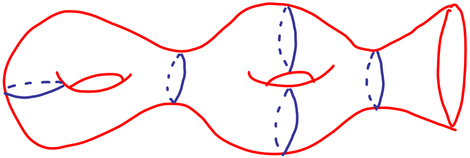
Geometric Microstates for the Three Dimensional Black Hole?



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Workshop: Applications of AdS/CFT to QCD and CMT

19-10-15

The Problem:

Gravity in 2+1 dimensions is more-or-less exactly solvable:

- No local degrees of freedom
- Global degrees of freedom associated with topology

Let's quantize them!

We will study the simplest possible theory: Chiral gravity.

Can we find a bulk, geometric description of black hole microstates?

Can we make the AdS_3/CFT_2 correspondence completely explicit?

The Theory:

The coupling constants

- Newton constant G
- Cosmological constant $\Lambda \sim -1/\ell^2 < 0$
- Gravitational Chern-Simons coupling $\frac{1}{\mu} = \ell$

can be combined into a dimensionless ratio $k = \frac{\ell}{16G} \in \mathbb{Z}$.

The states are labelled by

$$\Delta = \frac{1}{\ell}(M + J) \in \mathbb{Z}, \qquad \bar{\Delta} = \frac{1}{\ell}(M - J) = 0$$

The theory should be dual to a chiral CFT:

- $c = c_L = 24k$ is the central charge
- Δ is the L_0 eigenvalue of a state

The Strategy:

We will identify a class of black hole microstate geometries:

- Finite number of degrees of freedom coming from topology hidden behind the horizon.

A microstate is a wave function on the space of geometries:

- Holomorphic section of a certain line bundle on the moduli space of Riemann surfaces.
- Count states using intersection theory on moduli space.

The result will be compared with the semi-classical BH Entropy

$$S_{BH}(k, \Delta) \approx 4\pi\sqrt{k\Delta}$$

Plan for Today:

- Microstate Geometries
- Phase Space & Symplectic Structure
- Counting States

Quotients of AdS

AdS_3 can be written as

$$ds^2 = -dt^2 + \cos^2 t \; d\Sigma^2$$

where $d\Sigma^2$ is the hyperbolic metric.

Solutions are quotients AdS_3/Γ for some discrete group

$$\Gamma \subset SL(2, \mathbb{R}) \times SL(2, \mathbb{R}).$$

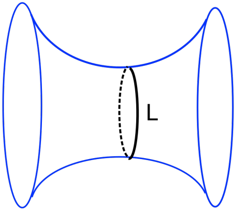
We will take Γ to be diagonal and hyperbolic:

- $d\Sigma^2$ is the negative curvature metric on a surface Σ_g.

These are (up to gauge equivalence) the most general known smooth solutions of Chiral gravity.

The BTZ Black Hole

The case where Σ is a cylinder is the eternal BTZ black hole:

$\Sigma =$ 

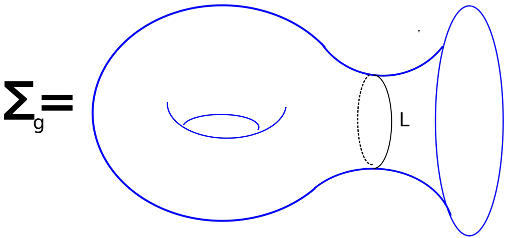
The two ends of the cylinder intersect the two asymptotic boundaries of BTZ.

The area of the black hole horizon is the length L of the minimum length geodesic on Σ.

The BTZ geometry does not describe a pure state in the Hilbert space of a single asymptotic observer, but rather a thermal mixed state.

Microstate Geometries

Take $\Sigma = \Sigma_g(L)$ to be a Riemann surface with one hole:



The geometry has a horizon, whose area is the length L of the minimum length geodesic around the hole.

The metric outside the horizon is *identical* to the BTZ black hole.

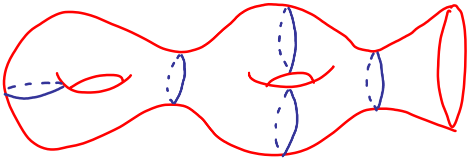
But the other asymptotic region has been replaced by topology behind the horizon.

This microstate geometry is dual to a pure state, not a mixed state.

The Space of Microstate Geometries

The configuration space is the moduli space $\mathcal{M}_g(L)$ of bordered Riemann surfaces.

If we decompose into pairs of pants we have a length L_i and a twist θ_i for each cuff.



So $\dim_{\mathbb{C}} \mathcal{M}_g(L) = 3g - 2$.

In Einstein gravity the configuration space would be two copies of Teichmuller space $(T_g \times T_g)/\mathrm{MCG}_g$, which has infinite volume.

But in chiral gravity the configuration space $\mathcal{M}_g(L) = T_g/\mathrm{MCG}_g$ is finite volume and compact (with a certain choice of compactification).

Symplectic Structure

The symplectic structure is the Weil-Peterson symplectic structure on $\mathcal{M}_g(L)$:

$$\omega = k \left(dL \wedge d\theta + \sum_i dL_i \wedge d\theta_i \right)$$

where L, θ are the length and twist parameters of the horizon.

The mass of the Black hole is

$$\Delta = kL^2 \ .$$

This is a function on phase space which becomes an operator upon quantization.

We would like to compute its spectrum.

Quantization of Black Hole Masses

The area L is conjugate to a twist parameter θ.

The geometry is invariant under Dehn twists $\theta \sim \theta + L$, so L is canonically conjugate to a periodic variable

$$[L, \theta] = k^{-1}$$

So the area spectrum is quantized

$$\Delta = kL^2 \in \mathbb{Z}$$

A similar argument gives the quantization of k.

Boundary Gravitons

Two metrics describe the same state only if they are related by a diffeomorphism which vanishes at infinity.

- We must quantize not the usual moduli space, but the space of metrics modulo *trivial* diffeomorphisms (a la Brown-Henneaux).

Quantizing the space of non-trivial diffeomoprhisms $diff(S^1)$ at the boundary gives the Virasoro descendants, i.e. boundary gravitons.

Quantizing the internal moduli of $\Sigma_g(L)$ gives new primaries.

These are the black hole microstates.

The Quantum States

The configuration space $\mathcal{M}_g(L)$ of *bordered* Riemann surfaces is symplectomorphic to the moduli space $\mathcal{M}_{g,1}$ of *punctured* Riemann surfaces:

$$\omega = k\kappa_1 + \Delta\psi_1$$

where

- κ is the Weil-Petersson class on $\mathcal{M}_{g,1}$
- ψ is the Chern class of the cotangent at the puncture

A black hole microstate is a holomorphic section of $\mathcal{L}_{k,\Delta}$ on $\mathcal{M}_{g,1}$, where

$$c_1(\mathcal{L}_{k,\Delta}) = k\kappa + \Delta\psi$$

The number of black hole microstates at genus g is

$$N_g(k,\Delta) = H^0(\mathcal{M}_{g,1}, \mathcal{L}_{k,\Delta})$$

.

Estimating $N_g(\Delta, k)$

The number of states can be estimated using intersection theory:

$$N_g(k,\Delta) \approx \int_{\mathcal{M}_{g,1}} e^{k\kappa+\Delta\psi} = \sum_{d=0}^{3g-2} \frac{k^{3g-2-d}\Delta^d}{(3g-2-d)!\ d!} \int_{\mathcal{M}_{g,1}} \kappa_1^{3g-2-d}\psi_1^d$$

Algorithms for computing these intersection numbers were given by Witten-Kontsevich, Mirzakhani, ...

- They can be computed explicitly for $g \lesssim 40$ (Zograf).
- We can make conjectures (supported by numerics) for intersection numbers at $g \to \infty$.

For $\Delta \gg g$ the number of states at fixed (large) genus

$$N_g(k,\Delta) \approx \frac{1}{(4g)!}\Delta^{3g-2}$$

is much smaller than the Black Hole entropy.

Large genus

The sum over genus diverges

$$N(k,\Delta) = \sum_{g=0}^{\infty} N_g(k,\Delta) = \infty$$

but (with a conjecture) the leading contribution

$$N(k,\Delta) \approx \sum_{d=0}^{\infty} \frac{1}{d!(2d+1)!!} \left(\frac{\pi^2}{2}\frac{\Delta}{k}\right)^d \left(\sum_{g \gg d} (2g)! k^{3g} + \dots\right)$$

gives an entropy linear in horizon area:

$$\log N(k,\Delta) = \pi\sqrt{\Delta/k} + C_g$$

The coefficient C_g is divergent but independent of Δ.

- Speculation: Non-perturbative effects $\implies$ C_g is finite.

This entropy is much smaller than the Bekenstein-Hawking entropy.

Conclusions

Quantizing topology behind the horizon leads to completely explicit, geometric black hole microstates:

At fixed genus we have no hope of accounting for black hole entropy.

- A large black hole can only be described by very complex topology behind the horizon.

The sum over genus diverges, and a natural regularization of this sum gives an entropy proportional to horizon area. But

$$\log N(k, \Delta) \approx \pi\sqrt{\Delta/k} \ll 4\pi\sqrt{k\Delta} = S_{BH}$$

Perhaps "pure" quantum gravity exists only when $k \approx \mathcal{O}(1)$.