Holographic QCD

(Review)

Shigeki Sugimoto (YITP, Kyoto Univ.)

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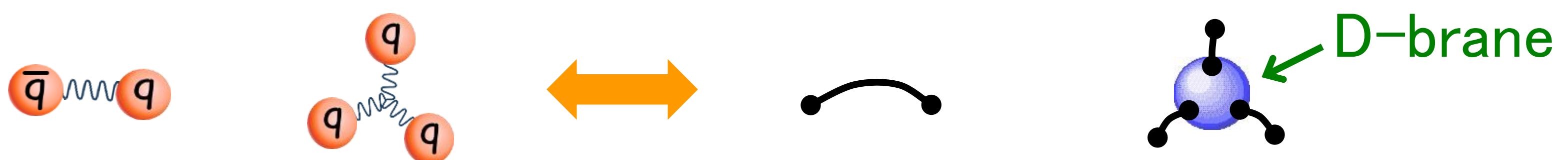
1 Introduction

Claim 1

Hadrons can be described by ***string theory***
without using quarks and gluons!

QCD ⟷ **dual !** (at low energy) ⟷ String theory (in a certain curved background)

“ ***Holographic QCD*** ”



meson baryon string D−brane with Nc strings

Claim 2

Effective theory of mesons (π, ρ, a1, …) is given by

5 dim U(N_f) YM–CS theory in a curved space–time

$$S_{5\text{dim}} \simeq S_{\text{YM}} + S_{\text{CS}}$$

$k(z) = 1 + z^2$

CS5–form

$$S_{\text{YM}} = \kappa \int d^4x dz \, \text{Tr}\left(\frac{1}{2} h(z) F_{\mu\nu}^2 + k(z) F_{\mu z}^2 \right) \qquad S_{\text{CS}} = \frac{N_c}{24\pi^2} \int_5 \omega_5(A)$$

$(\mu, \nu = 0 \sim 3)$

$\kappa \propto \lambda N_c$

$h(z) = (1 + z^2)^{-1/3}$

$(M_{\text{KK}} = 1 \text{ unit})$

- just one line
- only 2 parameters

$$\begin{cases} M_{\text{KK}} & \sim \text{cut off scale} \\ \lambda & \sim \text{bare coupling} \end{cases}$$

➡ a lot of predictions

cf) Traditional meson effective action

$$
\begin{aligned}
S_{\text{4dim}} &= \int d^4x \Bigg[\left.\begin{aligned}
&\frac{f_\pi^2}{4}\,\mathrm{tr}[D_\mu U^\dagger D^\mu U] \\
&+L_1\left(\mathrm{tr}[D_\mu U^\dagger D U]\right)^2 + L_2\,\mathrm{tr}[D_\mu U^\dagger D_\nu U]\mathrm{tr}[D^\mu U^\dagger D^\nu U] \\
&+L_3\,\mathrm{tr}[D_\mu U^\dagger D^\mu U D^\nu U^\dagger D^\nu U] \\
&-iL_9\,\mathrm{tr}[F^L_{\mu\nu} D^\mu U^\dagger D^\nu U^\dagger + F^R_{\mu\nu} D^\mu U^\dagger D^\nu U] + L_{10}\,\mathrm{tr}[U^\dagger F^L_{\mu\nu} U F^{R\,\mu\nu}]
\end{aligned}\right\}\ \text{pion} \\
&\qquad + \frac{1}{2}\mathrm{tr}F^v_{\mu\nu}F^{v\,\mu\nu} + m_\rho^2\,\mathrm{tr}[(v_\mu - g^{-1}\beta_\mu)^2]\Bigg] \quad \longleftarrow\ \rho\ \text{meson} \\
&\quad -\frac{N_c}{24\pi^2}\int_{\text{4dim}} \left[\begin{aligned}
&\mathrm{Tr}[(A_R dA_R + dA_R A_R + A_R^3)(U^{-1}A_L U + U^{-1}dU) - \text{p.c.}] \\
&+\mathrm{Tr}[dA_- dU^{-1} A_L U - \text{p.c.}] + \mathrm{Tr}[A_R(dU^{-1}U)^3 - \text{p.c.}] \\
&+\frac{1}{2}\mathrm{Tr}[(A_R dU^{-1}U)^2 - \text{p.c.}] + \mathrm{Tr}[U A_R U^{-1} A_L dU dU^{-1} - \text{p.c.}] \\
&-\mathrm{Tr}[A_R dU^{-1} U A_R U^{-1} A_L U - \text{p.c.}] + \frac{1}{2}\mathrm{Tr}[(A_R U^{-1} A_L U)^2]
\end{aligned}
\quad
\begin{aligned}
&+C_1\,\mathrm{tr}[\alpha_L^3\alpha_R - \alpha_R^3\alpha_L] + C_2\,\mathrm{tr}[\alpha_L\alpha_R\alpha_L\alpha_R] \\
&+C_3\,\mathrm{tr}[F^v\alpha_L\alpha_R\alpha_L\alpha_R] \\
&+C_4\,\mathrm{tr}[F^L(\alpha_L\alpha_R - \alpha_R\alpha_L) - F^R(\alpha_R\alpha_L - \alpha_L\alpha_R)]
\end{aligned}\right] \\
&\quad -\frac{N_c}{240\pi^2}\int_{\text{5dim}} \mathrm{Tr}(g dg^{-1})^5 \\
&\quad +(\text{many more terms!!})
\end{aligned}
$$

WZW term

$$
\begin{aligned}
D_\mu U &= \partial_\mu U - iA^L_\mu U + iUA^R_\mu \\
U &= \xi_L^\dagger \xi_R \\
\beta_\mu &= \frac{1}{2i}(\partial_\mu\xi_R\cdot\xi_R^\dagger + \partial_\mu\xi_L\cdot\xi_L^\dagger) \\
D_\mu\xi_L &= \partial_\mu\xi_L - igv_\mu\xi_L + i\xi_L A^L_\mu \\
D_\mu\xi_R &= \partial_\mu\xi_R - igv_\mu\xi_R + i\xi_R A^R_\mu \\
\alpha_{L\mu} &= \frac{1}{i}D_\mu\xi_L\cdot\xi_L^\dagger,\quad \alpha_{R\mu} = \frac{1}{i}D_\mu\xi_R\cdot\xi_R^\dagger
\end{aligned}
$$

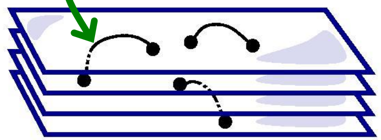
- very complicated
- lots of parameters

- This action can be obtained from the previous 5 dim action!
- Predicted masses and couplings are in reasonably good agreement with the experimental data!

★ Key idea : Gauge / string duality

[Maldacena 1997, …]

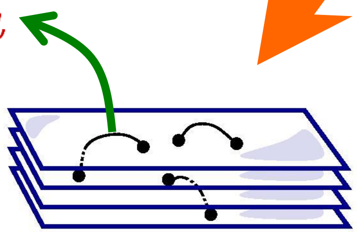
open string



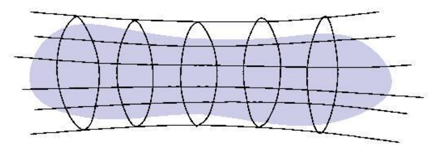
D-brane

gauge field

A_μ



corresponding
curved background



Gauge theory on the D-brane	$\longleftrightarrow$ dual !	String theory in a curved background
4 dim		**10 dim**

Note: SUSY, conformal sym. are not essential in this idea.

★ What is nice?

- String theory and gauge/string duality can be tested by experiments.

- New techniques and understanding in hadron physics.

- "Derivation" of old models of hadrons

  String model
  Skyrme model
  Vector meson dominance model
  Hidden local symmetry model
  Gell-Mann Sharp Wagner model

Plan of Talk

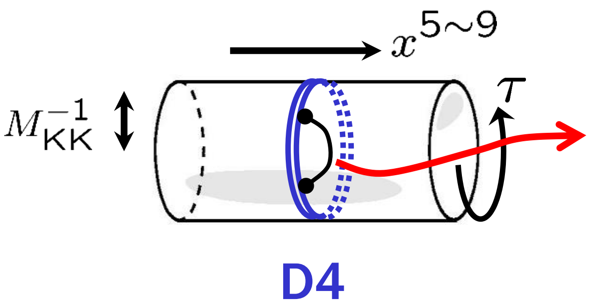
✓ 1. Introduction
2. Quick review of the model
3. Mesons
4. Baryons
5. Conclusion and discussion

② Quick review of the model

★ QCD realized in string theory

- D4-brane $\times N_c$ on $S^1 \ni \tau$ with $\psi(x^\mu, \tau + 2\pi M_{KK}^{-1}) = -\psi(x^\mu, \tau)$ $\mu = 0 \sim 3$

[Witten 1998]

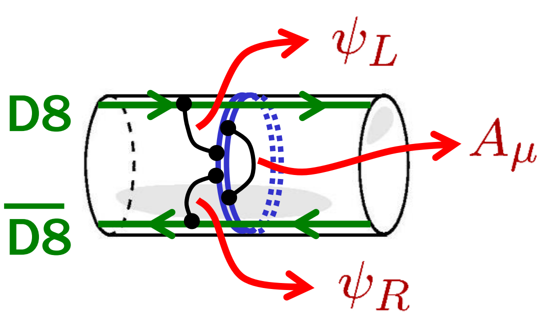


A_μ, $\underbrace{A_\tau,\ \phi \times 5,\ \psi \times 4}$ ~~SUSY~~

acquire mass $\sim M_{KK}$

4 dim pure Yang-Mills (at low energy)

- The above D4 system + D8-$\overline{\text{D8}}$ pair $\times N_f$ [Sakai-S.S. 2004]



U(Nc) QCD with Nf massless quarks

(at low energy)

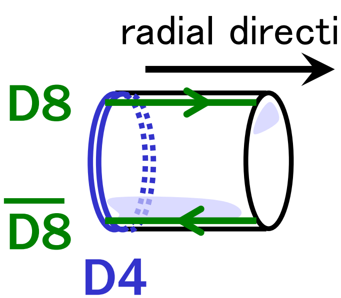
★ Holographic description

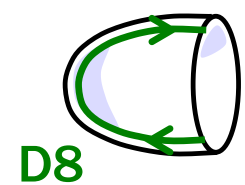
[Sakai-S.S. 2004]

- Here we assume $N_c \gg N_f$ and use "probe approximation". [Karch-Katz 2002]

$\left\{ \begin{array}{l} \text{• D4–branes are replaced with the corresponding background.} \\ \text{• D8–}\overline{\text{D8}}\text{ pairs are treated as probes.} \end{array} \right.$

D4-brane on S^1 + N_f D8-$\overline{\text{D8}}$ pairs → **String theory in the D4 background + N_f D8-branes**





"Holographic QCD"

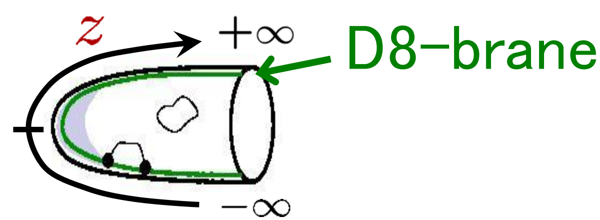
$\lambda = g_{\rm YM}^2 N_c \longleftrightarrow 1/\alpha'$ $\quad (M_{\rm KK} = 1$ unit$)$

$\lambda^{3/2}/N_c \longleftrightarrow g_s$

★ Can we find hadrons?

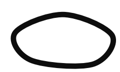
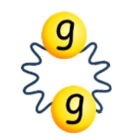
The topology of the D4 background is

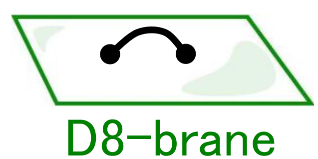
$$\underset{x^\mu}{\mathbf{R}^{1,3}} \times \underset{(y,z)}{\mathbf{R}^2} \times S^4$$



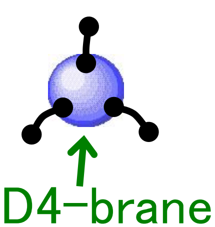
D8−branes are extended along $(x^\mu, z) \times S^4$

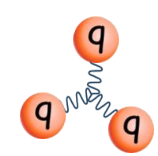
particles in $\mathrm{R}^{1,3}$:

- Closed strings  → glueballs 

- Open strings on D8 

→ mesons 

- D4 wrapped on S^4 

→ baryons 

③ Mesons

★ Meson effective theory

- We have N_f D8−branes extended along $(x^\mu, z) \times S^4$

  ➡ **open string eff. theory is a 9 dim $U(N_f)$ gauge theory**

- We only consider the states invariant under $SO(5) \curvearrowright S^4$
  (SO(5) non−inv. states are unwanted artifacts of the model)

  ➡ **reduced to 5 dim**

5 dim $U(N_f)$ YM−CS theory in a curved space−time

$$S_{\text{5dim}} \simeq S_{\text{YM}} + S_{\text{CS}}$$

$k(z) = 1 + z^2$

CS5−form

$$S_{\text{YM}} = \kappa \int d^4x dz \, \text{Tr} \left(\frac{1}{2} h(z) F_{\mu\nu}^2 + k(z) F_{\mu z}^2 \right) \qquad S_{\text{CS}} = \frac{N_c}{24\pi^2} \int_5 \omega_5(A)$$

$\kappa \propto \lambda N_c$ $\qquad h(z) = (1 + z^2)^{-1/3}$ $\qquad (M_{\text{KK}} = 1 \text{ unit})$

★ 5 dim YM-CS theory = 4 dim meson theory

$$A_\mu(x^\mu, z) = \sum_{n\geq 1} B_\mu^{(n)}(x^\mu)\psi_n(z)$$

$$A_z(x^\mu, z) = \sum_{n\geq 0} \varphi^{(n)}(x^\mu)\phi_n(z)$$

complete sets

Chosen to diagonalize kinetic & mass terms of $B_\mu^{(n)}, \varphi^{(n)}$

$\varphi^{(0)} \sim$ pion $\quad B_\mu^{(1)} \sim \rho$ meson $\quad B_\mu^{(2)} \sim a_1$ meson $\quad \cdots$

$$\boxed{S_{5\text{dim}}(A) \;=\; S_{4\text{dim}}(\pi, \rho, a_1, \rho', a_1', \cdots)}$$

- Reproduces old phenomenological models
  - Vector meson dominance [Sakurai 1960, Gell-Mann -Zachariasen 1961, …]
  - Gell-Mann Sharp Wagner model [Gell-Mann -Sharp-Wagner 1962]
  - Hidden local symmetry [Bando-Kugo-Uehara-Yamawaki-Yanagida 1985]
  - Skyrme model [Skyrme 1961]
- Masses and couplings roughly agree with experiments.

★ Mesons masses

- To diagonalize Kinetic & mass terms, we choose:

$$\left[\begin{array}{ll} -K^{1/3}\partial_z(K\partial_z\psi_n) = \lambda_n\psi_n & \phi_n(z) = \partial_z\psi_n(z) \quad (n \geq 1) \\ \kappa\int dz\, K^{-1/3}\psi_n\psi_m = \delta_{nm} & \phi_0(z) = \dfrac{c}{K(z)} \end{array}\right]$$

Then, we obtain

$F_{\mu\nu}^{(n)} \equiv \partial_\mu B_\nu^{(n)} - \partial_\nu B_\mu^{(n)}$

eaten

massless pion

$$S_{D8}^{DBI} \sim \sum_{n\geq 1}\int d^4x \,\mathrm{Tr}\left[\underbrace{\frac{1}{2}F_{\mu\nu}^{(n)2} + \lambda_n M_{KK}^2\left(B_\mu^{(n)} - \partial_\mu\varphi^{(n)}\right)^2}_{\text{massive vector meson}}\right] + \int d^4x \,\mathrm{Tr}\,\partial_\mu\varphi^{(0)2}$$

$$+\,(\text{interaction terms})$$

- **Mass**

mass	ρ	a_1	ρ'
exp.(MeV)	776	1230	1465(?)
our model	[776]	1189	1607
ratio	[1]	1.03	0.911

(∃ other candidates 1570? 1720?
see my talk on Monday)

input $\quad (M_{KK} \simeq 949\ \text{MeV})$

★ Coupling constants

coupling		fitting m_ρ and f_π	experiment
f_π	$1.13 \cdot \kappa^{1/2} M_{\rm KK}$	[92.4 MeV]	92.4 MeV
L_1	$0.0785 \cdot \kappa$	0.584×10^{-3}	$(0.1 \sim 0.7) \times 10^{-3}$
L_2	$0.157 \cdot \kappa$	1.17×10^{-3}	$(1.1 \sim 1.7) \times 10^{-3}$
L_3	$-0.471 \cdot \kappa$	-3.51×10^{-3}	$-(2.4 \sim 4.6) \times 10^{-3}$
L_9	$1.17 \cdot \kappa$	8.74×10^{-3}	$(6.2 \sim 7.6) \times 10^{-3}$
L_{10}	$-1.17 \cdot \kappa$	-8.74×10^{-3}	$-(4.8 \sim 6.3) \times 10^{-3}$
$g_{\rho\pi\pi}$	$0.415 \cdot \kappa^{-1/2}$	4.81	5.99
g_ρ	$2.11 \cdot \kappa^{1/2} M_{\rm KK}^2$	0.164 GeV^2	0.121 GeV^2
$g_{a_1\rho\pi}$	$0.421 \cdot \kappa^{-1/2} M_{\rm KK}$	4.63 GeV	$2.8 \sim 4.2$ GeV

- Pion decay constant

$$S_{\rm 5dim} \sim \int d^4x \left[\frac{{f_\pi}^2}{4} {\rm Tr}(U^{-1}\partial_\mu U)^2 + \frac{1}{32 e_S^2} {\rm Tr}[U^{-1}\partial_\mu U, U^{-1}\partial_\nu U]^2 \right] + \cdots$$

$$U(x) = e^{2i\pi(x)/f_\pi}$$

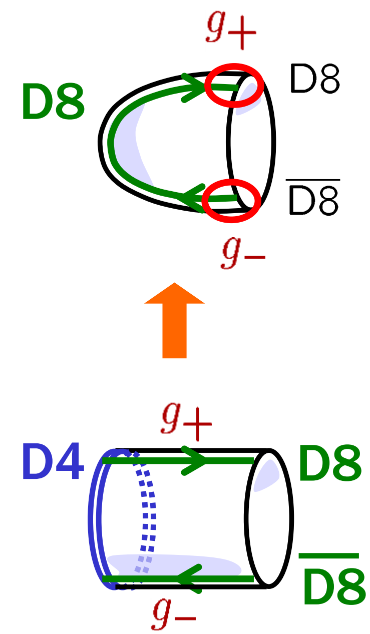
- Couplings including ρ meson

$$S_{\rm 5dim} \sim \int dx^4 \left[\cdots + 2 g_{\rho\pi\pi} {\rm Tr}(\rho_\mu [\pi, \partial^\mu \pi]) - 2 g_\rho {\rm Tr}(\rho_\mu Q) A_\mu^{\rm em} + \cdots \right]$$

$$\rho_\mu = B_\mu^{(1)} \qquad Q = \frac{1}{3} \begin{pmatrix} 2 & \\ & -1 \end{pmatrix}$$

for $N_f = 2$

★ Chiral symmetry



The gauge transformation at $z = \pm\infty$ corresponds to the chiral symmetry

$$\boxed{(g_+, g_-) \in U(N_f)_L \times U(N_f)_R}$$

$$\left(\, g_\pm \equiv \lim_{z \to \pm\infty} g(x^\mu, z) \, \right)$$

- Define $\boxed{U(x^\mu) \equiv P \exp\left\{ - \int_{-\infty}^{\infty} dz' A_z(x^\mu, z') \right\}}$

➡ transforms as

$$U(x^\mu) \to g_+ U(x^\mu) g_-^{-1}$$

➡ interpreted as the pion field in chiral Lagrangian

$$U(x^\mu) = e^{2i\pi(x^\mu)/f_\pi}$$

★ Pion effective action

Inserting the mode expansion into the 5 dim action, we obtain

$$S_{\rm D8}^{\rm DBI} \simeq \int d^4x \left[\frac{f_\pi^2}{4} {\rm Tr}(U^{-1}\partial_\mu U)^2 + \frac{1}{32e_S^2} {\rm Tr}[U^{-1}\partial_\mu U, U^{-1}\partial_\nu U]^2 \right] + \cdots$$

$$f_\pi^2 = \frac{4\kappa}{\pi} M_{\rm KK}^2 \qquad e_S^{-2} \simeq 2.51 \cdot \kappa$$

This is the Skyrme model action.

- Note: $\frac{1}{32e_S^2} {\rm Tr}[U^{-1}\partial_\mu U, U^{-1}\partial_\nu U]^2 \;=\; L_1 P_1 + L_2 P_2 + L_3 P_3$ (for $N_f = 3$)

$$\left[\begin{array}{ll} P_1 = {\rm Tr}(\partial_\mu U^{-1}\partial^\mu U)^2 & L_1 = 1/(32e_S^2) \\ P_2 = {\rm Tr}(\partial_\mu U^{-1}\partial_\nu U){\rm Tr}(\partial^\mu U^{-1}\partial^\nu U) & L_2 = 1/(16e_S^2) \\ P_3 = {\rm Tr}(\partial_\mu U^{-1}\partial^\mu U\,\partial_\nu U^{-1}\partial^\nu U) & L_3 = -3/(16e_S^2) \end{array} \right]$$

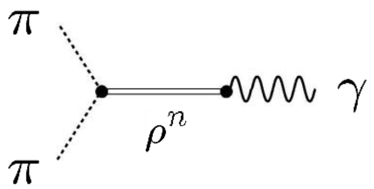
- If we fix $M_{\rm KK}$ & κ to fit m_ρ & f_π, we obtain

	L_1	L_2	L_3
exp.($\times 10^{-3}$)	0.4 ± 0.3	1.4 ± 0.3	-3.5 ± 1.1
our model	0.584	1.17	-3.51

★ *Pion form factor*

[⇒ See talk by Boschi-Filho]

- pion form factor is computed as



$$F_\pi(k^2) = \sum_{n\geq 1} \frac{g_{\rho^n} g_{\rho^n\pi\pi}}{k^2 + m_{\rho^n}^2}$$

"*vector meson dominance*"

[Sakurai 1960, Gell-Mann –Zachariasen 1961, ...]

- charge radius

$$F_\pi(k^2) \simeq 1 - \frac{1}{6}\langle r^2\rangle^{\pi^\pm} k^2 + \cdots$$

$$\Rightarrow \quad \langle r^2\rangle^{\pi^\pm} = 6 \sum_{n\geq 1} \frac{g_{\rho^n} g_{\rho^n\pi\pi}}{m_{\rho^n}^4} \simeq 11.0\, M_{\mathrm{KK}}^{-2}$$

If we fix M_{KK} by fitting the ρ meson mass, we obtain

$$\langle r^2\rangle^{\pi^\pm} \simeq (0.690\ \mathrm{fm})^2 \qquad \text{(our model)}$$
$$\simeq (0.672\ \mathrm{fm})^2 \qquad \text{(experiment)}$$

★ Couplings from CS-term

Inserting the mode exp. into the CS–term, we obtain

$$S_{\mathrm{D8}}^{\mathrm{CS}} \simeq -\frac{N_c\, i}{4\pi^2 f_\pi^2}\int_4 \mathrm{Tr}\left[\pi\, d\rho^n d\rho^m c_{nm} + \cdots\right]$$

$$\rho_\mu^n = B_\mu^{(2n-1)}$$

$$c_{nm} = \frac{1}{\pi}\int dz\, K^{-1}\psi_{2n-1}\psi_{2m-1}$$

➡ π-ω-ρ coupling

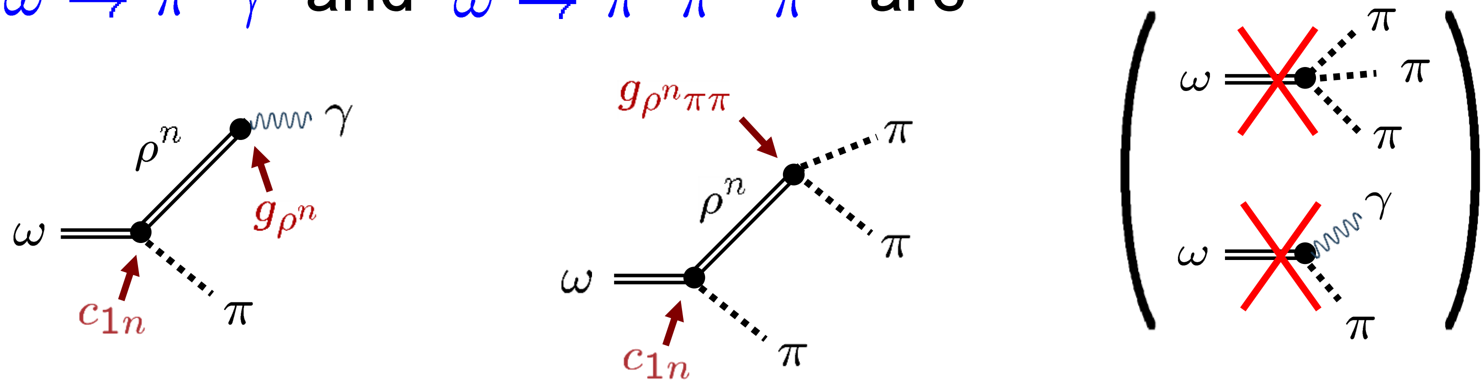
$$\rho = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{1}{\sqrt{2}}(\omega+\rho^0) & \rho^+ \\ \rho^- & \frac{1}{\sqrt{2}}(\omega-\rho^0) \end{pmatrix}$$

Moreover, one can show

- Complete vector meson dominance
- Terms with more than one pion field vanish.

★ ω meson decay ($\omega \to \pi^0\gamma$ and $\omega \to \pi^0\pi^+\pi^-$)

- Our model predicts that the relevant diagrams for $\omega \to \pi^0\gamma$ and $\omega \to \pi^0\pi^+\pi^-$ are



➡ Exactly the same as the ***GSW model*** !

[Gell-Mann -Sharp-Wagner 1962]

- Furthermore, we find

$$\Gamma(\omega \to \pi^0\gamma) = \frac{N_c^2}{3}\frac{\alpha}{64\pi^4 f_\pi^2}\left(\sum_{n=1}^{\infty}\frac{c_{1n}\, g_{\rho^n}}{m_{\rho^n}^2}\right)^2 |\mathbf{p}_\pi|^3 = \frac{N_c^2}{3}\frac{\alpha}{64\pi^4 f_\pi^2}\, {g_{\rho\pi\pi}}^2 |\mathbf{p}_\pi|^3$$

➡ reproduces the proposal given by Fujiwara et al !

[Fujiwara-Kugo-Terao-Uehara-Yamawaki 1985]

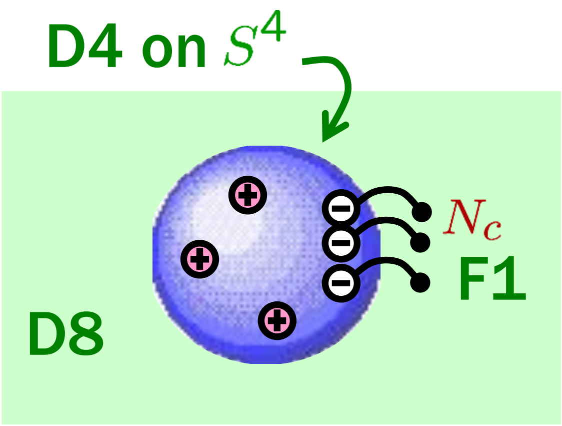
4 Baryons

★ Baryon as wrapped D4-brane

[Witten, Gross-Ooguri 1998]

- Consider a D4–brane wrapped on the S^4

RR flux $\frac{1}{2\pi}\int_{S^4} dC_3 = N_c$ induces N_c units of ele charge on the D4



➡ N_c **F-strings should be attached to cancel this charge**

➡ **Bound state of N_c quarks**

➡ **Baryon**

Baryon $\simeq$ D4-brane wrapped on the S^4

Baryon mass ($\propto$ vol. of S^4) is generated by the geometry!

cf) Skyrme model

[Skyrme 1961]

$$S = \int d^4x \left[\frac{f_\pi^2}{4} \mathrm{Tr}(U^{-1}\partial_\mu U)^2 + \frac{1}{32e_S^2} \mathrm{Tr}[U^{-1}\partial_\mu U, U^{-1}\partial_\nu U]^2 \right]$$

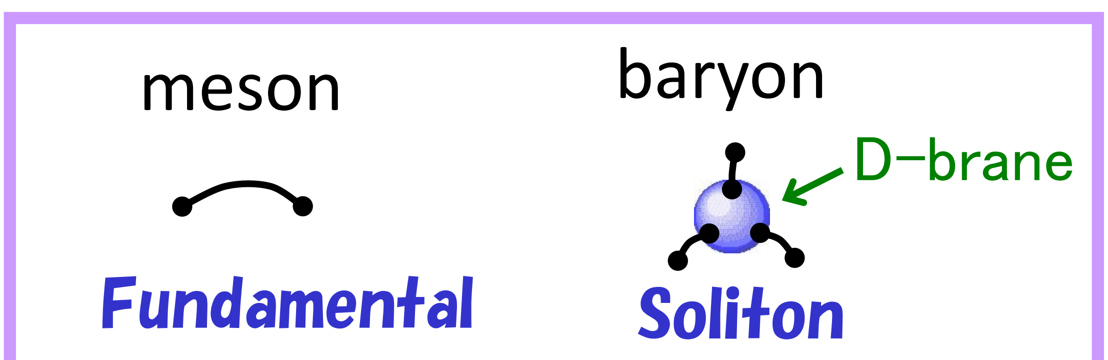
$U(x^\mu)$: U(N_f)-valued ➡ classified by $\pi_3(U(N_f)) \simeq \boldsymbol{Z}$

winding number : $N_B = \frac{1}{24\pi^2} \int_{\boldsymbol{R}^3} \mathrm{Tr}\left((U^{-1}dU)^3\right)$ ➡ baryon number

Solitons with $N_B \neq 0$ are called “Skyrmions”

$$\boxed{\text{Baryon = Soliton in meson theory}}$$

- Our description of baryons is a generalization of this idea!

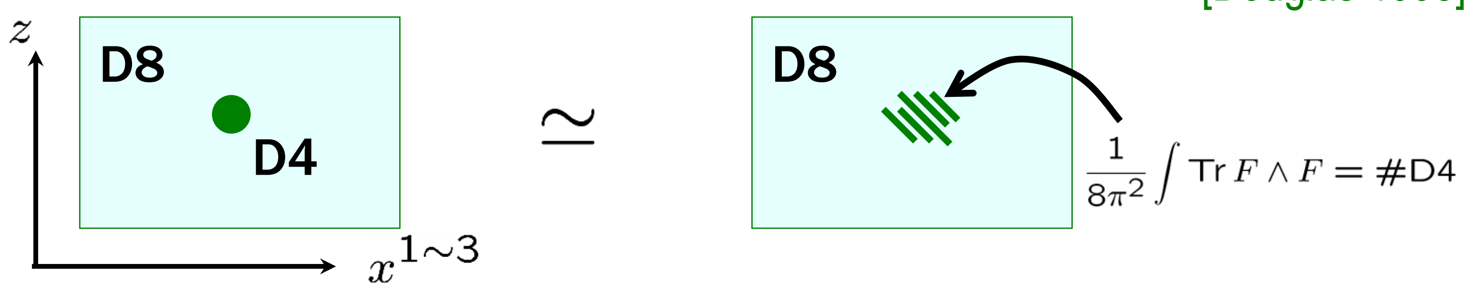


★ Baryon as instanton [Sakai-S.S. 2004]

- In our model, the wrapped D4 are embedded in D8.

D4 within D8 $\simeq$ **instanton on D8**

[Douglas 1995]



- The instanton is related to Skyrmion by

$$\boxed{U(x^\mu) \equiv P\exp\left\{-\int_{-\infty}^{\infty} dz\, A_z(x^\mu, z)\right\}}$$

Skyrmion instanton

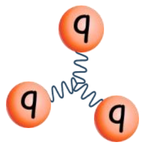
[Atiyah-Manton 1989]

$$N_B = \frac{1}{24\pi^2}\int_{\boldsymbol{R}^3}\mathrm{Tr}\left((U^{-1}dU)^3\right) = \frac{1}{8\pi^2}\int_{\boldsymbol{R}^4}\mathrm{Tr}(F\wedge F)$$

This is exactly the relation we used to write down the chiral Lagrangian!

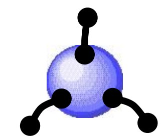
Four descriptions of Baryons

bound state of N_c quarks



$$N_B = \frac{1}{N_c}\#\text{quarks}$$

D4-brane on S^4



$$N_B = \#\text{D4-branes}$$

Skyrmion

$$N_B = \frac{1}{24\pi^2}\int_{\boldsymbol{R}^3} \mathrm{Tr}\left((U^{-1}dU)^3\right)$$

instanton

$$N_B = \frac{1}{8\pi^2}\int_{\boldsymbol{R}^4} \mathrm{Tr}(F \wedge F)$$

Now, they are all connected !

★ Quantization of solitons

[Hata-Yamato-Sakai-S.S. 2007]

- Consider a slowly moving (rotating) baryon configuration.
  Use the moduli space approximation method :

  Instanton moduli $\mathcal{M} \ni (X^\alpha) \rightarrow (X^\alpha(t))$ $(\alpha = 1, 2, \cdots, \dim \mathcal{M})$

  $\uparrow$ time

  $\boxed{A_M(t,x) \sim A_M^{\mathrm{cl}}(x; X^\alpha(t))}$

  $S_{\mathrm{5dim}} \rightarrow$ Quantum Mechanics for $X^\alpha(t)$

- For SU(2) one instanton,

$$\mathcal{M} \simeq \{(\underbrace{\vec{X}, Z}_{\text{position}}, \rho)\} \times SU(2)/\mathbf{Z}_2 \qquad \mathbf{Z}_2 : \mathbf{a} \to -\mathbf{a}$$

size: ρ; $SU(2) \ni \mathbf{a}$ ← SU(2) orientation

$$\rightarrow \quad L_{\mathrm{QM}} = \frac{G_{\alpha\beta}}{2}\dot{X}^\alpha \dot{X}^\beta - U(X^\alpha) \qquad U(X^\alpha) = 8\pi^2\kappa\left(1 + \lambda^{-1}\left(\frac{\rho^2}{6} + \frac{3^6\pi^2}{5\,\rho^2} + \frac{Z^2}{3}\right) + \mathcal{O}(\lambda^{-2})\right)$$

<u>Note</u> $(\vec{X}, \mathbf{a})$: genuine moduli

(ρ, Z) : massive modes. But, we keep them,
since they are lighter than the other massive modes.

- Solving the Schrodinger equation for this quantum mechanics, we obtain the wave functions for the baryons.

Example   Nucleon wave function:

$$\psi(\vec{X},\mathbf{a},\rho,Z) \ \propto \ e^{i\vec{p}\cdot\vec{X}} R(\rho)\psi_Z(Z)T(\mathbf{a})$$

$$\left(\begin{array}{lll} R(\rho)=\rho^{\tilde{l}}e^{-A\rho^2} & \tilde{l}=-1+2\sqrt{1+N_c^2/5} & A=\frac{8\pi^2\kappa}{\sqrt{6}} \\ \psi_Z(Z)=e^{-AZ^2} & T(\mathbf{a})=a_1+ia_2 \text{ for } |p\uparrow\rangle & \text{etc.}\end{array}\right)$$

- Evaluating the currents, we can calculate various quantities,

e.g., charge radius, magnetic moments, axial coupling, ele-mag form factors, etc.

$$J_\mu \simeq \delta S_{\rm 5dim}/\delta A^\mu_{\rm bdry}\Big|_{\rm instanton\ solution}$$

$$j_\mu(\vec{x}) = \langle B|J_\mu(\vec{x};\vec{X},Z,\rho,\mathrm{a})|B\rangle$$

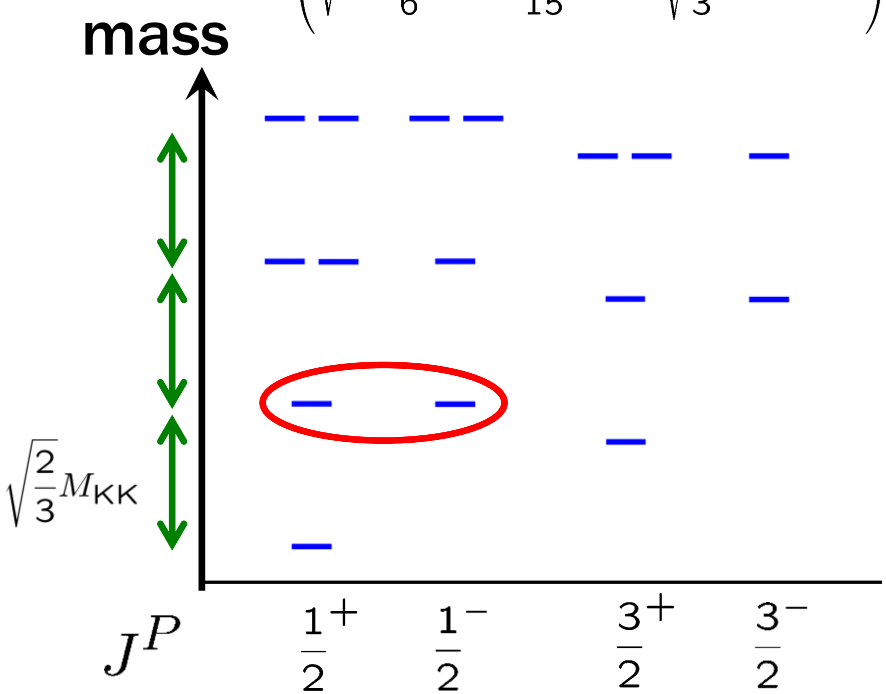
baryon state

$$\langle r^2\rangle = \int d^3x\, r^2 j^0(\vec{x})$$ etc.

★ Baryon spectrum

Theory

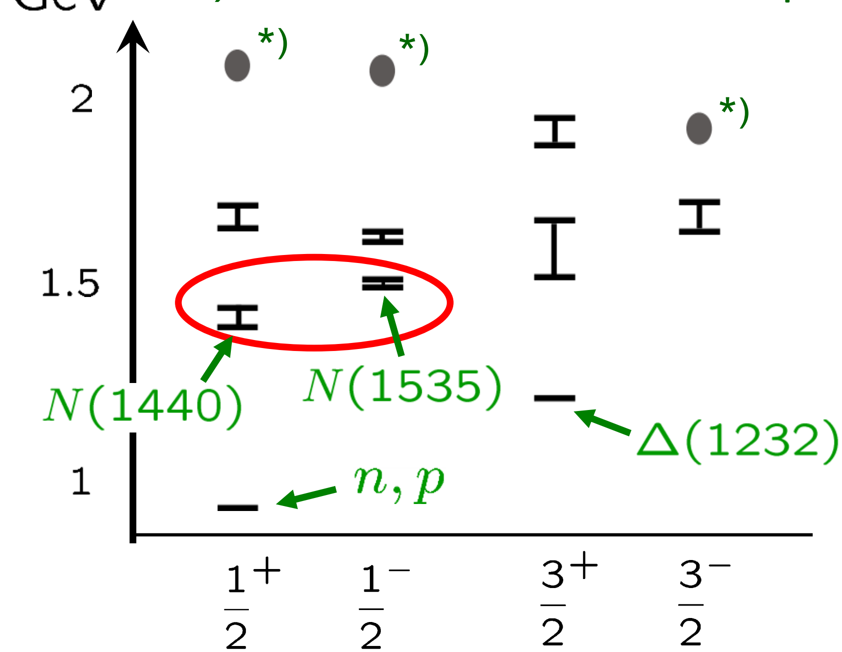
$$M \simeq M_0 + \left(\sqrt{\frac{(l+1)^2}{6} + \frac{2}{15} N_c^2} + \sqrt{\frac{2}{3}} (n_\rho + n_z) \right) M_{KK}$$



Experiment

($I = J$ states from PDG)

*) Evidence for existence is poor



Note:

- We only consider the mass difference, since $\mathcal{O}(N_c^0)$ term in M_0 is not known.
- $M_{KK} \simeq 949$ MeV (fixed by ρ-meson mass) is a bit too large. It looks better if M_{KK} were around 500 MeV.

★ $\langle r^2 \rangle$ and g-factors

[Hashimoto-Sakai-S.S. 2008]

Adkins-Nappi-Witten
for Skyrmion

	our result	exp.	ANW
$\langle r^2 \rangle^{1/2}_{I=0}$	0.742 fm	0.806 fm	0.59 fm
$\langle r^2 \rangle^{1/2}_{I=1}$	0.742 fm	0.939 fm	∞
$\langle r^2 \rangle^{1/2}_{A}$	0.537 fm	0.674 fm	—
$g_{I=0}$	1.68	1.76	1.11
$g_{I=1}$	7.03	9.41	6.38
g_A	0.734	1.27	0.61

[See also,
Hong-Rho-Yee-Yi 2007,
Hata-Murata-Yamato 2008,
Kim-Zahed 2008,
Pomarol-Wulzer 2008,
Panico-Wulzer 2008, ...]

- We can also evaluate these for excited baryons such as $\Delta(1232), N(1440), N(1535), \cdots$

★ Form factors

$$\langle N, \vec{q}/2 | J^0_{\rm em}(0) | N, -\vec{q}/2 \rangle = G_E(\vec{q}^2)\, \chi^\dagger_{s'} \chi_s$$

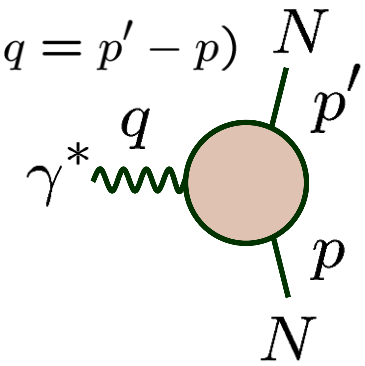
Electric form factor

$$\langle N, \vec{q}/2 | J^i_{\rm em}(0) | N, -\vec{q}/2 \rangle = \frac{i}{2m_N} G_M(\vec{q}^2)\, \chi^\dagger_{s'} (\vec{q} \times \vec{\sigma}) \chi_s$$

Magnetic form factor

$(\vec{p}\,' = -\vec{p} = \vec{q}/2$: Breit frame$)$

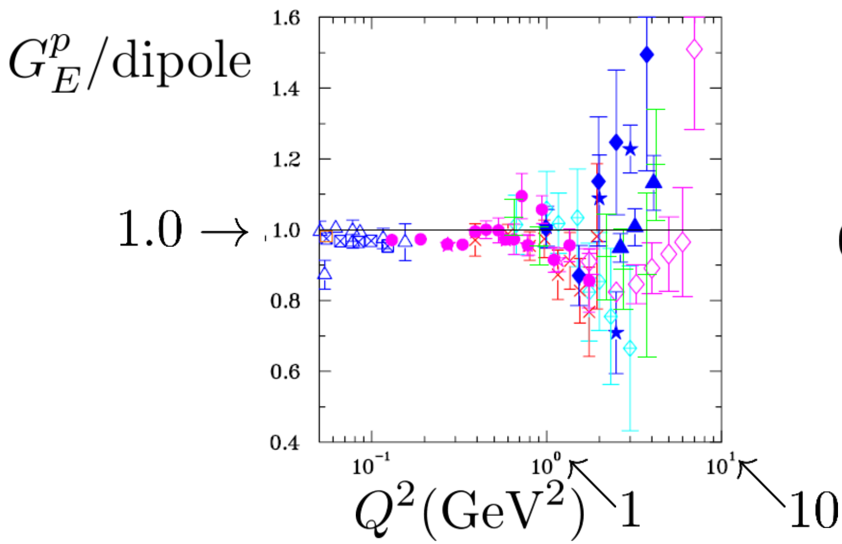
$(q = p' - p)$

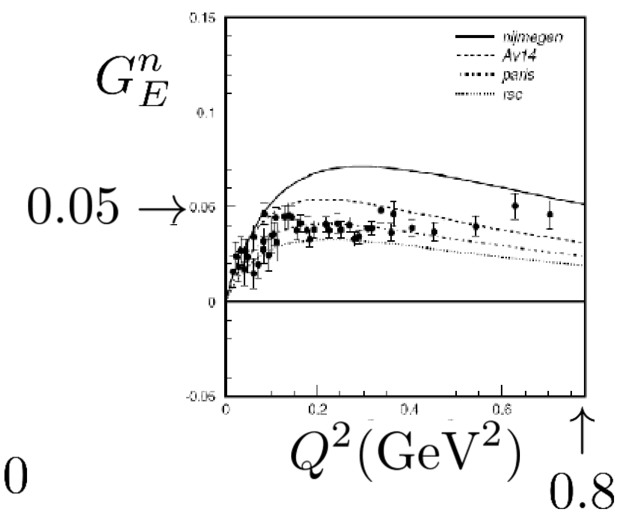


- Experimental data suggest

dipole $(\Lambda \simeq 0.71\ \mathrm{GeV}^2)$

$$G^p_E(Q^2) \simeq \frac{1}{\mu_p} G^p_M(Q^2) \simeq \frac{1}{\mu_n} G^n_M(Q^2) \simeq \frac{1}{(1 + \frac{Q^2}{\Lambda^2})^2} \qquad G^n_E(Q^2) \simeq 0$$



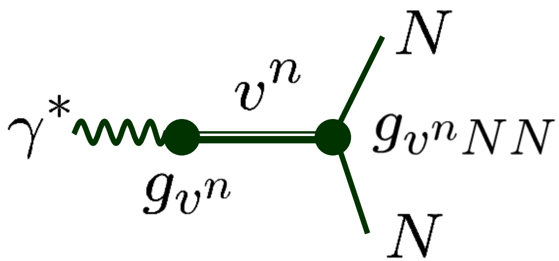


Figures taken from Perdrisat et al hep-ph/0612014

Our result

[Hashimoto-Sakai-S.S. 2008]

$$G_E^p(Q^2) = \frac{1}{\mu_p} G_M^p(Q^2) = \frac{1}{\mu_n} G_M^n(Q^2) = \sum_{n\geq 1} \frac{g_{v^n} g_{v^n NN}}{Q^2 + m_n^2} \qquad G_E^n(Q^2) = 0$$

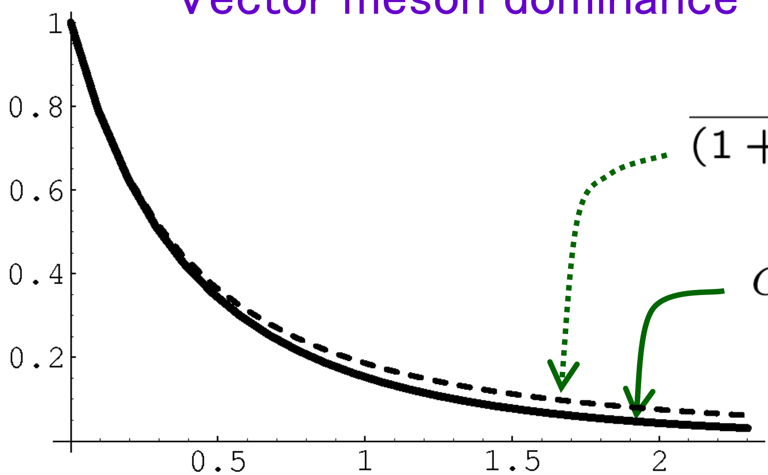


with

$$g_{v^n} = -2\kappa (k(z)\partial_z \psi_{2n-1})\Big|_{z=+\infty}$$

$$g_{v^n NN} = \langle \psi_{2n-1}(Z) \rangle$$

Vector meson dominance



$$\frac{1}{(1+Q^2/\Lambda^2)^2}$$ dipole ($\simeq$ experiment)

$$G_E^p(Q^2) = \sum_{n\geq 1} \frac{g_{v^n} g_{v^n NN}}{Q^2 + m_n^2}$$ our result

Q^2 [GeV^2]

$$G_E^p(Q^2) \simeq 1 - 2.38Q^2 + 4.02(Q^2)^2 - 6.20(Q^2)^3 + 9.35(Q^2)^4 - 14.0(Q^2)^5 + \cdots$$

$$\frac{1}{(1+Q^2/\Lambda^2)^2} \simeq 1 - 2.38Q^2 + 4.24(Q^2)^2 - 6.71(Q^2)^3 + 9.97(Q^2)^4 - 14.2(Q^2)^5 + \cdots$$

with $\Lambda^2 = 0.758\ \mathrm{GeV}^2$ $\qquad (M_{\mathrm{KK}} = 1\ \text{unit})$

4 Conclusion and discussion

- Though the approximation is still very crude,
  our model catches various features of QCD
  and provides new insights in hadron physics.

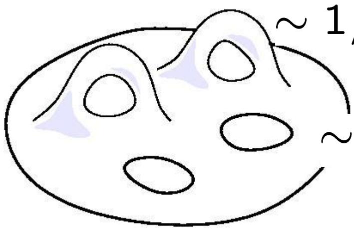
  " ***much better than expected !*** "

- A lot of qualitative properties in QCD can be understood
  from the geometry of the background.

  - **Confinement**
  - **Chiral symmetry breaking**
  - **Phase transition**
  - **Origin of baryon mass**
  - **etc ...**

- It is in principle possible to improve the approximation.

QCD		**String theory**
$1/N_c$ correction	⟷	loop correction
$1/\lambda$ correction	⟷	α' correction



$\sqrt{\alpha'} = l_s$: string length

- To make $M_{\rm KK}$ large, we have to go beyond SUGRA approximation

$$M_{\rm KK} \to \infty \ , \quad \lambda \to 0$$

$$m_\rho = M_{\rm KK} \underline{f(\lambda)} \quad \text{: fixed}$$

to be determined

Questions

- How can we approach $\lambda \to 0$ regime?
- Complete understanding of QCD phase diagram

  introducing T, μ, B, E, m_q, $\theta \cdots$
- A better description for $T > T_c$
- Time dependent processes

Thank you！