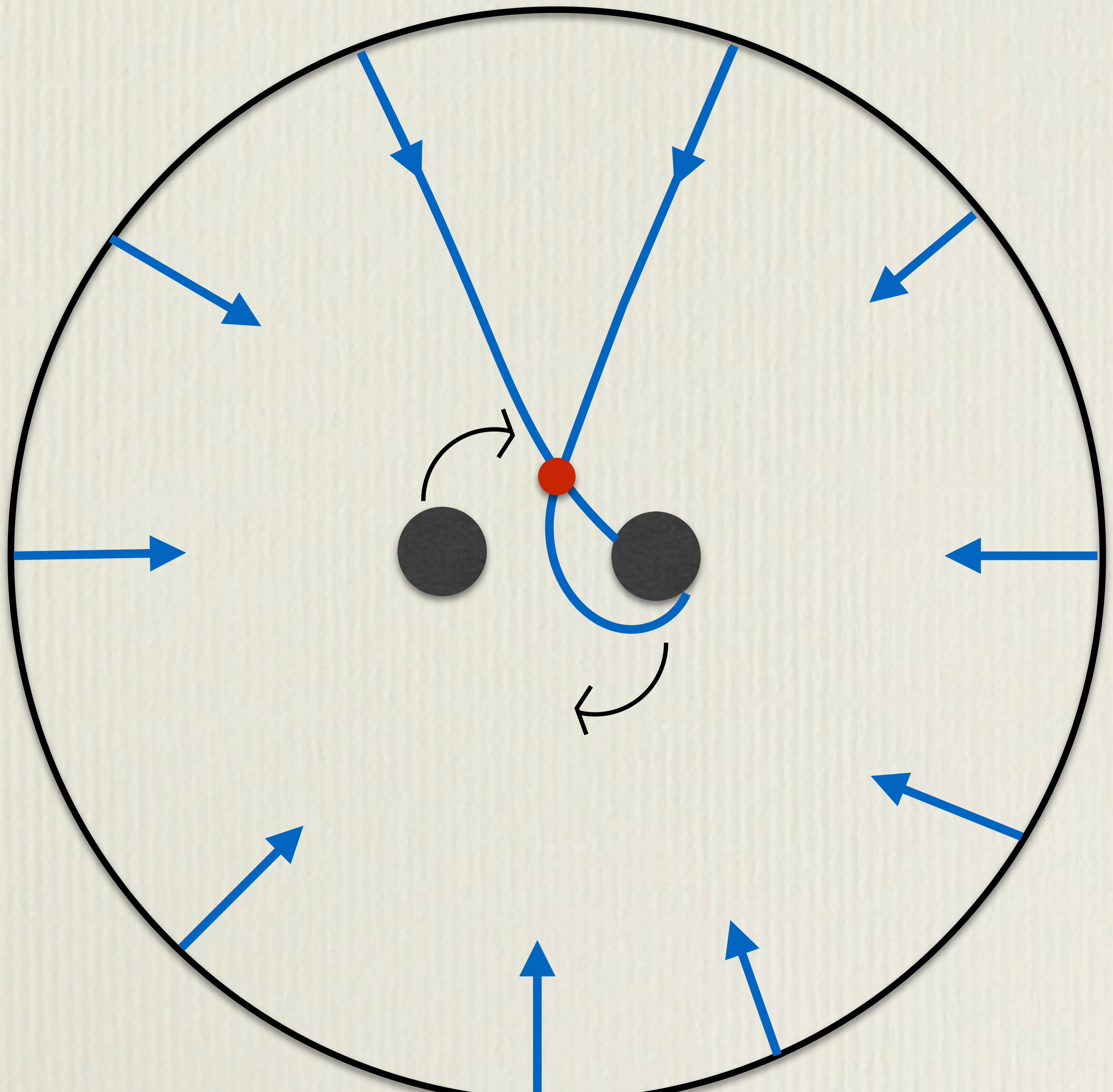
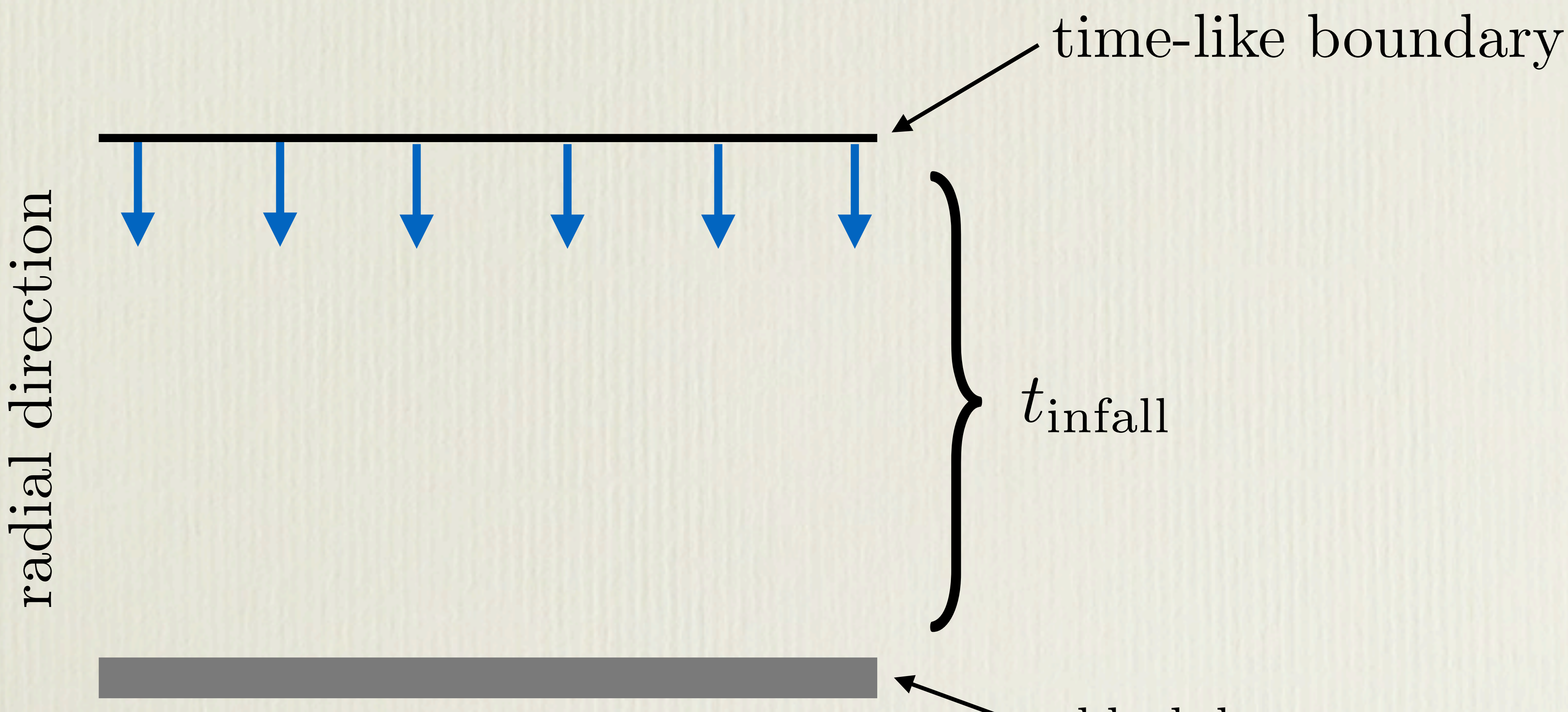


caustics (outside horizon)
= coordinate singularities

characteristic formulation (4)

- works for wide range of holographic QFT problems, but can fail if shortest relevant length scale < dissipative time scale



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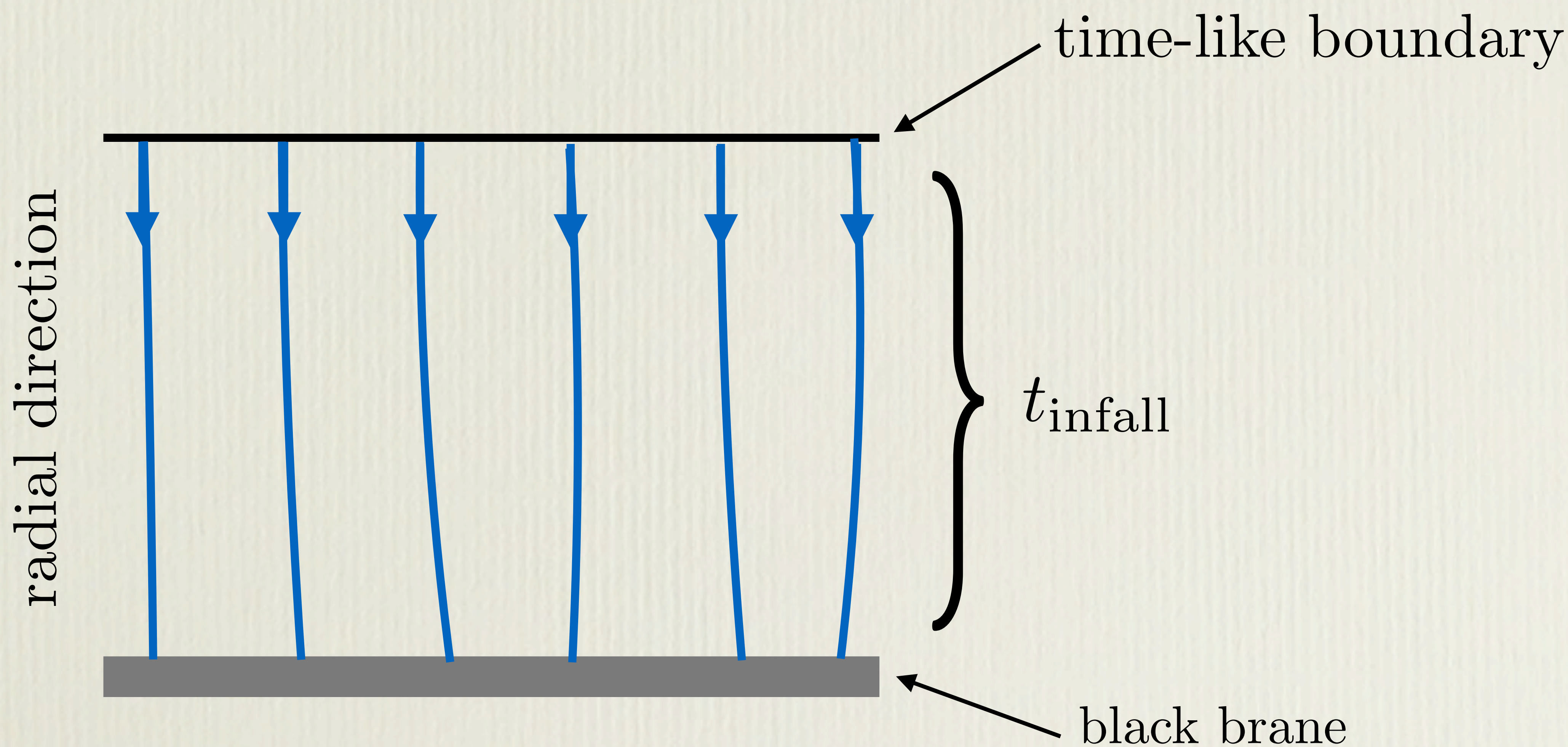

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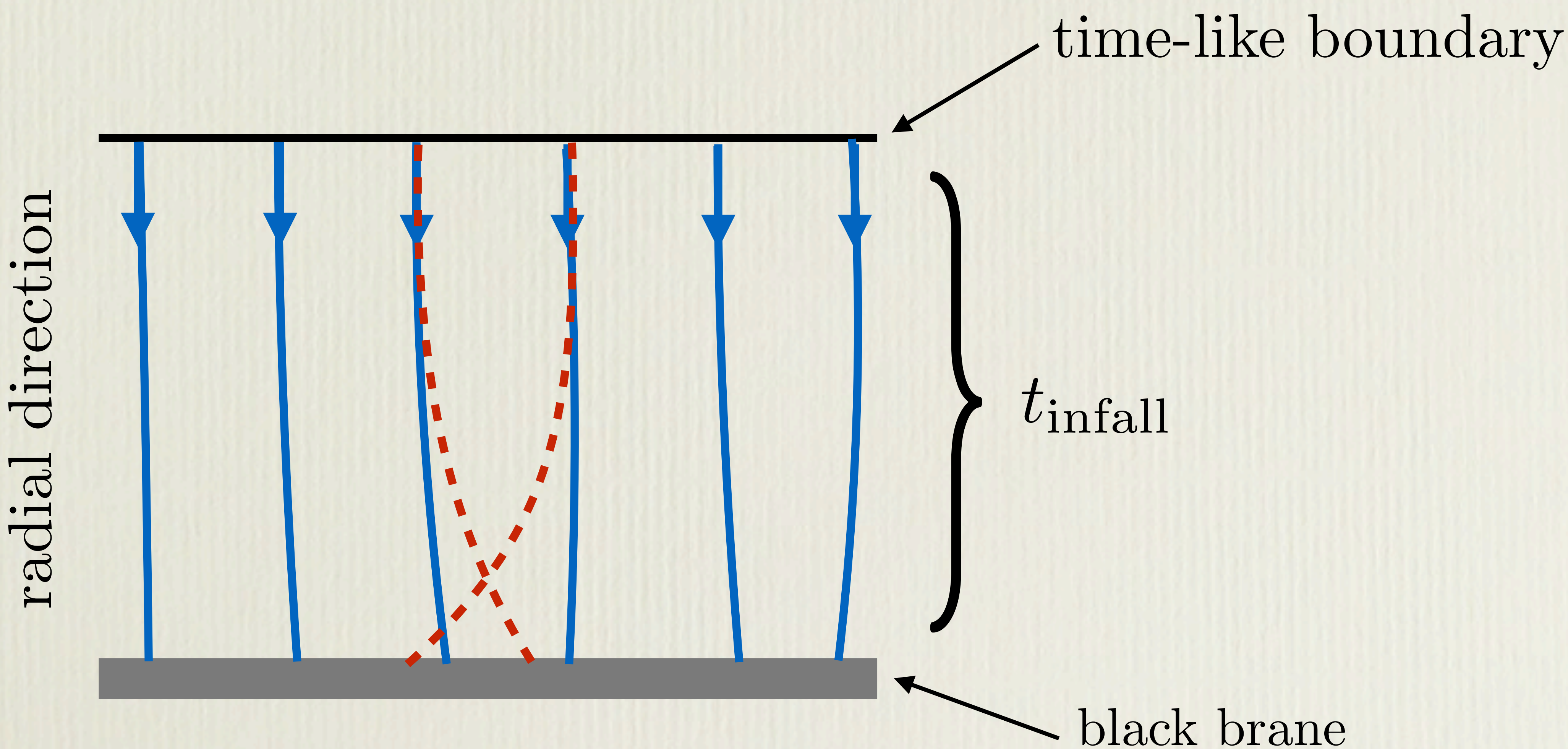

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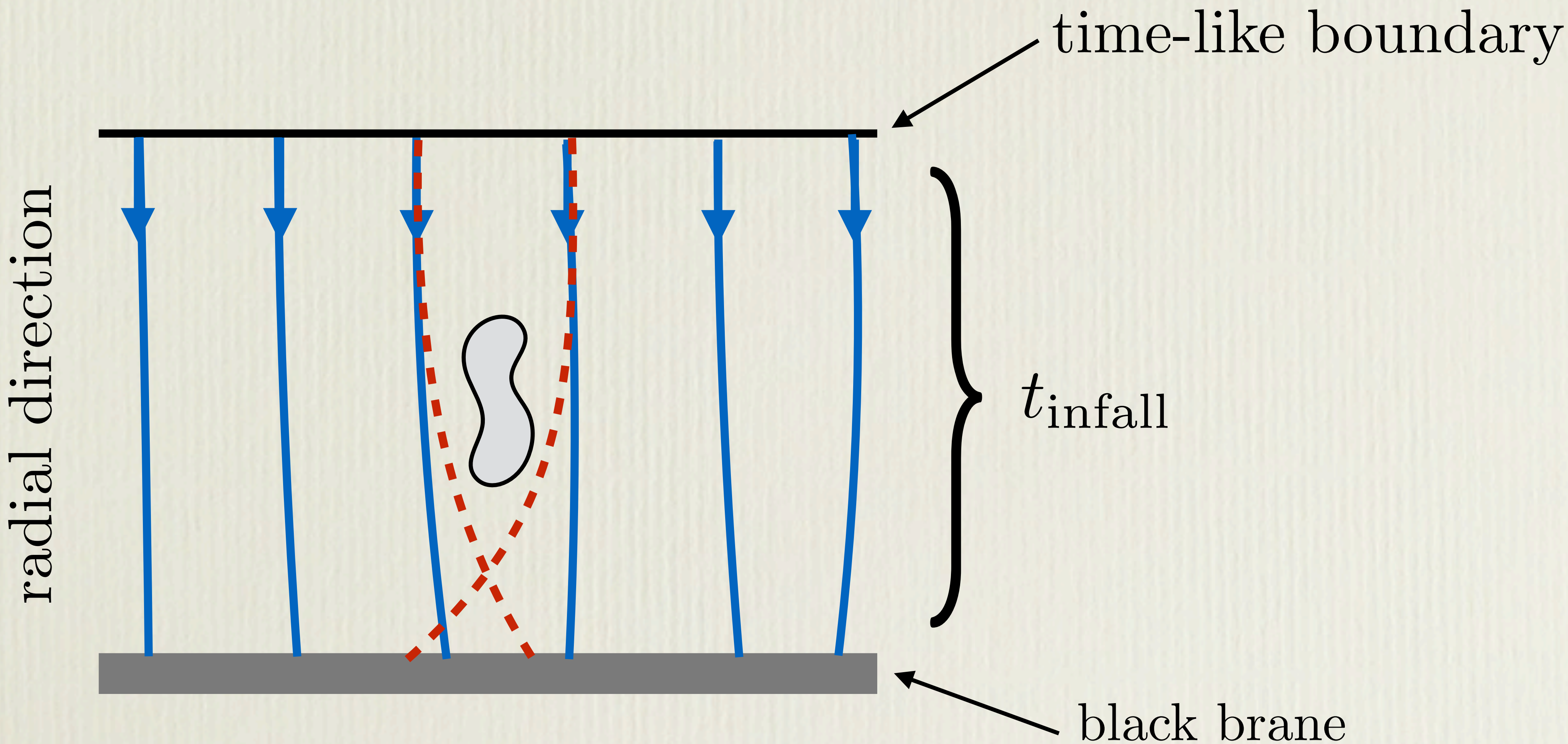

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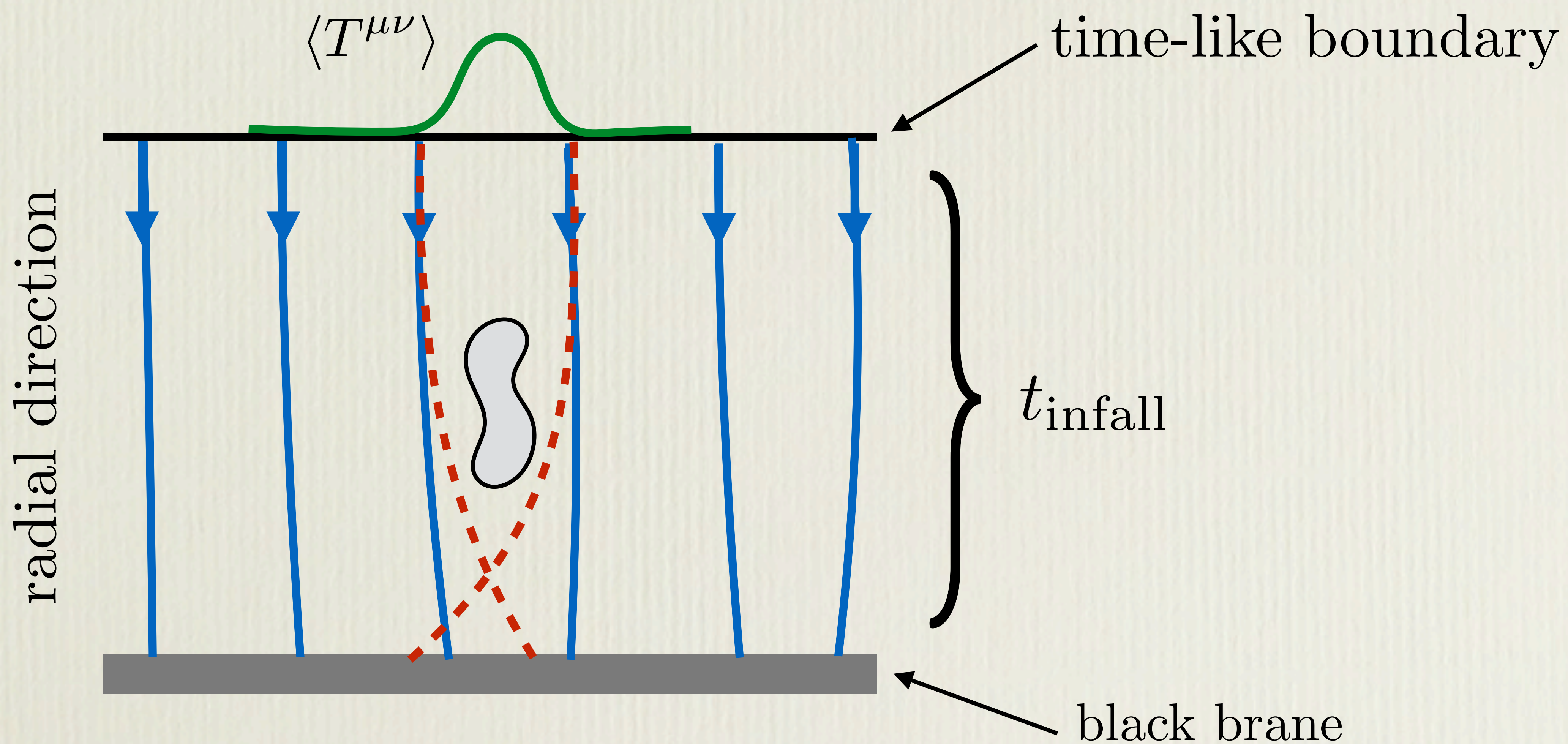

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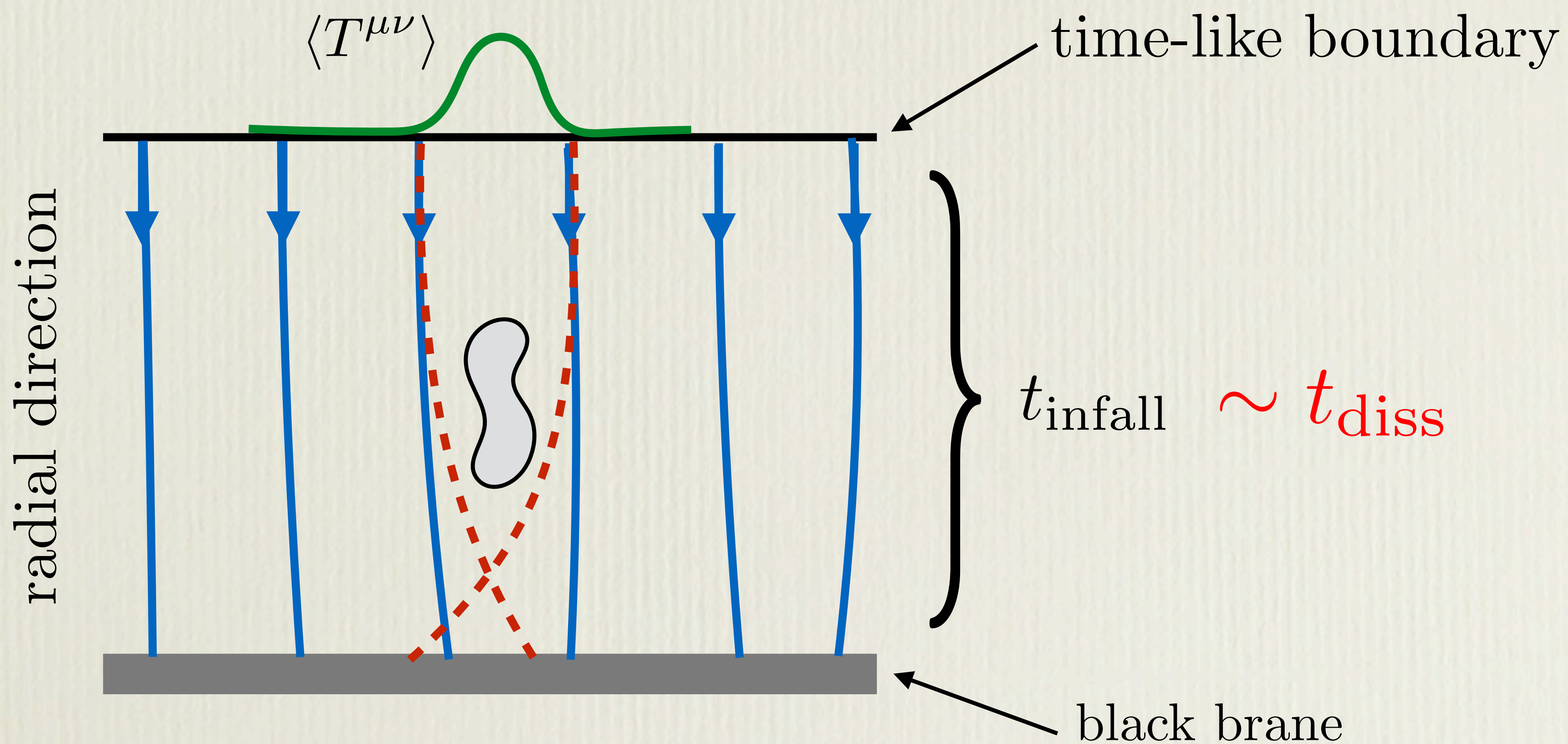

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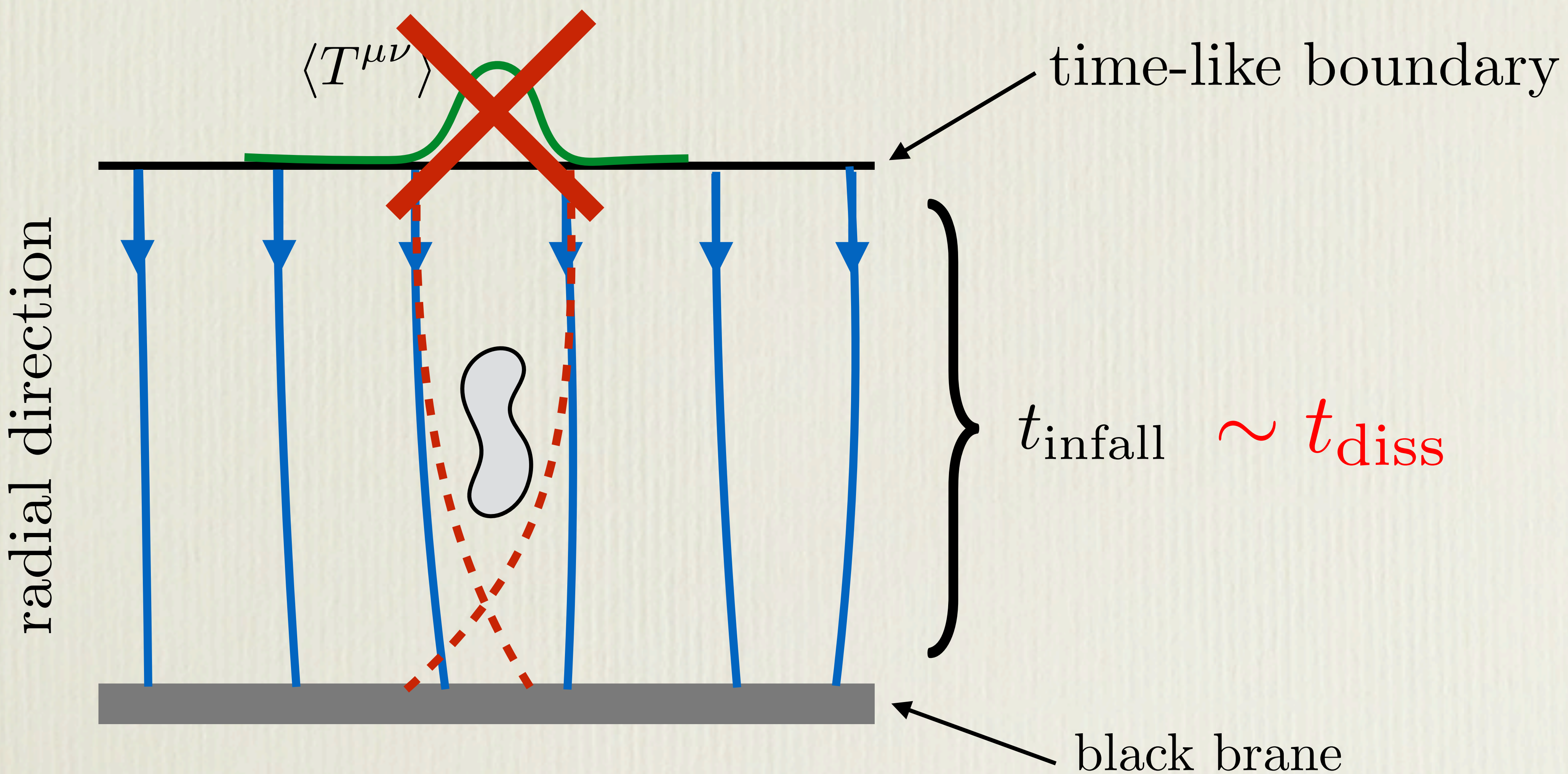

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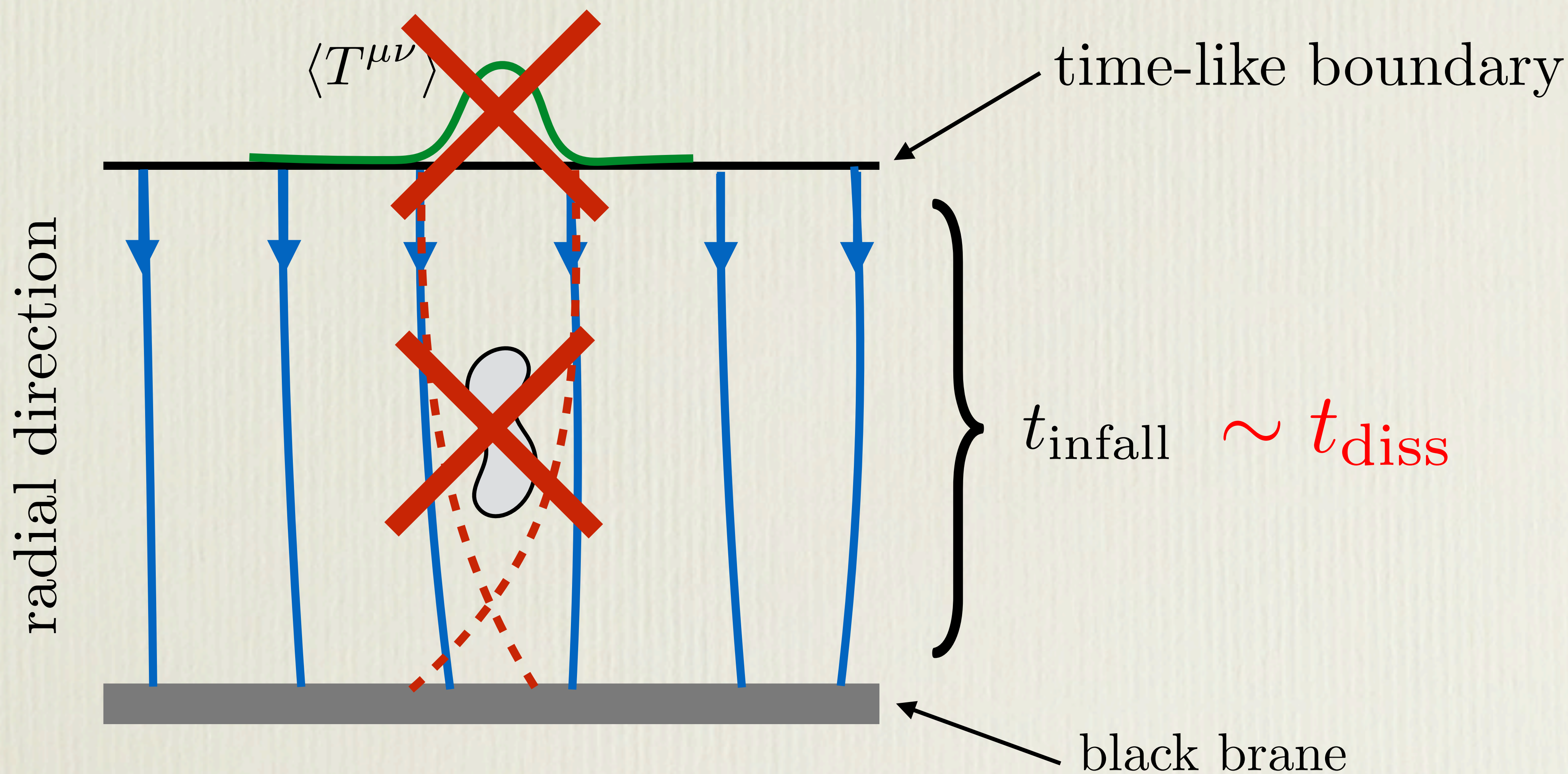
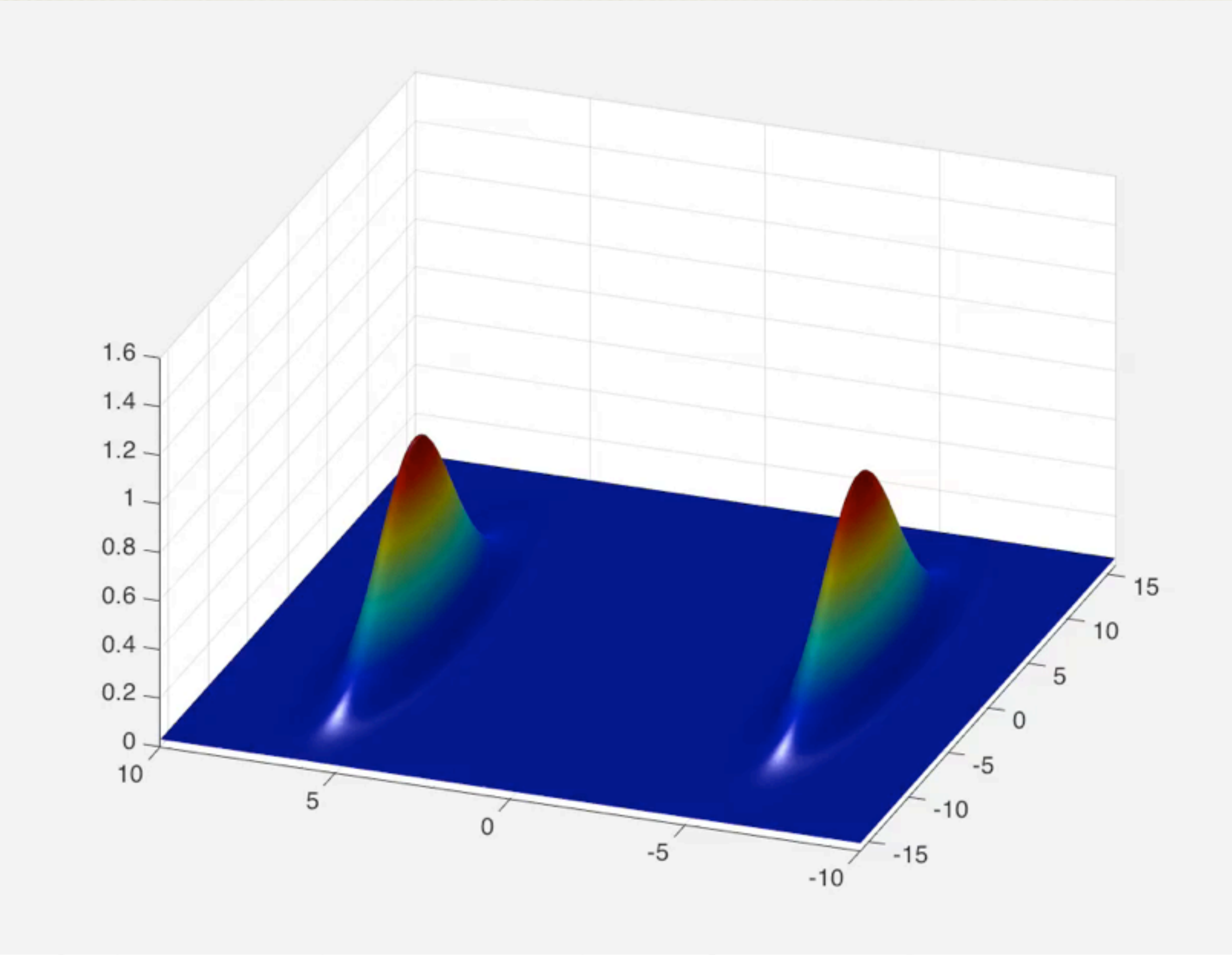


Off-center collisions

energy density

energy density



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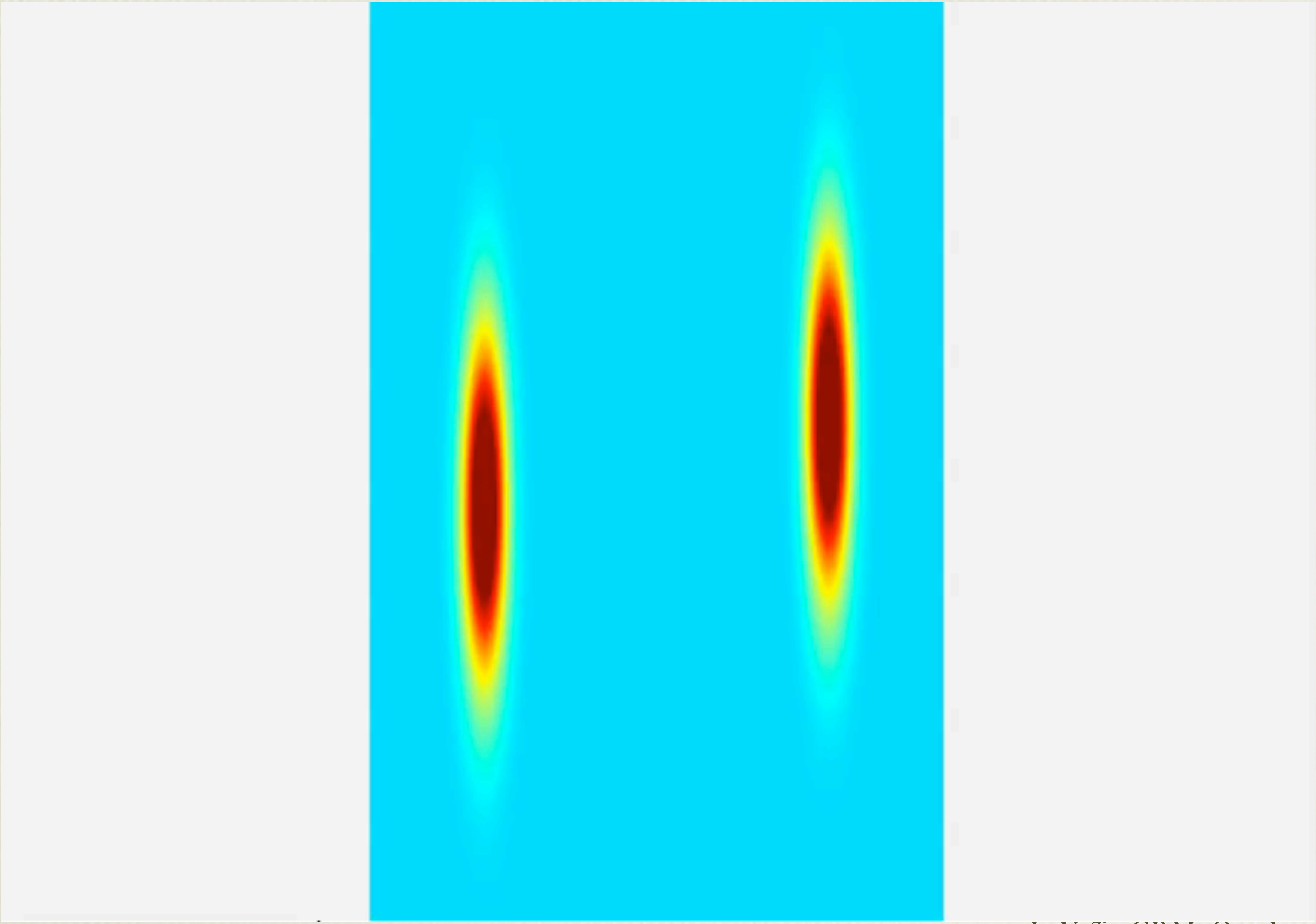
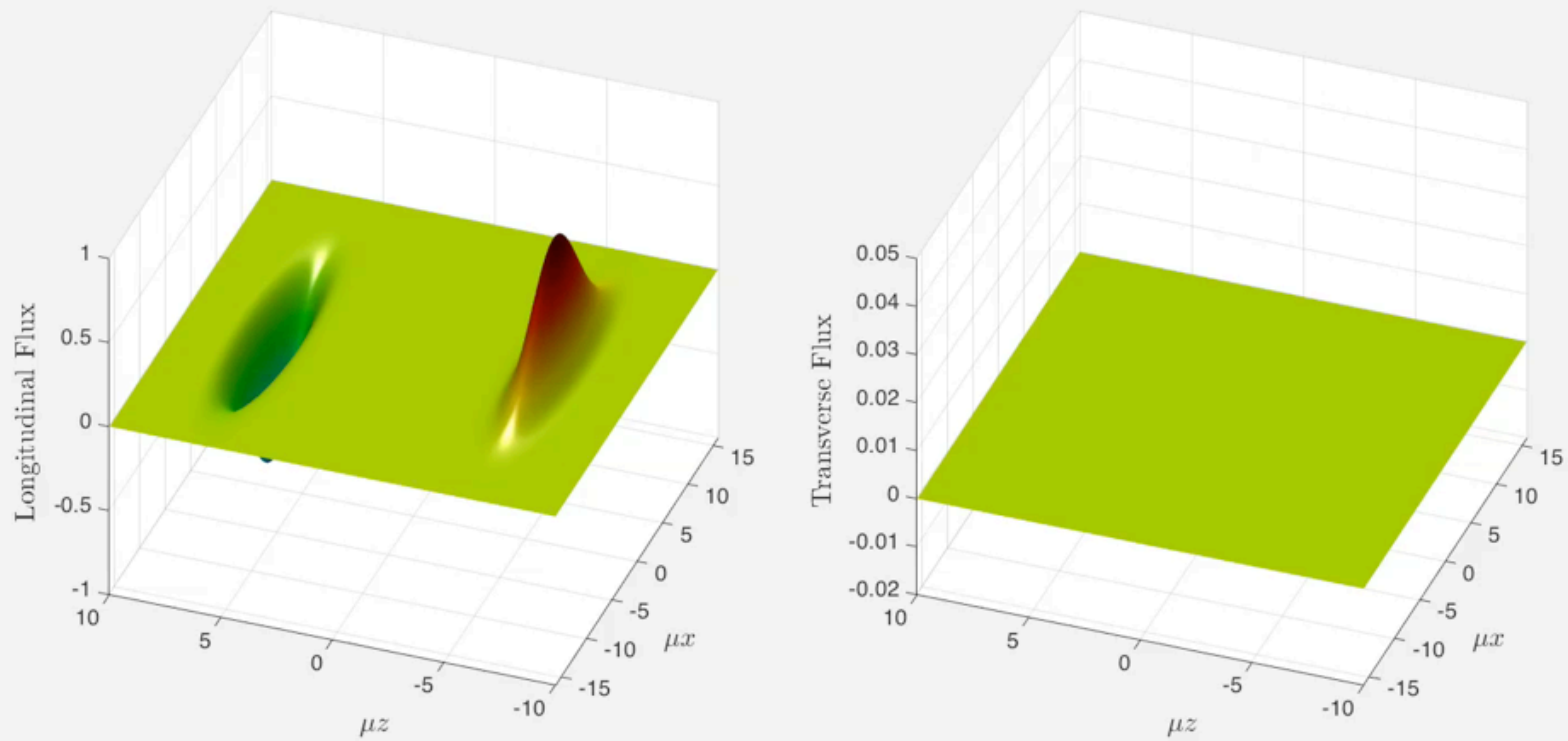


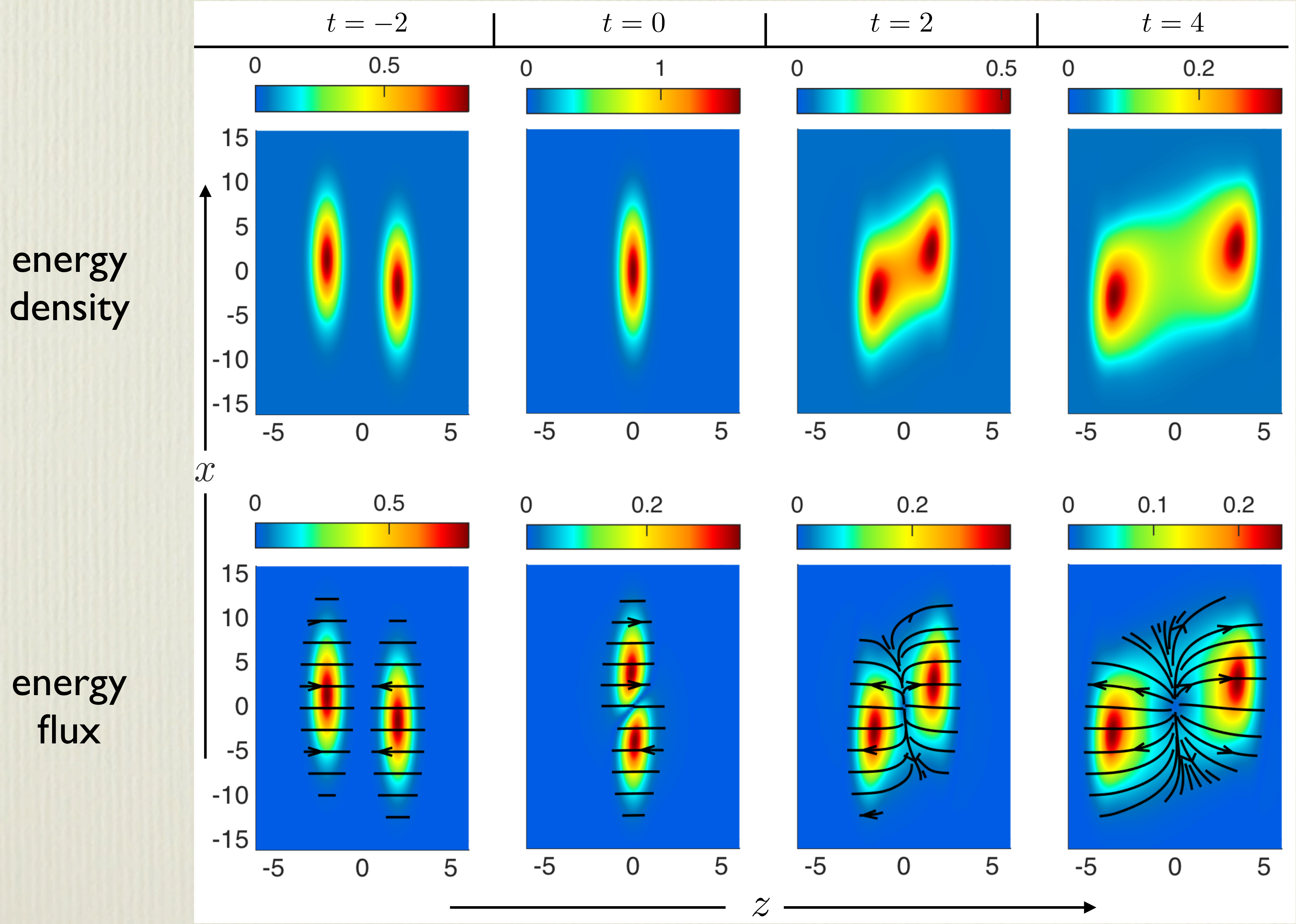
energy flux

energy flux

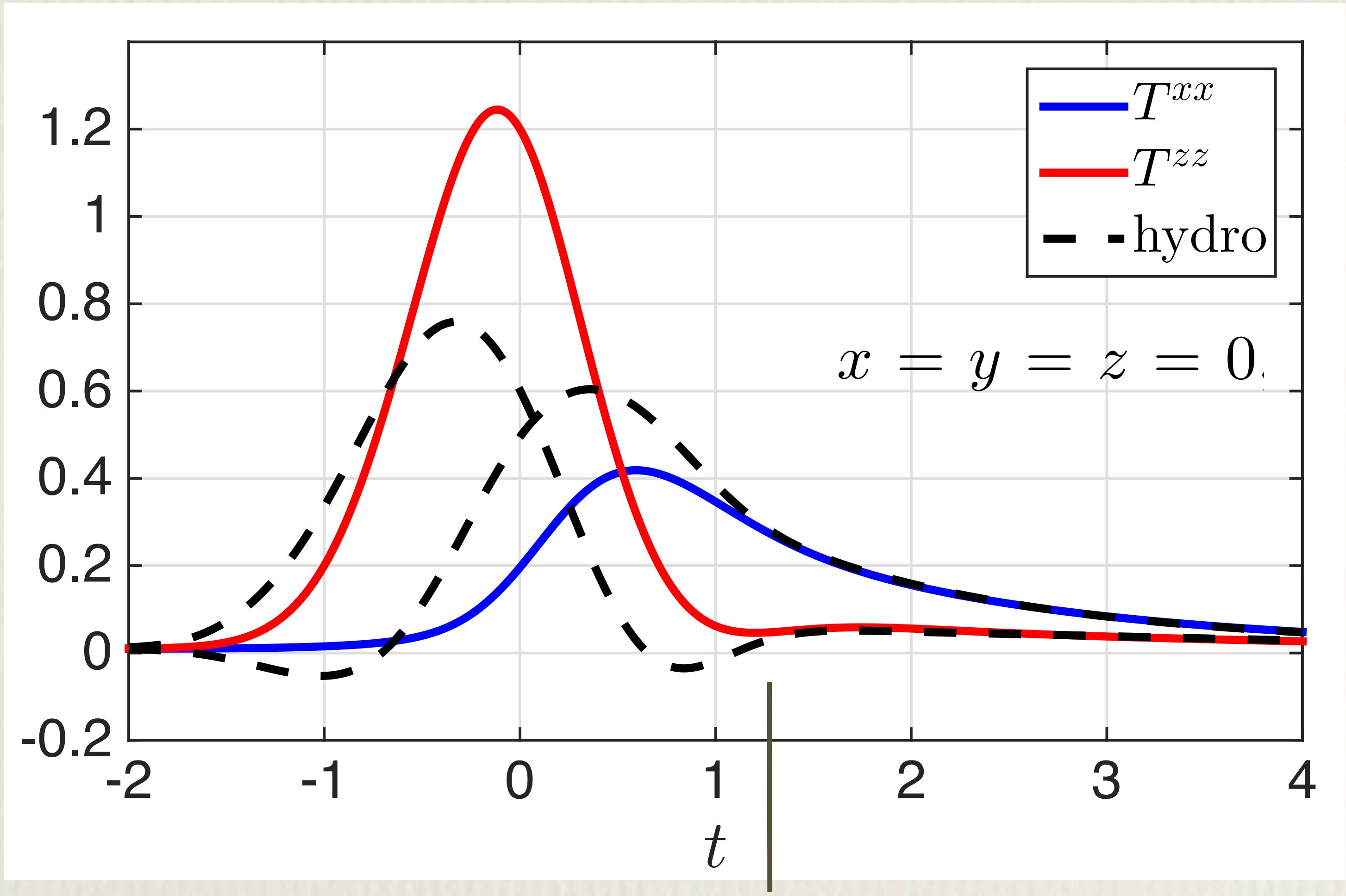


Longitudinal Flux
1
0.5
0
-0.5
-1
10
5
0
-5
-10
μz
15
10
5
0
-5
-10
-15
μx
Transverse Flux
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
10
5
0
-5
-10
μz
15
10
5
0
-5
-10
-15
μx

snapshots



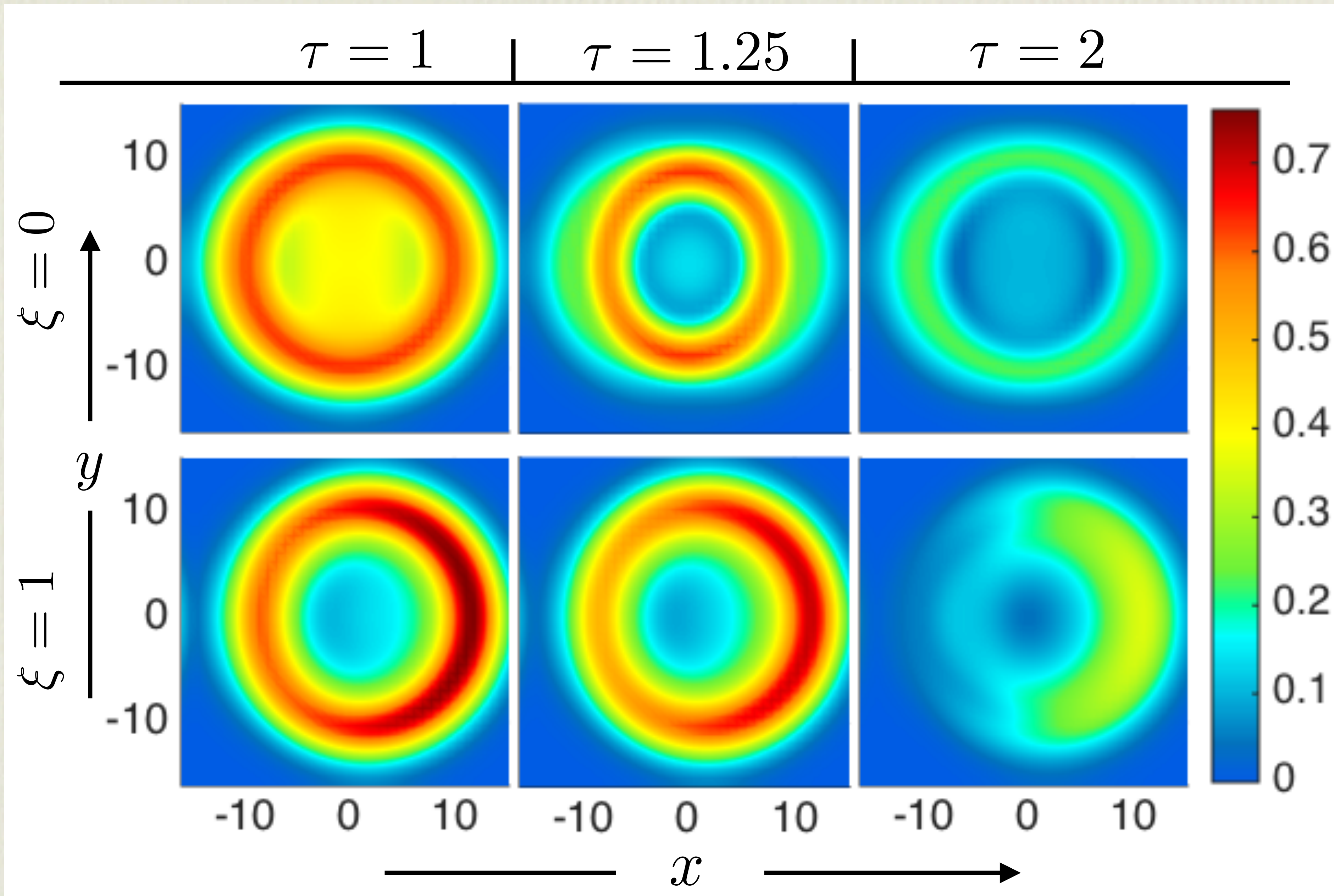
transverse & longitudinal pressure



$t_{\text{hydro}} \approx 1.25$

hydro onset ≈ 30% faster than for planar shocks

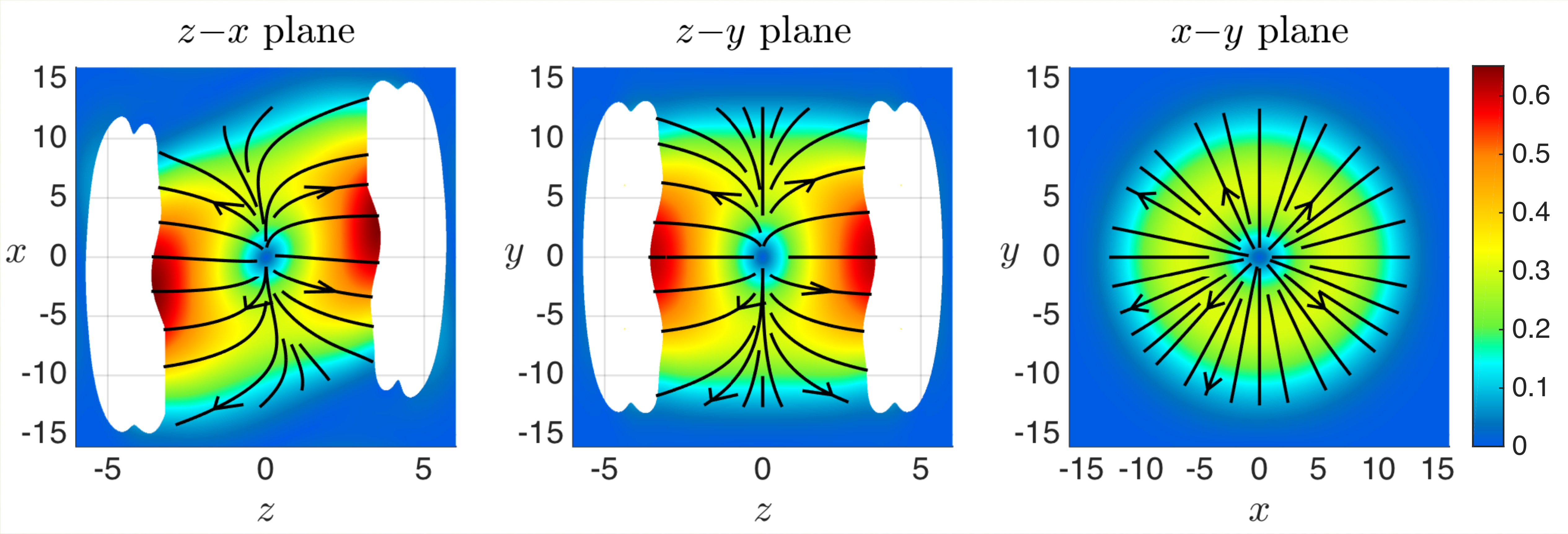
hydrodynamic residual



$\Delta \equiv (1/\bar{p})\sqrt{\Delta T_{\mu\nu}\Delta T^{\mu\nu}}$, $\Delta T^{\mu\nu} \equiv T^{\mu\nu} - T^{\mu\nu}_{\rm hydro}$ $\bar{p} \equiv \epsilon/3$

flow velocity

$t = 4$ non-hydro regions excised

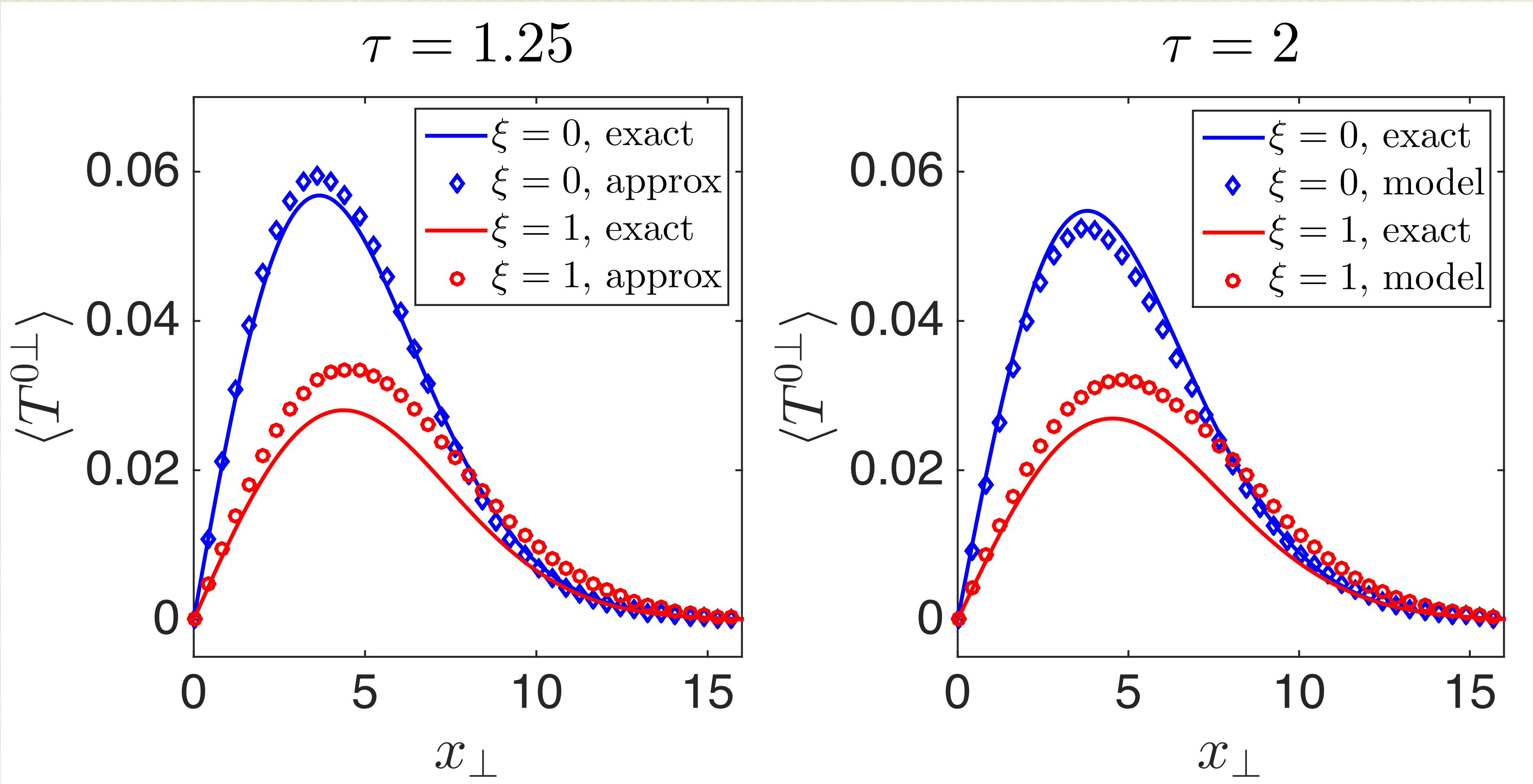


substantial radial flow:

$$v_\perp(x_\perp = 5) \approx 0.3$$

$$v_\parallel^{\max} \approx 0.64$$

radial flow



Vredevoogd & Pratt: "universal flow" model (assumes boost invariance & transverse rotational symmetry):

$$T^{0x} = -\tfrac{t}{2}\,\partial_x \epsilon$$
$$T^{0y} = -\tfrac{t}{2}\,\partial_y \epsilon$$

elliptic flow?

- no evident "almond" shape to fluid droplet
- transverse flow nearly symmetric
- negligible transverse pressure anisotropy: $\dfrac{|T_{xx} - T_{yy}|}{\frac{1}{2}(T_{xx} + T_{yy})} < 1\%$
- but:

elliptic flow?

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- but:
    - Gaussian choice of initial energy density profile
    - overlap function: $$\begin{aligned} \varepsilon_+(\vec{x})\,\varepsilon_-(\vec{x}) &\propto e^{-\frac{1}{2}(\mathbf{x}_\perp - \mathbf{b}/2)^2} e^{-\frac{1}{2}(\mathbf{x}_\perp + \mathbf{b}/2)^2} \\ &= e^{-(\mathbf{x}_\perp^2 + (\mathbf{b}/2)^2)} \end{aligned}$$

lessons

- successful proof-of-principle: holographic calculation of colliding "nuclei" without (over)simplifying symmetry assumptions
- numerical solution of 5D gravitational initial value problems feasible with desktop computing resources (and good methods)
- substantial radial flow develops very early
- faster hydro onset in non-planar collisions
- much more to do:
    - variation w. impact parameter, longitudinal thickness, transverse size
    - more realistic non-Gaussian energy density profile
    - fluctuations in initial profile
    - confining theories