Gregory-Laflamme as the confinement/deconfinement transition in holographic QCD

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QCD Applications of AdS/CFT to QCD and condensed matter physics,

October 20, 2015

Based on: hep-th/1107.4048 (with T Morita), 1507.08949 (with H Isono and T Morita)

Plan of the talk:

1. Witten's model: gravity dual of pure Yang-Mills
2. Deconfinement transition as “Hawking-Page”
3. Deconfined YM is phase separated from Black hole! problem
4. Lessons from lower dimensional gauge theories and lattice
5. Way out: Fermion b.c. and new phase diagram
6. Gregory-Laflamme as deconfinement transition
7. Consequences

Related work:

Phase diagram of Sakai-Sugimoto

O. Aharony, J. Sonnenschein and S. Yankielowicz, "A holographic model of deconfinement and chiral symmetry restoration," Annals Phys. **322** (2007) 1420 [arXiv:hep-th/0604161].

Phase diagram of low D gauge theory

G. Mandal, M. Mahato, T. Morita, "Phases of one dimensional large N gauge theory in a 1/D expansion," JHEP **1002**, 034 (2010). [arXiv:0910.4526 [hep-th]].

G. Mandal and T. Morita, "Phases of a two dimensional large N gauge theory on a torus," arXiv:1103.1558 [hep-th].

Centre symmetry and large N volume independence

P. Kovtun, M. Unsal, L. G. Yaffe, "Volume independence in large N(c) QCD-like gauge theories," JHEP **0706** (2007) 019. [hep-th/0702021 [HEP-TH]].

Some consequences

A. Rebhan, "The Witten-Sakai-Sugimoto model: A brief review and some recent results," arXiv:1410.8858 [hep-th].

M. Hanada, Y. Matsuo and T. Morita, "Instanton dynamics in finite temperature QCD via holography," arXiv:1505.04498 [hep-th].

Phase transitions in AdS/CFT: $\mathcal{N}=4$ SYM

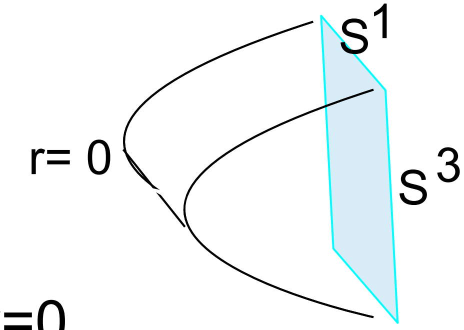
N_{SUSY} = 4 SYM on 3-sphere (radius L) x thermal circle

$$F = -N^2 T f(\lambda, x), \;\; x = TL$$

Thermal= Antiperiodic fermions along thermal circle, breaks SUSY

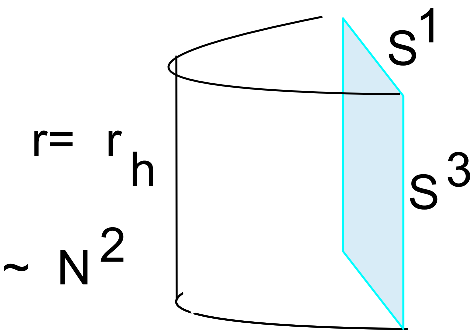
At a high temperature, approximately pure YM in 3D, which has a deconfined phase.

AdS(5) with thermal circle

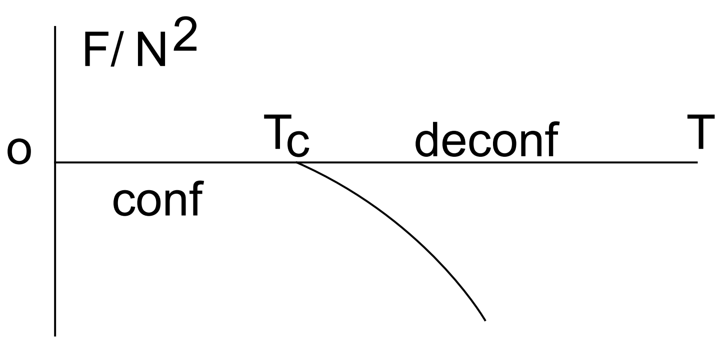


No horizon
Hence Entropy=0
(at order N^2)

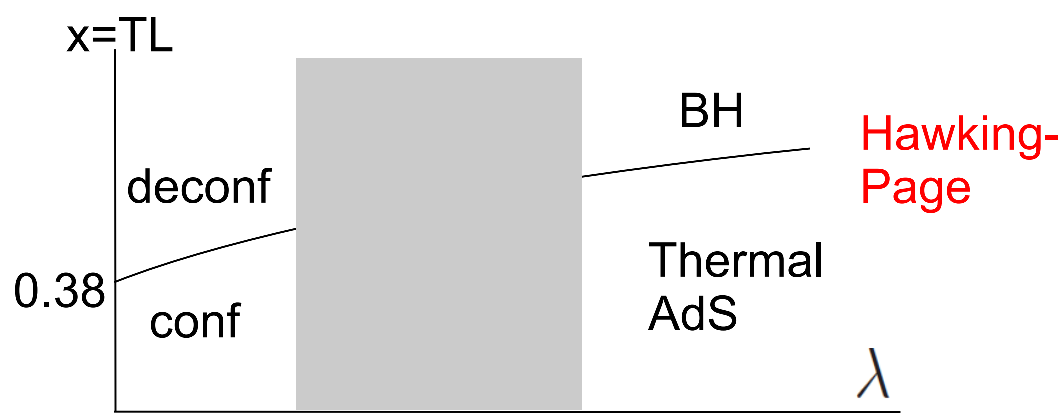
AdS-BH



Entropy= S(BH) ~ N^2



Witten 1998



Aharony, Minwalla et al 2003, Sundborg 2003

Pure Yang-Mills in 3+1

The coupling constant runs,
yielding a mass scale $\Lambda_{YM} = \Lambda e^{-1/N g_{YM}^2(\Lambda)}$

The free energy has a different non-trivial scaling form:

$$F(g_{YM}, \Lambda, T, N) = N^2 T^4 f(T/\Lambda_{YM})$$

$x_c = O(1)$ $\qquad\qquad$ x

Deconfinement transition $T_c \sim \Lambda_{YM}$

Cannot study deconfinement transition using weak-coupling perturbative theory at all (unlike for ***N***=4 SYM)

Tool to study phase diagram: Lattice or AdS/CFT
[in low dimensional gauge theory, large flavour limit (see later)]

Deconfinement transition: order parameters

Confinement

No free quarks

$$W_0 = \exp[-S_q] =$$

$$\mathrm{Tr} P(\exp[i \int dt \; A_0]) = 0$$

Deconfinement

Free quarks

$$W_0 \neq 0$$

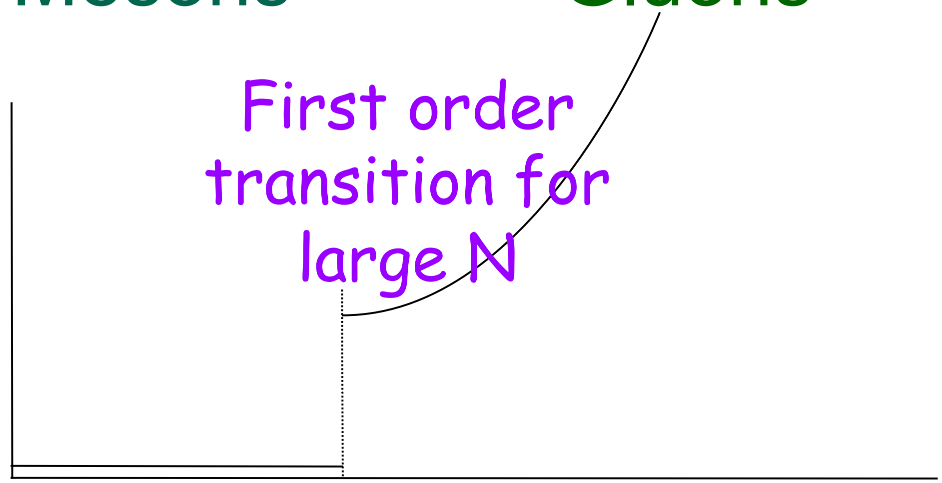
Order parameter for centre symmetry: symmetry under gauge transformations quasiperiodic up to a Z_N element h

$$g(\theta + 2\pi, x) = hg(\theta, x)$$

$$g_{Ad}(\theta + 2\pi, x) = hg_{Ad}(\theta, x)h^{-1} = g_{Ad}(\theta, x)$$

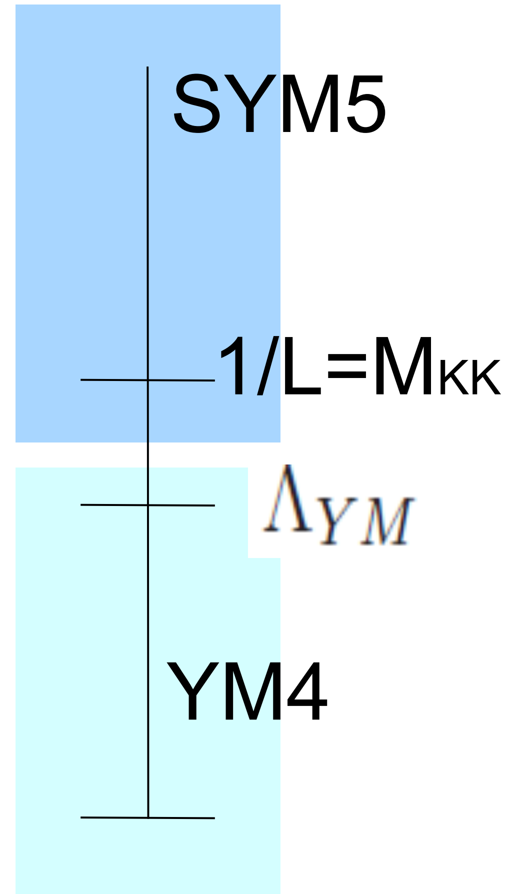
Deconfinement transition: order parameters

Confinement	Deconfinement
$W_0 = 0$	$W_0 \neq 0$
$S = O(1)$	$S = O(N^2)$
Glueballs, Mesons	Gluons



Witten's proposal (Witten 1998)

Start with 5D SYM (non-conformal) on
and shrink . Take AP boundary condition on
fermions along both circles.



Fermion mass = (tree)

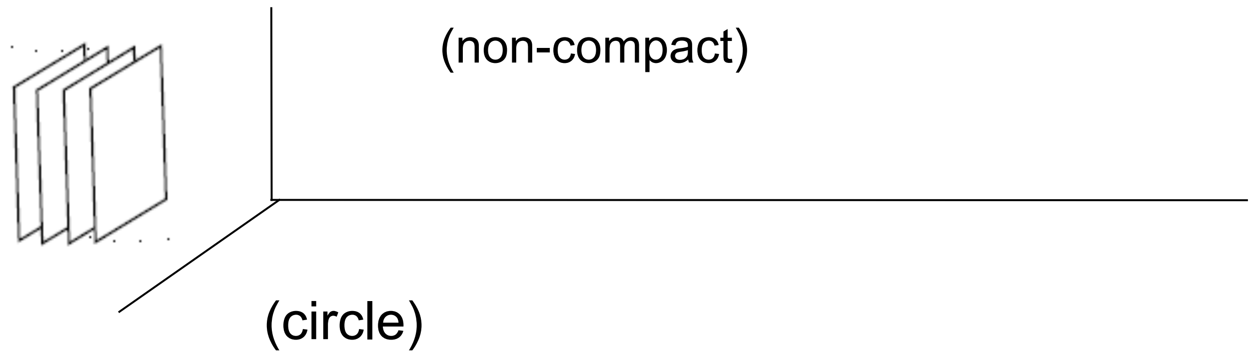
Scalar mass = (one-loop)

For <<1, we can ignore KK modes; this
results in pure YM theory in

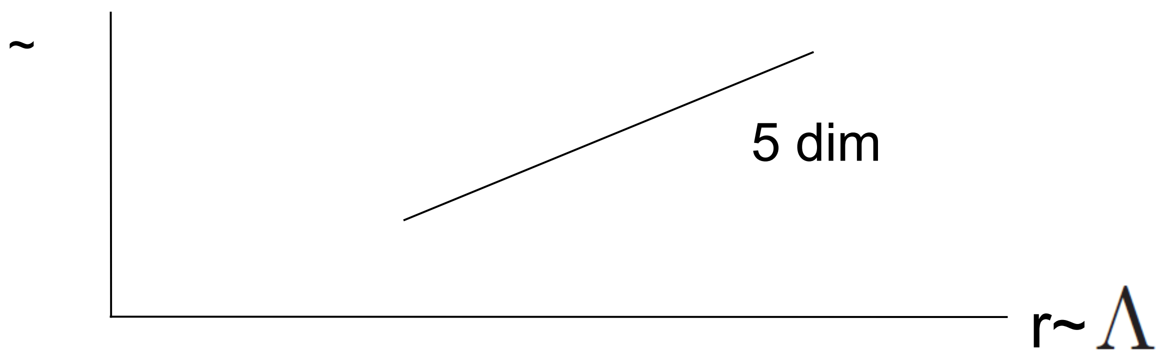
Λ_{YM} = 1/L exp[-1/]

Towards a gravity dual

5D SYM arises from a stack of N D4 branes



In the large N limit, the stack of D4 branes is replaced by a gravitational background. It has a nontrivial dilaton



$$g_{eff} \sim r = \Lambda$$

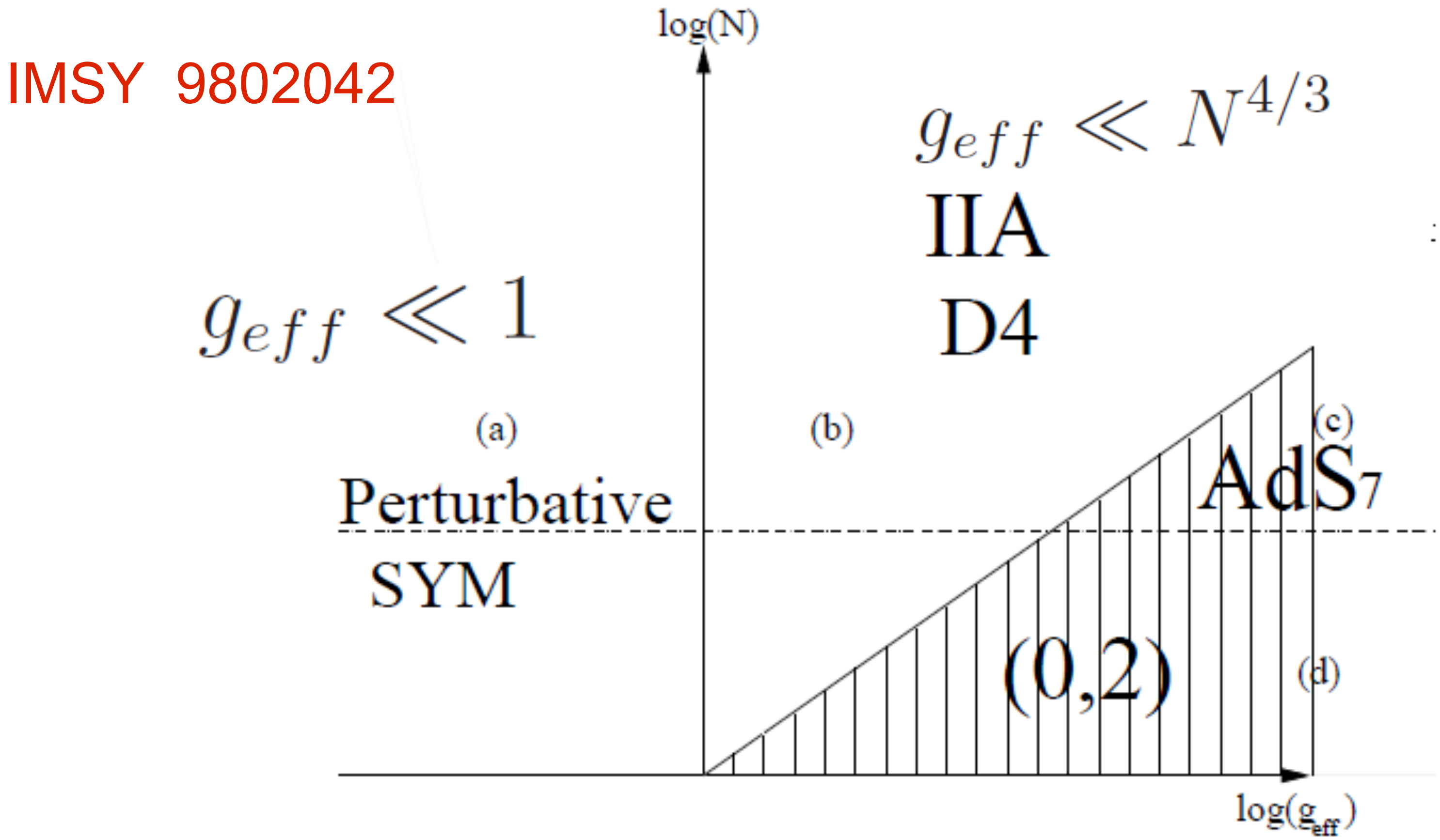
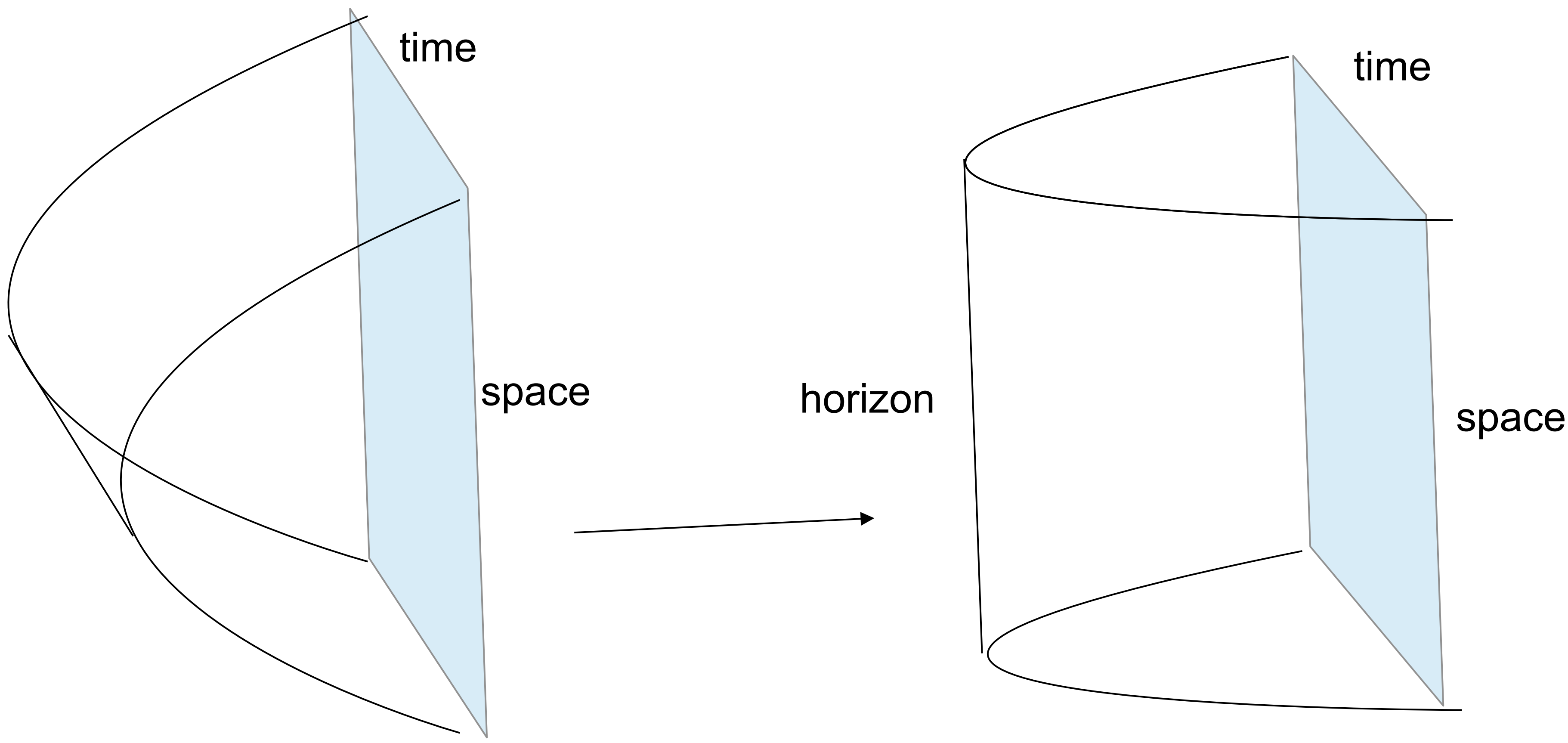


Figure 4: The D4-brane map: The horizontal dashed line separates between the small N region and the large N region. The UV region is described by a super-conformal theory on a circle (the marked region) (c,d), which is dual for large N to M-theory on a background $AdS_4 \times S^7$ (c) (with an identification). In the IR the theory is described by "perturbative" super-Yang-Mills (a). For large N we have the intermediate region (b) described by the IIA D4 brane solution.

Two solutions

Scherk-Schwarz Transition



Low temperature solution:
Soliton

High temperature solution:
Black hole

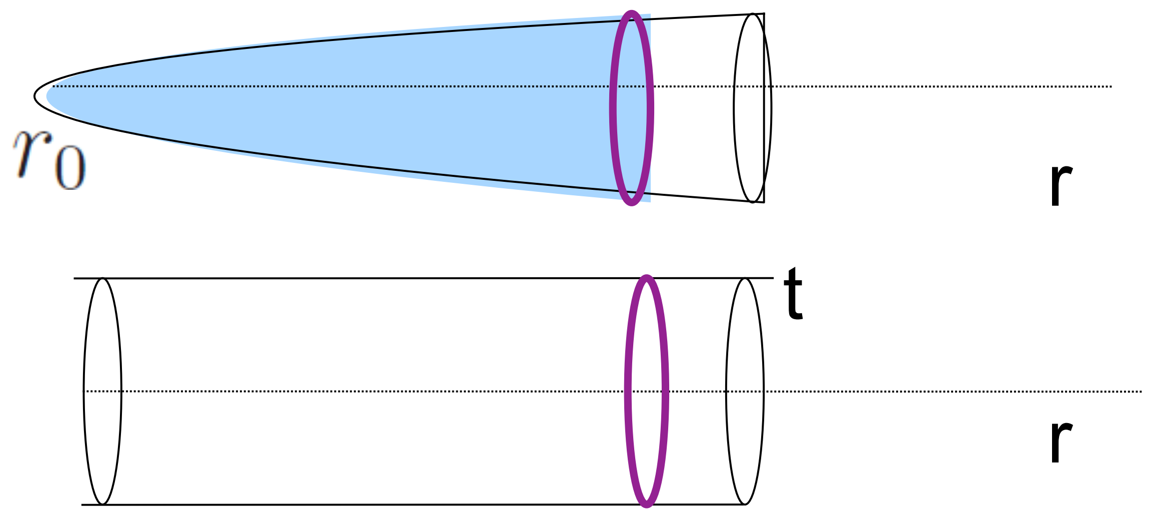
Geometry of the solution relevant at low T: D4 soliton

$$ds^2 = r^{3/2}[dt^2 + \sum_{i=1}^{3} dx_i^2 + f(r)dx_4^2] + \frac{1}{r^{3/2}}[\frac{dr^2}{f(r)} + r^2 d\Omega_4^2]$$

where

Note that the radial direction ends at

$W_4 \neq 0$



$W_0 = 0$

Classical action

Free energy is independent of temperature, hence S= 0

Confinement? $W_0 = 0$, S= O(1) ✓

Pure YM4? $W_4 \neq 0$. **Essential for KK reduction of gauge theory** ✓

However, gravity description is good only for

Gravity gives an analog of the strong coupling expansion.
Need to ensure no phase transition.

Other solutions => Black D4 brane

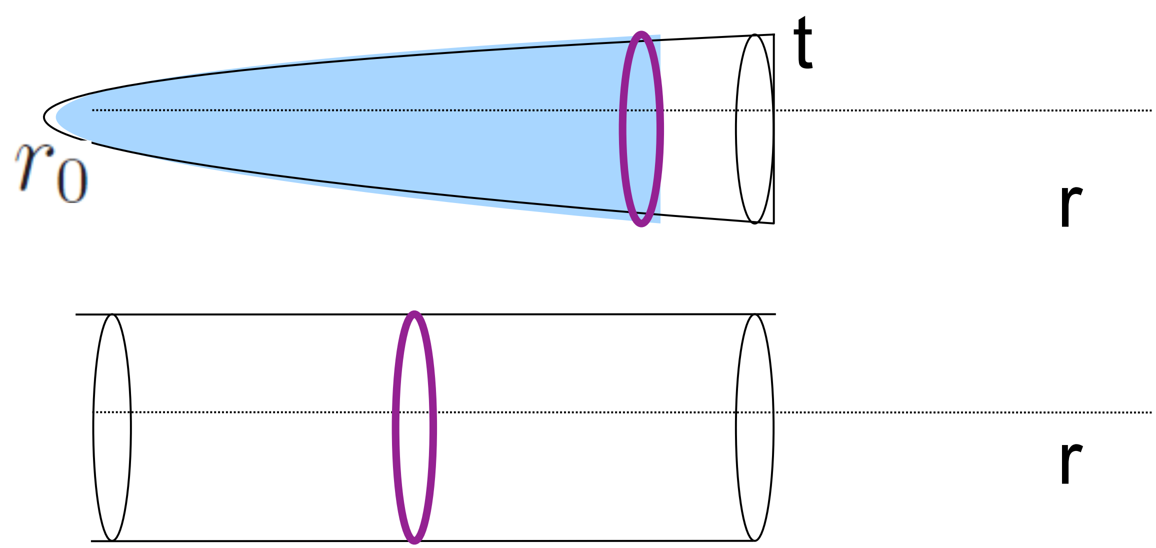
$$ds^2 = r^{3/2}[dx_4^2 + \sum_{i=1}^{3} dx_i^2 + f(r)dt^2]$$
$$+ \frac{1}{r^{3/2}}[\frac{dr^2}{f(r)} + r^2 d\Omega_4^2]$$

This is related to the solitonic D4 by $t \leftrightarrow x_4$.

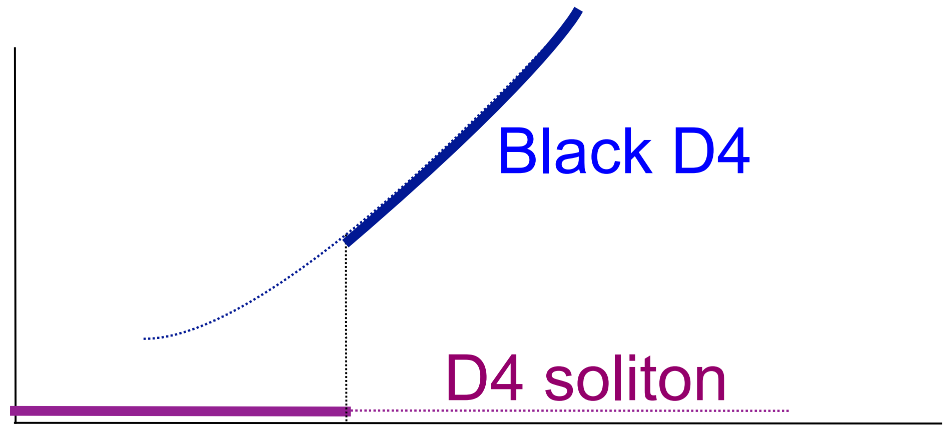
is now the black hole horizon

$W_0 \neq 0$

$W_4 = 0$



Classical action



Pure YM4 $\subset$ SYM5
(deconfinement phase)

Black D4

$W_0 \neq 0$

$W_0 \neq 0$ ✓

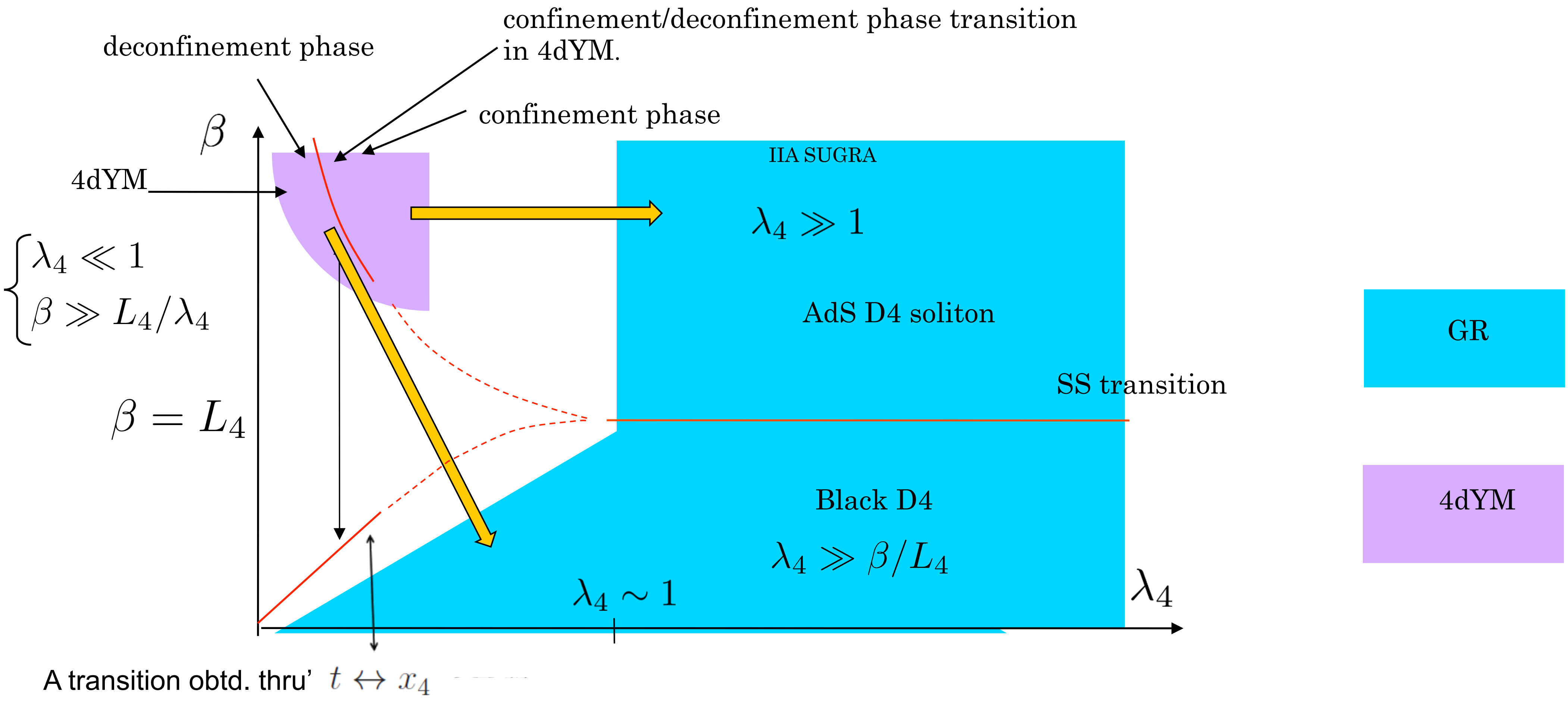
✓

$W_4 \neq 0$

Phase boundary

$W_4 = 0$ X

(In this slide)



There is a phase boundary between YM4 (deconfinement) and Black D4 phase. There are no phase boundaries between YM4(confinement) and the solitonic D4 phase. (GM, Morita 2011, 2012; See also Aharony, Sonnenschein, Yankielowicz 2006)

Intermediate phase in gauge theory also consistent with large N cascade (Narayanan-Neuberger 2003-2007)

Summary so far:

We have a strong coupling continuation of the confinement phase of pure YM4 (in terms of the solitonic D4). However, the black brane as the strong coupling continuation of the deconfinement phase of pure YM4 is problematic.

Is there a way out?

The solitonic D4 and the black D4 are the only solutions with AP boundary conditions for fermions on both the and t cycles.

Recall that AP is necessary along the cycle, in order to break supersymmetry. Naively, it would appear that the AP boundary condition along the t cycle is also imperative, to describe thermal fermions. However,.....

Gauge theory in 1+1 dimensions

Consider adjoint scalar QCD (massless) in 1+1 dimensions, with D flavours.
(= D+2 dim pure YM in a box)

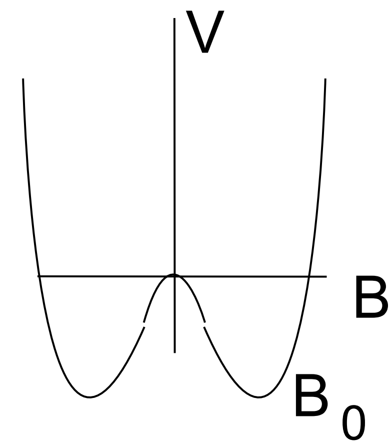
$$S=\int_0^\beta dt\int_0^L dx\ \mathrm{Tr}\left(F_{\mu\nu}^2+\frac{1}{2}\sum_{I=1}^{D}(D_\mu Y^I)^2-\frac{g^2}{4}\sum_{I,J}[Y^I,Y^J][Y^I,Y^J]\right)$$

The SO(D) singlet sector can be described by a change of variable

$$Y_a^I Y_b^I \rightarrow B_{ab}$$

The adjoint scalars are then described by a potential

$$V(B)=-D\ln\mathrm{Det}[D_\mu^2-B]+B^2/g^2$$



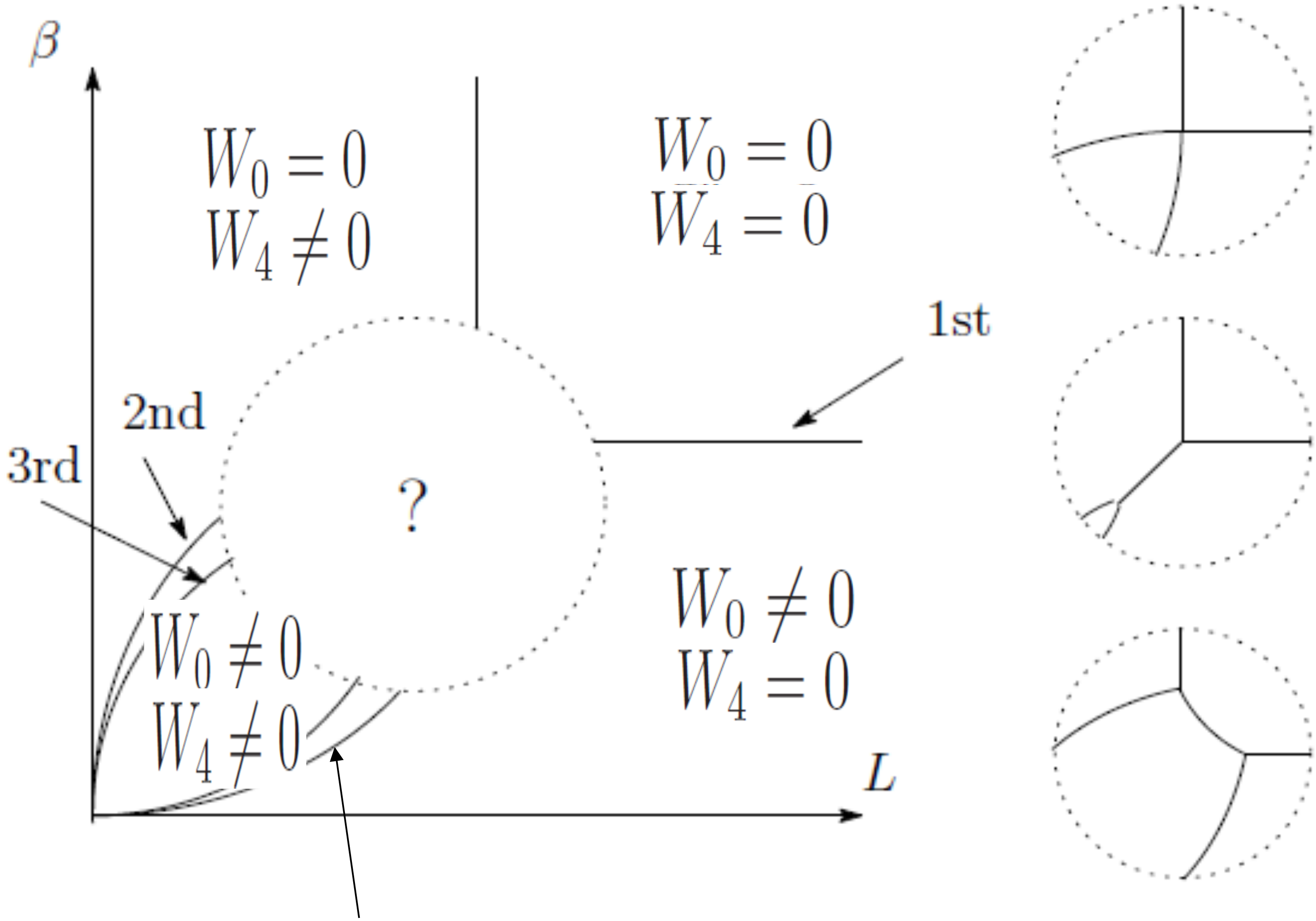
At large D, B is frozen at the minimum, leading to a dynamical mass generation of the scalars.

The scalars can be integrated out, leaving an effective action of gluons which can be solved at small and large values of L and (compared to the scale set by the Yang Mills coupling).

GM, Mahato, Morita, 2009-11

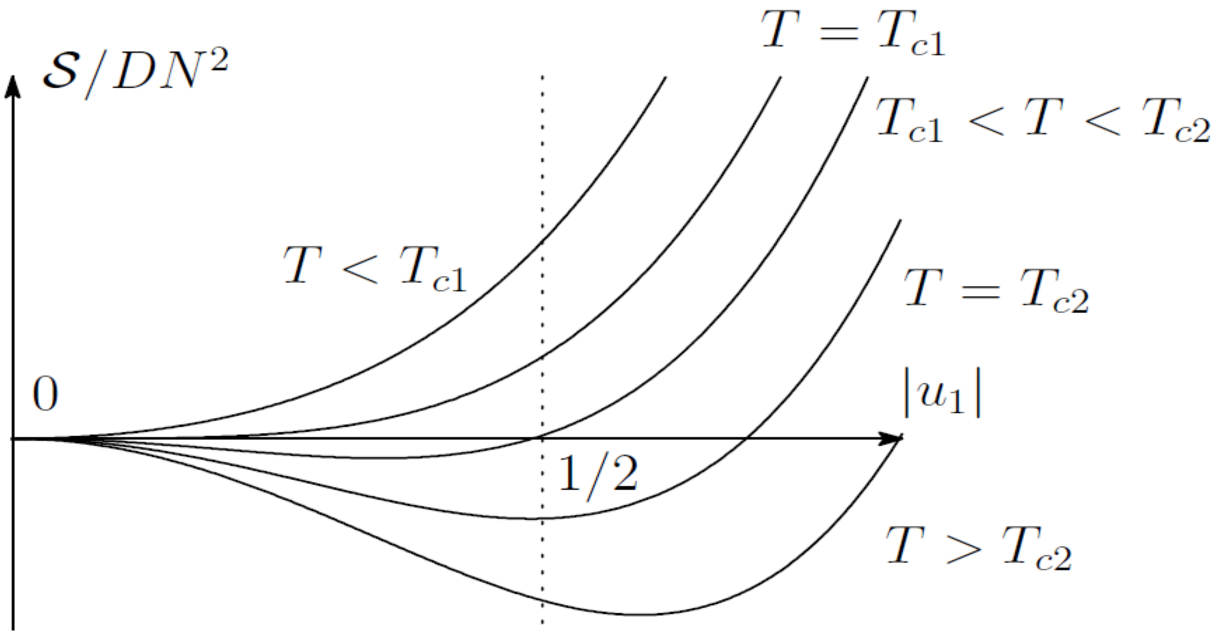
Gauge theory in 1+1 dimensions

GM, Morita 2011, 2012



Transition temperatures agree with numerical calculations of Nishimura et al (2007), up to $O(1/D^2)$.

[gravity diagram]



$$\beta_{c1}\tilde{\lambda}^{1/3} = \log D\left(1 + \frac{1}{D}\left(\frac{203}{160} - \frac{\sqrt{5}}{3}\right)\right).$$

$$\beta_{c2}\tilde{\lambda}^{1/3} - \beta_{c1}\tilde{\lambda}^{1/3} = \frac{\log D}{D}\left[-\frac{1}{6} + \frac{1}{D}\left(\left(-\frac{499073}{460800} + \frac{203\sqrt{5}}{480}\right)\log D - \frac{1127\sqrt{5}}{1800} + \frac{85051}{76800}\right)\right].$$

		T_{c1}	T_{c2}
Nishimura	Numerical result (D=9)	0.8761	0.905
GM, Mahato, Morita	Leading large D result	0.947	0.964
	Large D including $1/D$ effect	0.895	0.917

Gravity dual: D2 brane theory

D2 branes at weak coupling correspond to N=4 SYM theory in 2+1 dimensions. Euclieanize, and compactify on circles. Time =   ,   x1= L, x2= L2.

Apply antiperiodic boundary condition to fermions along L2. Fermions acquire a mass 1/L2. For L2 << 1/   (mass scale set by the dimensionful coupling), we are left with 1+1 dimensional adjoint scalar QCD.

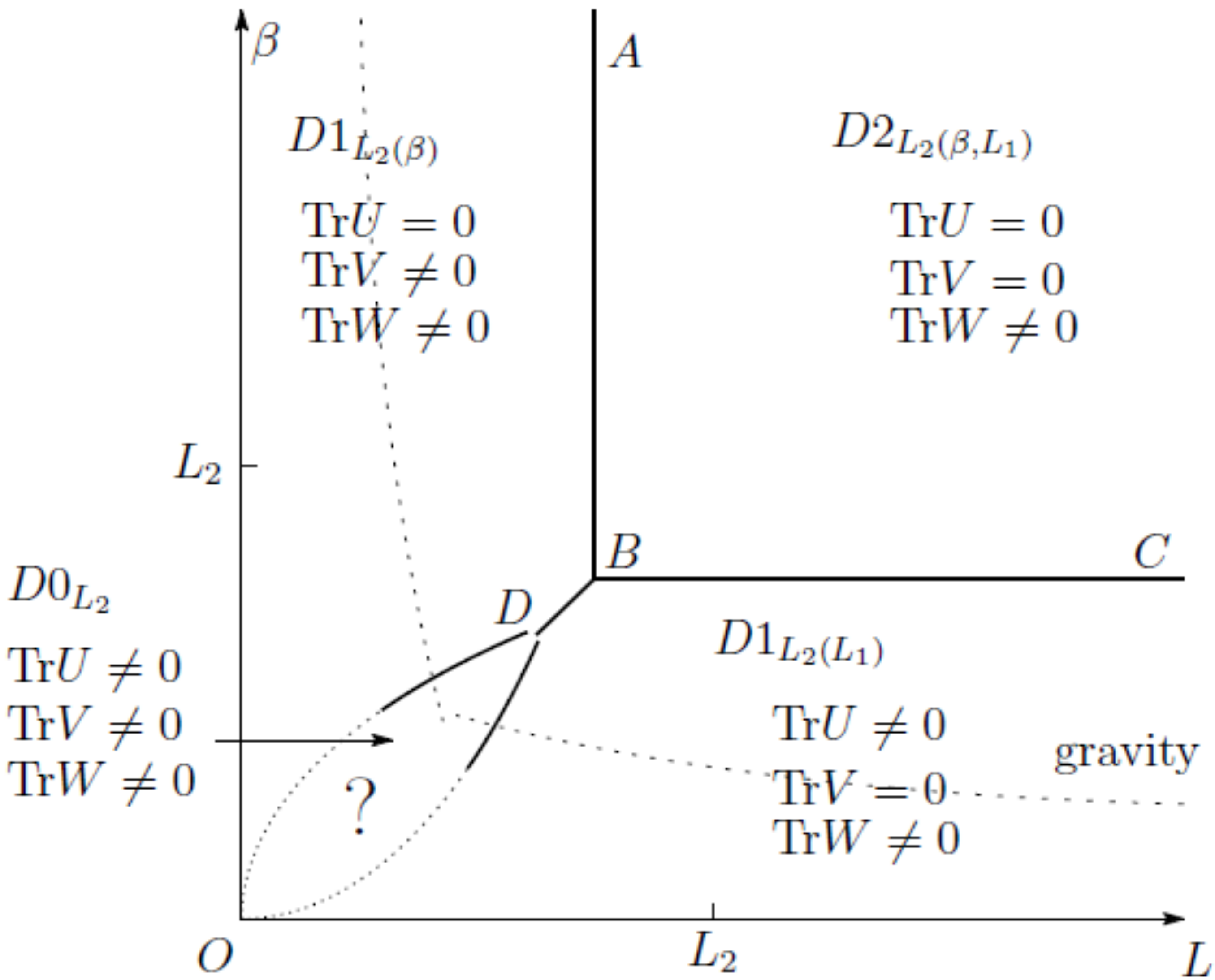
At strong coupling, we have reliable supergravity. However, the QCD-scale goes above the KK scale, hence the KK particles come back into the spectrum.

The supergravity description is not a strict dual of the 1+1 dimensional bosonic gauge theory, but rather a strong coupling continuation.

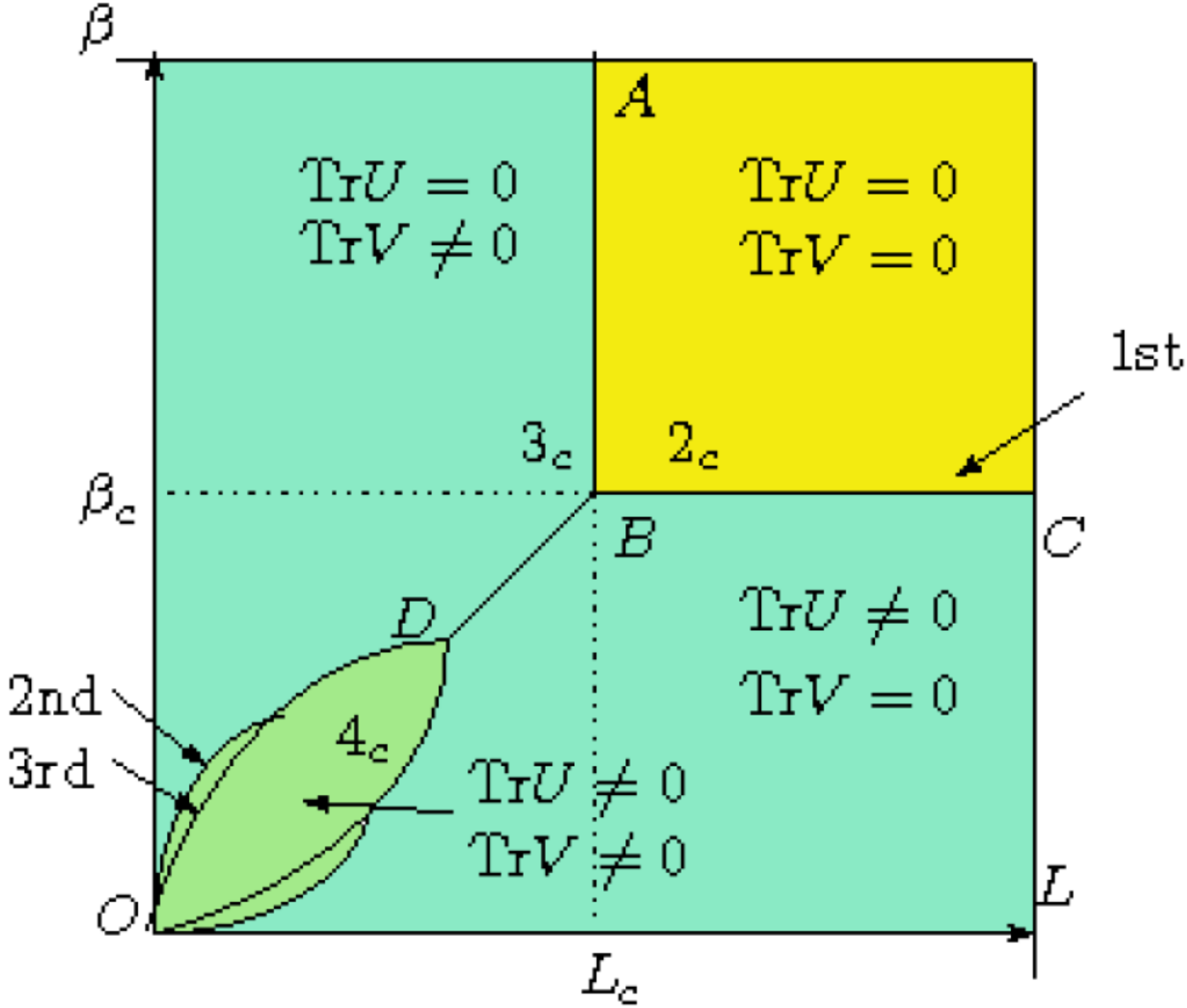
In the supergravity, must allow ALL four possible b.c. for fermions along time and x1: (AP, AP), (P, AP), (AP, P), (P,P). At weak coupling, the fermions decouple; hence any one of the four can descend to 1+1 gauge theory, in principle.

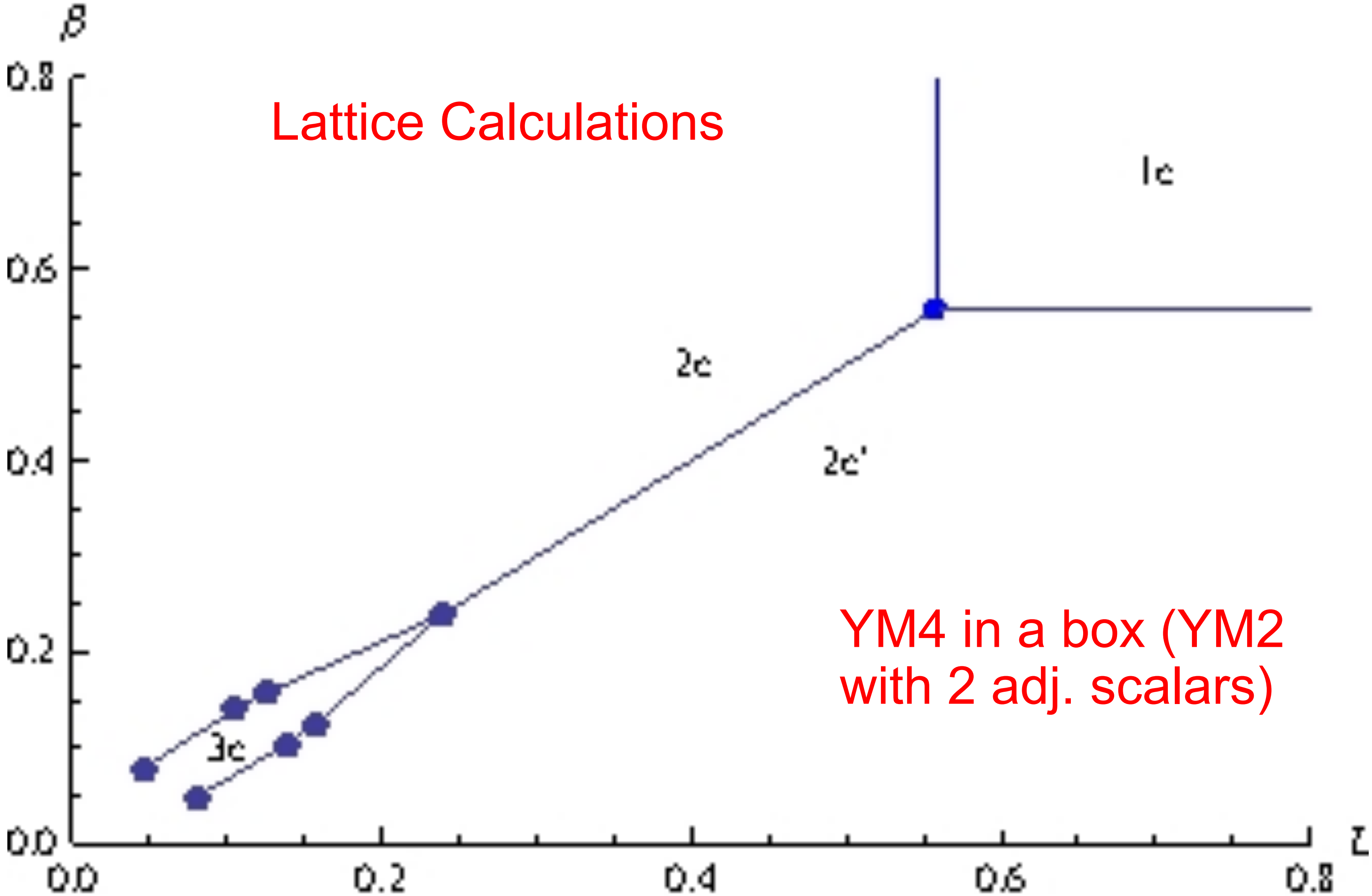
(P,AP) b.c

(P,AP) b.c. along (time, x2) (and P along x1) has a smooth extraplation to the gauge theory phase diagram



Combined phase diagram





Narayanan, Neuberger 2003-2007,
GM, Morita, Narayanan 2010

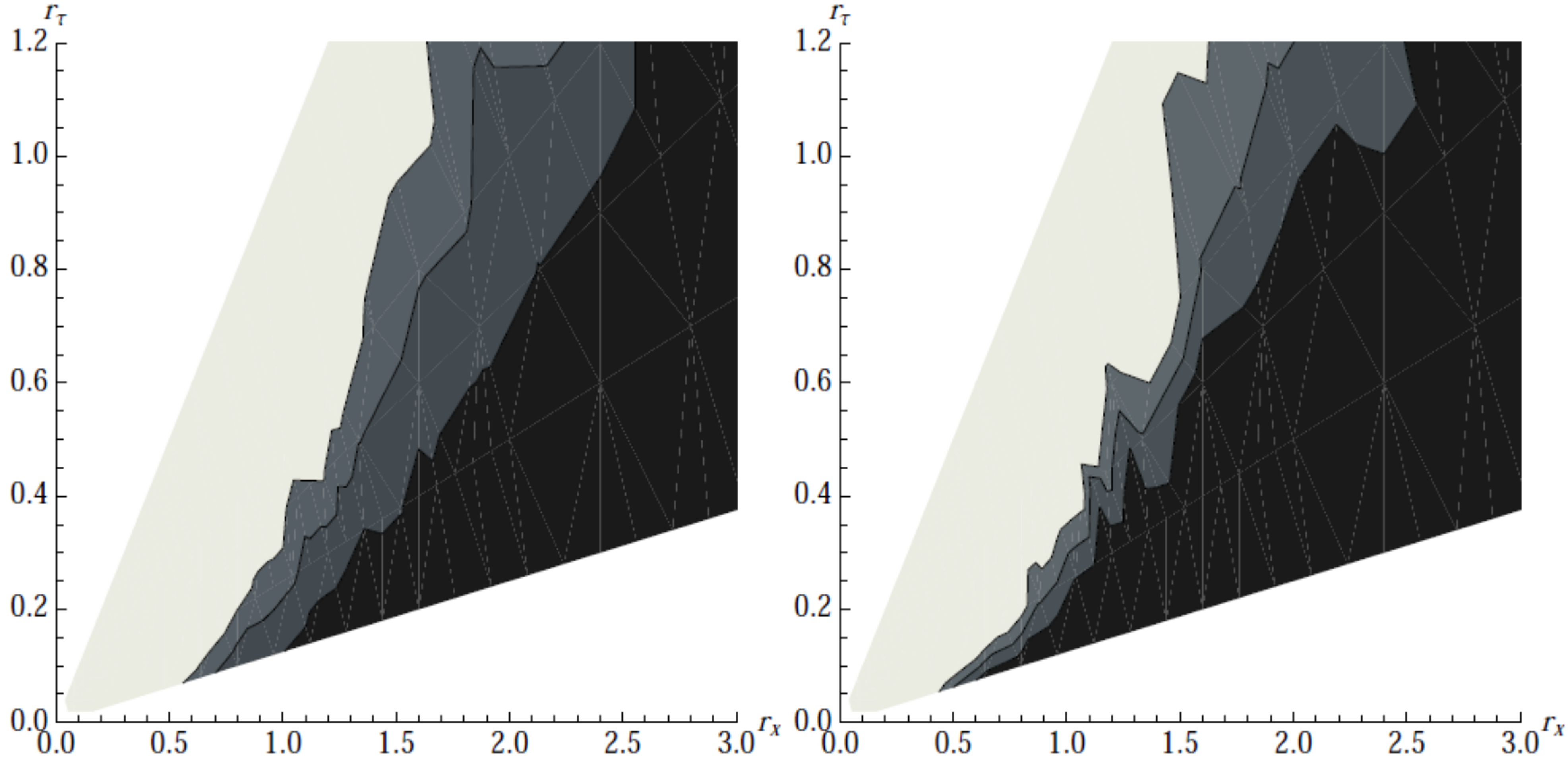


Figure 7: Plot of contours of the expectation of the spatial Polyakov line P_x over the r_x, r_τ plane. The left frame shows $SU(3)$, and the right $SU(4)$. The three contours plotted are 0.4, 0.5, 0.6, and the simulation data collates and interpolates runs made on lattices 2×16, 2×8, 3×8, 4×4 and 4×8 therefore giving a variety of aspect ratios r_τ / r_x.

(Catteral, Joseph, Wiseman 2010) [back]

Back to pure YM in 3+1

As in the lower dimensional case, in the pure YM4 limit of SYM5, all fermions decouple from the system. So no need to insist on antiperiodic fermions on the thermal cycle.

In other words, periodic fermions along the thermal circle gives a twisted partition function (Witten index). However, in the pure YM4 limit since fermions do not contribute (F=0 effectively), the difference between Witten index and the normal partition function disappears!

$$Z^{\rm SYM5}_{\rm (AP,AP)} = {\rm Tr}\, e^{-\beta H_{\rm SYM5}} \to {\rm Tr}\, e^{-\beta H_{\rm YM4}}, \qquad (\lambda_4 \to 0, \frac{L_4}{\lambda_4 \beta} \to 0),$$

$$Z^{\rm SYM5}_{\rm (P,AP)} = {\rm Tr}(-1)^F e^{-\beta H_{\rm SYM5}} \to {\rm Tr}\, e^{-\beta H_{\rm YM4}}, \qquad (\lambda_4 \to 0, \frac{L_4}{\lambda_4 \beta} \to 0).$$

New Proposal

(GM, Morita 2011)

Look for gravity solutions with (P, AP) boundary conditions along

At high temperature, a new solution appears (clumped soliton)! And an old solution – the black hole – disappears!

As T goes up, the thermal cycle shrinks, giving rise to light winding modes, at around $T \sim \sqrt{\lambda_4} / L$

In the (AP, AP) case the transition to the black D4 brane happens at T=1/L before reaching such temperatures.

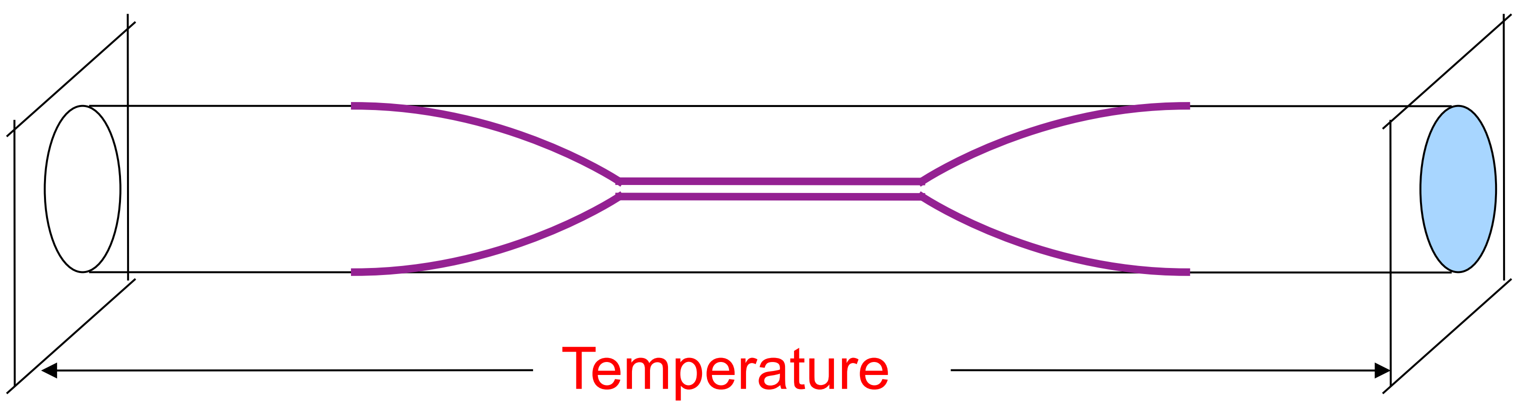
However with the (P, AP) boundary condition, the black D4 brane is not an allowed geometry since the black hole admits only AP spin structure along the thermal circle.

New Proposal (contd.)

In order for the gravity solution to remain valid, we must switch to the T-dual geometry: the type IIA solitonic D3 brane, which is uniformaly distributed (smeared) along the (dual) time-circle t', of radius

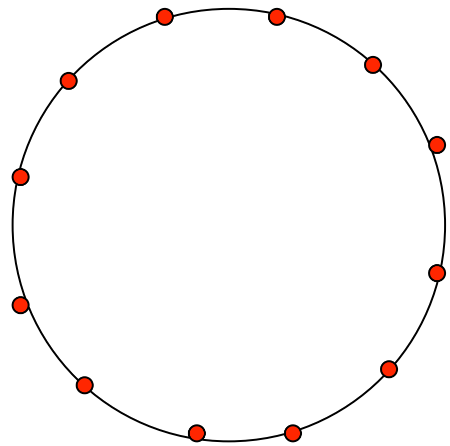
As the temperature rises further, increases, which signals a new instability.

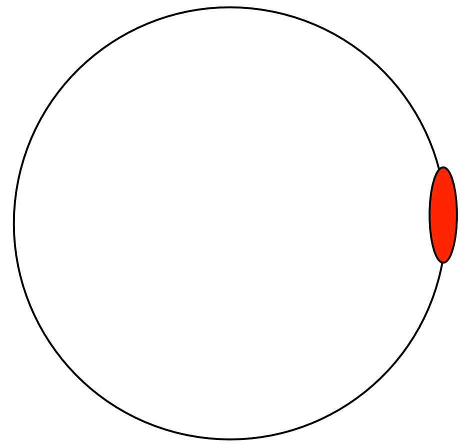
A black string wrapped on a large enough circle pinches to zero size to form a black hole [Gregory and Laflamme, 1994].



D-brane interpretation

Uniform black brane → (Gregory-Laflamme) → Localized black brane





Gauge theory (IIB version)

Eigenvalues of compact scalar uniform → Eigenvalues of clumped

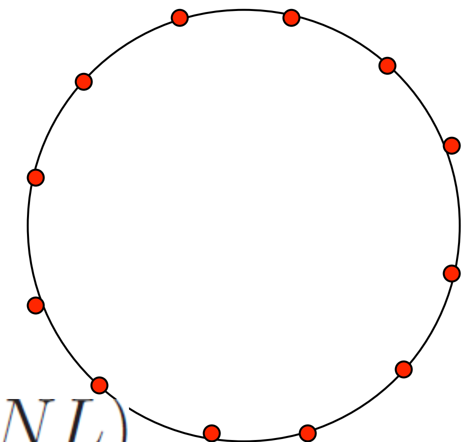
Gauge theory (IIA version)

Eigenvalues of uniform → (Deconfine-ment) → Eigenvalues of clumped

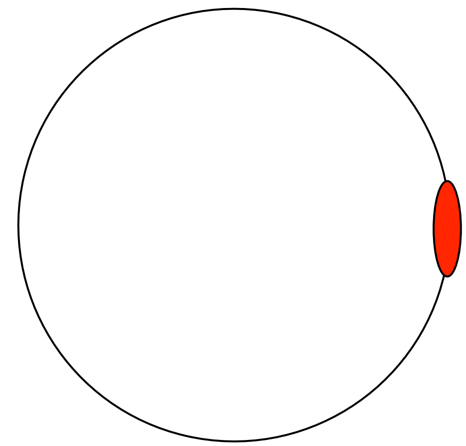
KK reduction in gauge theories [SU(N)]

Uniform eigenvalues of Wilson line=centre symmetry=fractional periodicity

Non-uniform eigenvalues=broken centre symmetry=integral periodicity



$$p = 2\pi n/L \to$$
$$p + A,\ A = 2\pi k/(NL)$$



Effective KK radius= N L
(Eguchi-Kawai, Kovtun-Unsal-Yaffe,…)

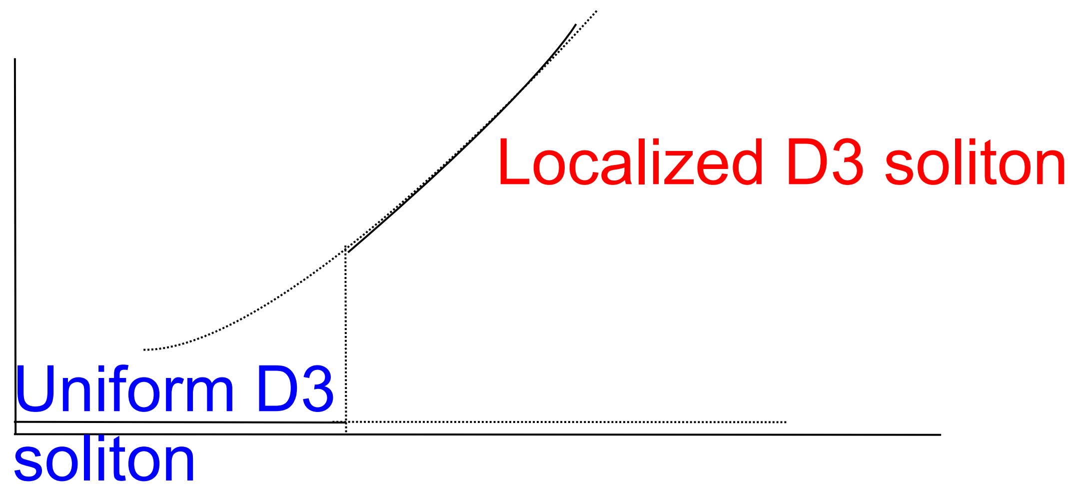
KK radius = L

In the large N limit, effective KK mass = 1/(NL) -> 0.
Circle decompactifies (Eguchi-Kawai equivalence, large N volume independence)

(back)

New Proposal (contd.)

In the present case, the GL transition gives rise to a localized solitonic D3 brane solution.



Pure YM4 $\subset$ SYM5 (deconfinement phase)

$W_0 \neq 0$

$W_4 \neq 0$

Localized D3 soliton

$W_0 \neq 0$ ✓

✓

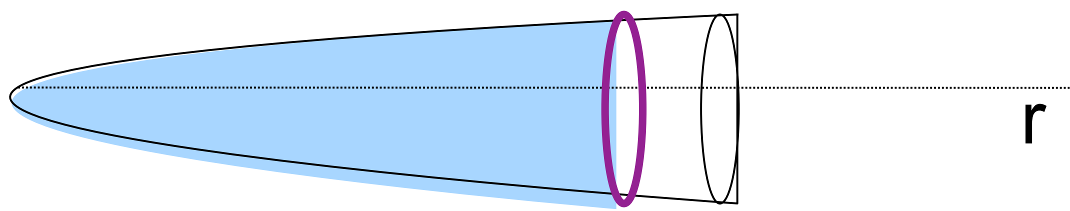
$W_4 \neq 0$ ✓

The high temperature (localized) phase

$$ds^2 = \rho^2 \left(\sum_i^3 dx_i^3 + f(\rho) dx_4^2 \right) + \rho^{-2} \left(d\rho^2 / f(\rho) + \rho^2 d\Omega_5^2 \right)$$

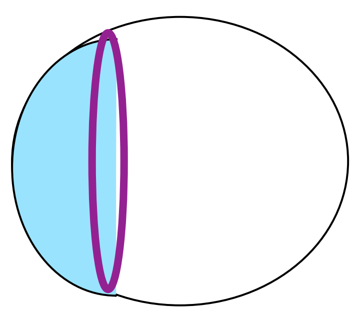
Classical action
Hence there is a non-zero entropy.

$W_4 \neq 0.$

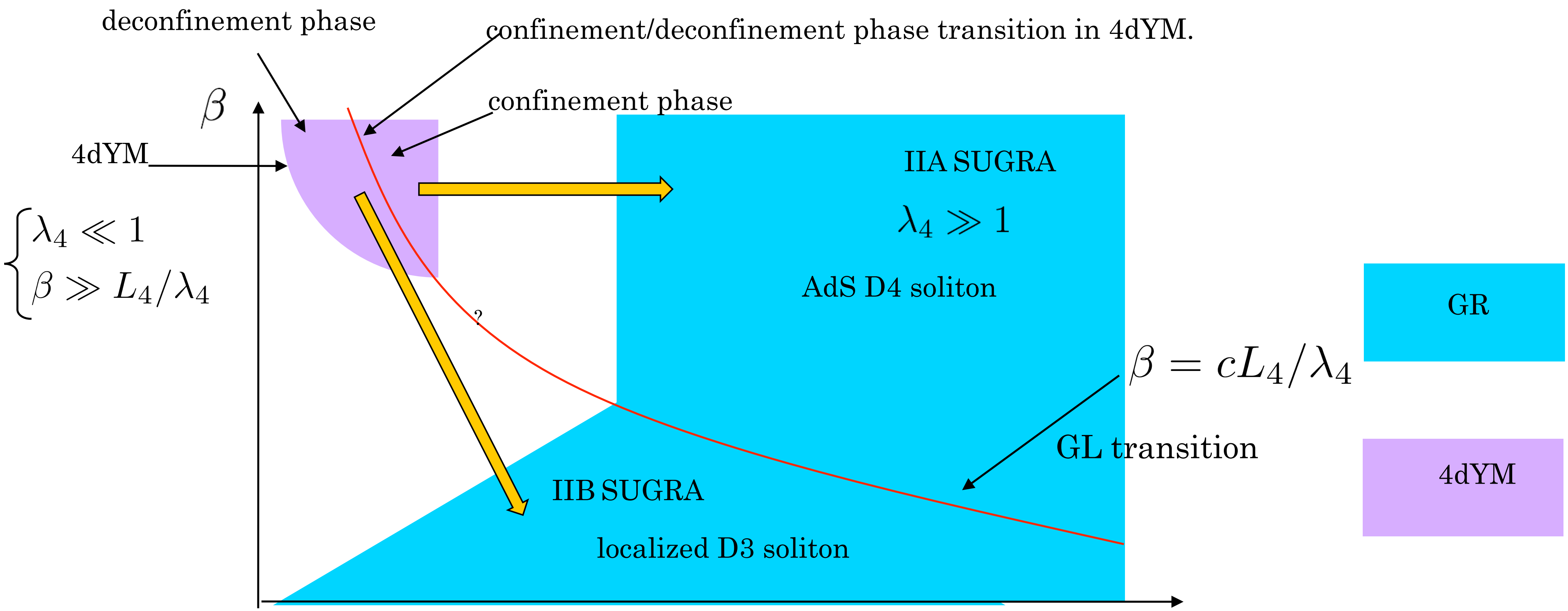


Note that the (x4, r) plane is still a cigar in the solitonic solutions, whether D4 or D3, (uniform or localized).

$W_0 \neq 0$



◆ Phase structure of 5dSYM in the periodic boundary condition



A smooth continuation exists between gravity and pure YM4 both the low temp and in the high temp phases.

Our proposal is therefore suitable for a strong coupling expansion, unlike the proposal of black D4 as deconfinement phase.

Some applications:

Deconfinement Temerature

$$T_c = \frac{M_{KK}}{2\pi}$$ = 151 , Hawking Page transition temperature (Witten's model)

Too low

$$T_c = \frac{\lambda_4 M_{KK}}{2\pi \times 8.54}$$ Gregory Laflamme transition temperature (our model)

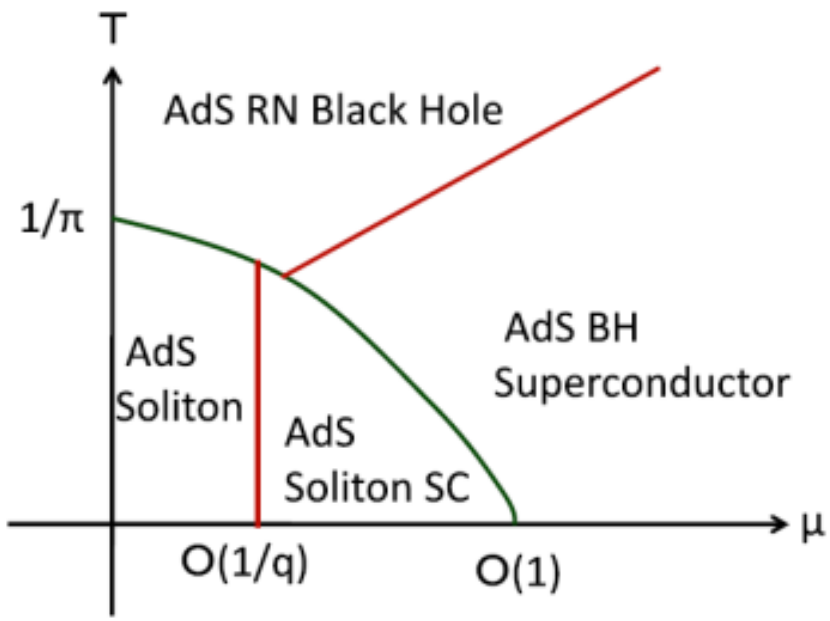
M_{KK} = 949 , λ_4 = 16.6 (12.5) , T_c = 294 (222)

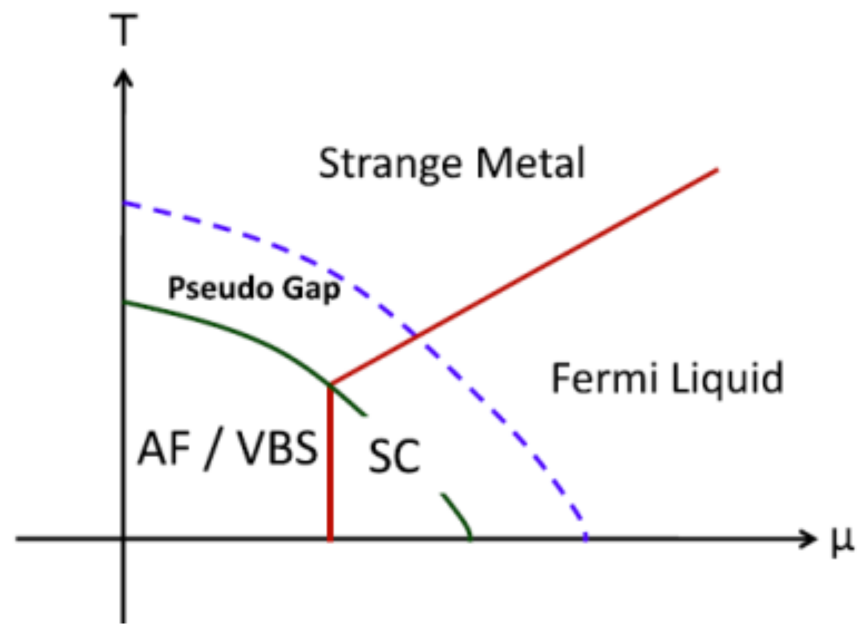
Right ballpark !

(Rebhan 2014)

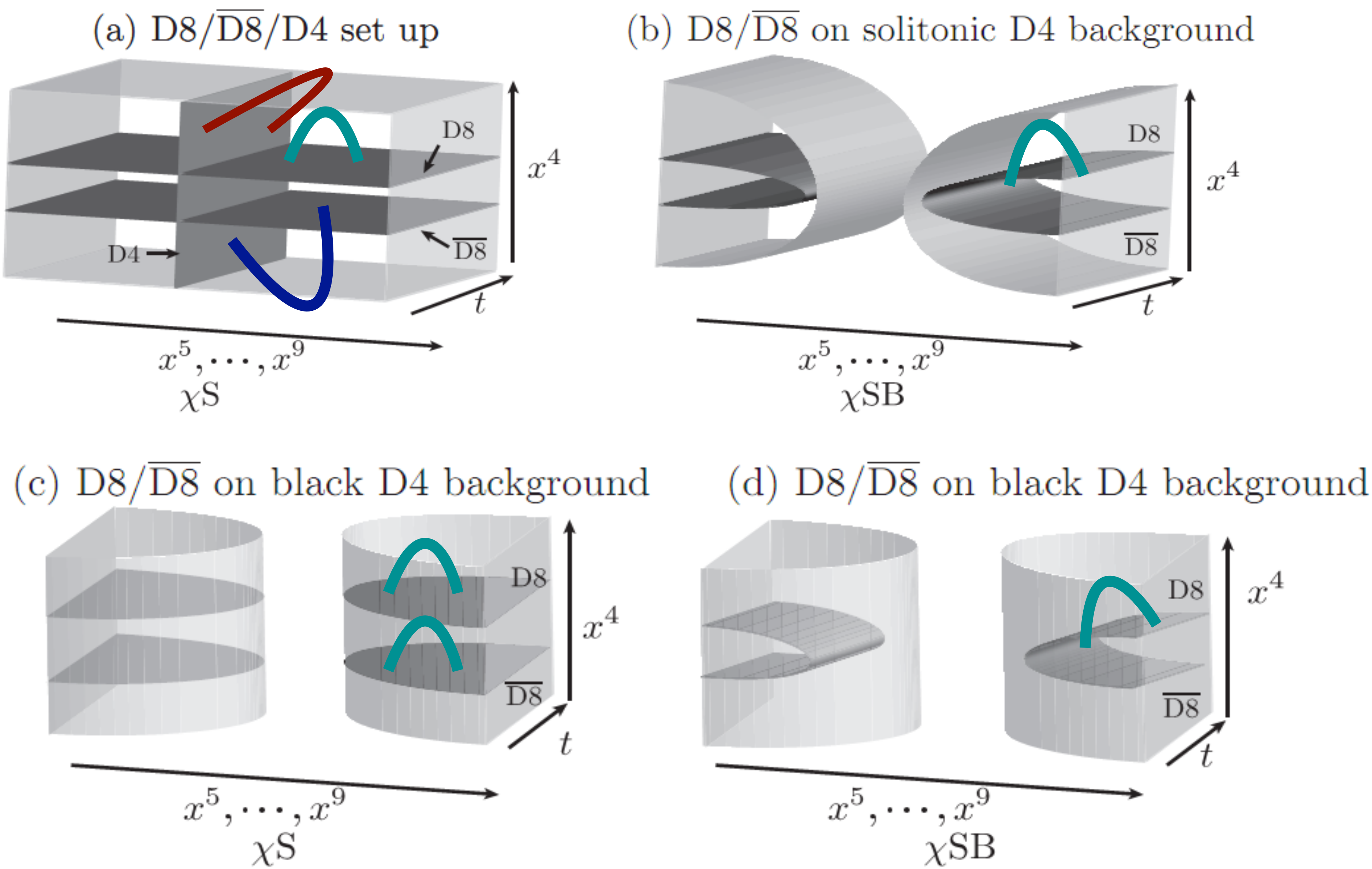
Insulator/Superconductor transition

Nishioka-Ryu-Takayanagi



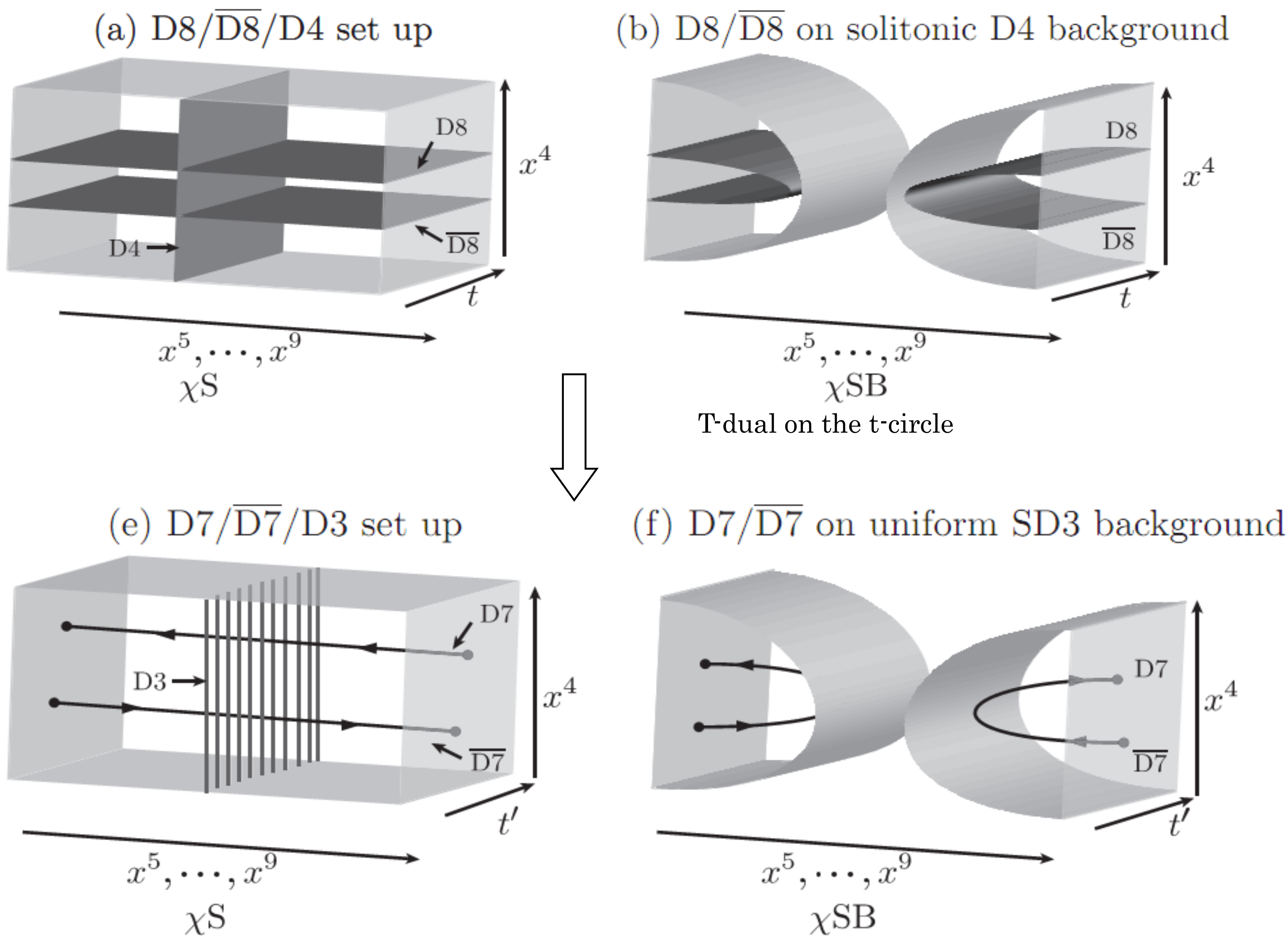


Chiral Symmetry breaking in Sakai-Sugimoto model

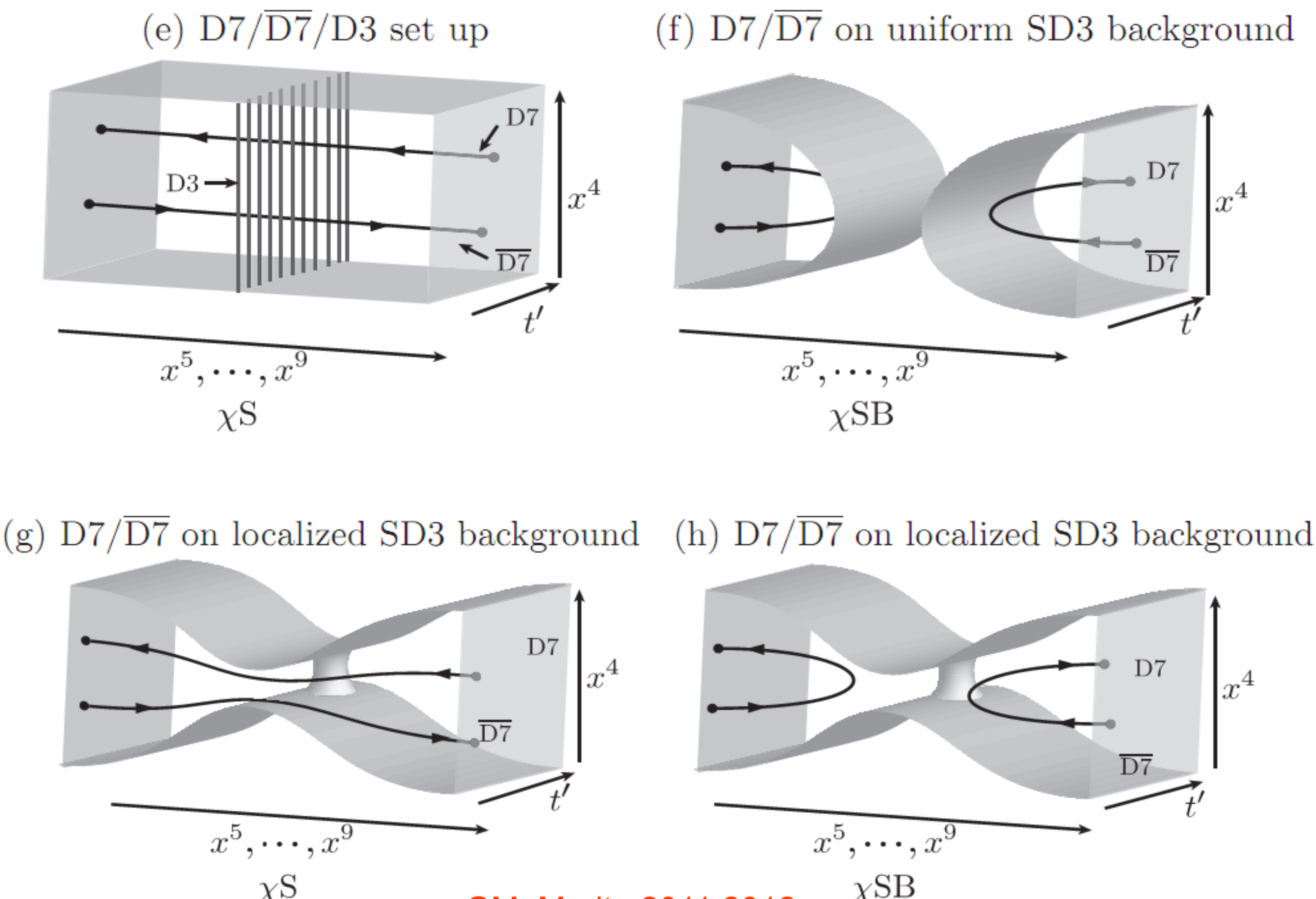


Aharony, Sonnenschein, Yankielowicz 2006

Chiral Symmetry breaking in Sakai-Sugimoto model

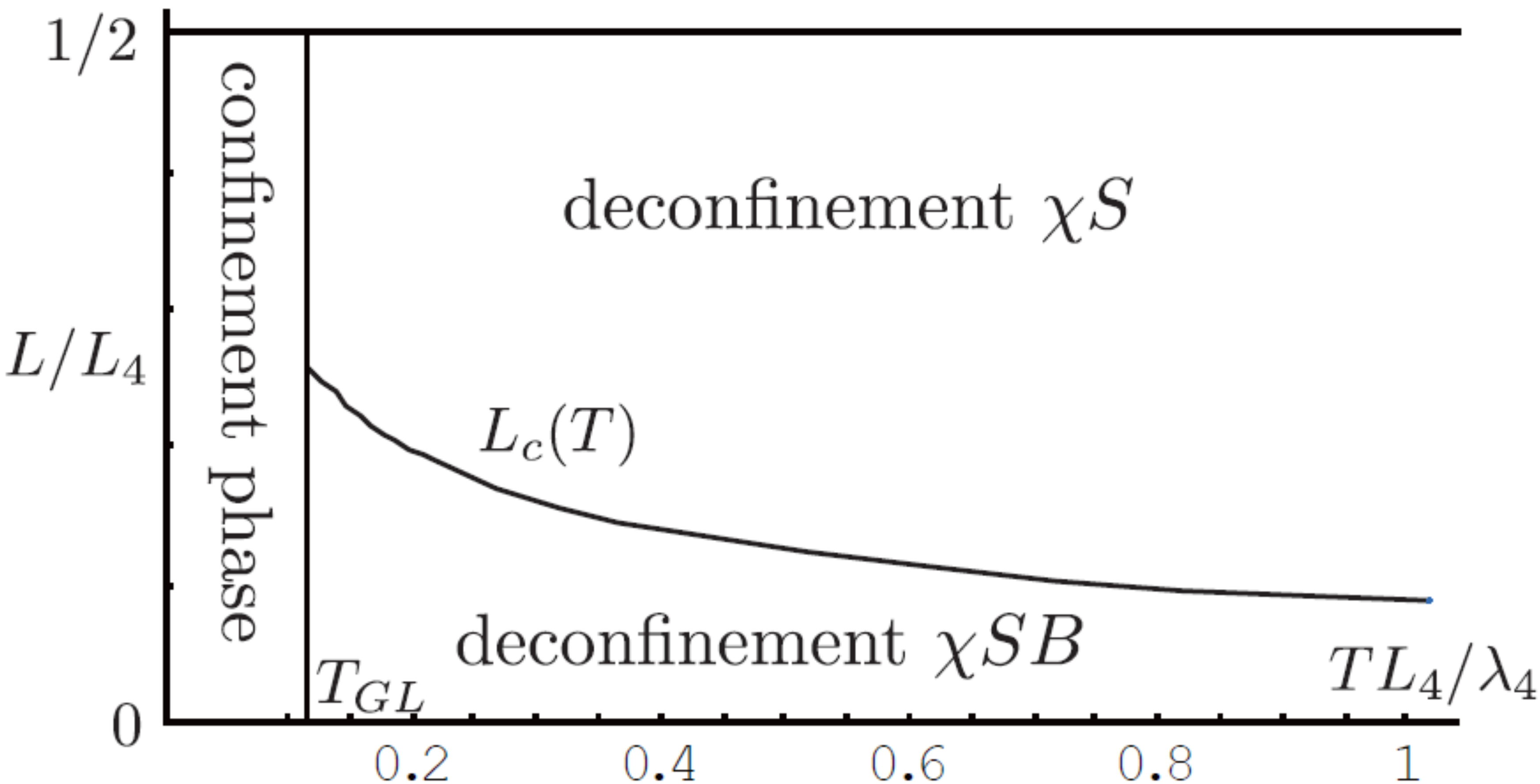


Chiral Symmetry breaking in Sakai-Sugimoto model



GM, Morita 2011,2012

Deconfinement followed by Chiral Symmetry Restoration



(GM, Morita 2011, 2012)

Conclusion

AdS/CFT can provide important information about phases of gauge theories, including nonsupersymmetric gauge theories.

Conventional gravity dual of deconfinement phase of 4D YM, namely the black brane solution, has problems (symmetries do not match).

We proposed Gregory-Laflamme transition (between uniform and clumped soliton) as the gravity dual of confinement/deconfinement transition; the

We presented (partial) phase diagrams of YM4/SYM5 in both the scenarios.

Our conclusions are valid for general *d*. For YM1/SYM2, lattice calculations support our proposal (Catteral, Joseph, Wiseman 2010; Narayan 2010).

Our proposal is consistent with weakly coupled low dimensional gauge theory calculations (GM, Morita 2010,11,12).

We explained chiral symmetry breaking/restoration in our model and proved the result T(S) > T(deconfinement).

Open questions

To understand dissipation in our model of deconfinement.

Lorenzian interpretation of our solution (it is localized in the Euclidean time-direction).

The significance of the boundary condition (AP, AP) along (t, x4).

Our observation seems to be generic. Whenever we use the SS transition, the high temperature phase of gravity would have different symmetries from that of the dual gauge theory. Generalize to other cases, including AdS/CMT.

Dynamical phase transitions and equilibration in the low dimensional gauge theory. GM, Morita 2013

When quarks are present in the 4D theory, how does one understand their periodic BC along time? Morita, next talk