Backreaction effects of matter coupled higher derivative gravity

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(Based on arXiv:1409.8019 and ongoing work with Ramadevi)

Indian Institute of Technology, Bombay

Applications of AdS/CFT to QCD and condensed matter physics, October 22, 2015

Plan of the talk

- *Part One*
  - **Find the background solution for the gravity action**
  - **Make note of backreaction**
- *Part Two*
  - **Find shear viscosity and entropy density for this background**
  - **Effects of backreaction**

Introduction

Strongly coupled field theories

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Strongly coupled field theories

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  - AdS_5/CFT_4 **[Maldacena, Adv.Theor.Math.Phys.2:231-252,1998]**

Type IIB string theory on $AdS_5 \times S^5 \longleftrightarrow \mathcal{N} = 4$ **SYM theory**.

- **For most liquids it is known that the ratio of shear viscosity to entropy density(η/s) exhibits a minimum at phase transition temperature.**
- **However, for strongly coupled liquids(e.g. QGP) this behavior has not been understood well.**
- **It is expected that at the phase transition temperature, η/s will show a minimum for QGP.**

- **To locate transition temperature precise behavior of η/s with T is needed.**

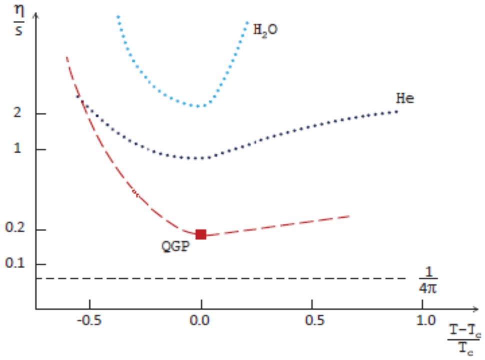


Figure: [arXiv:1206.3581]

- **The figure shows expectation taking analogy with H_2O and He plots.**

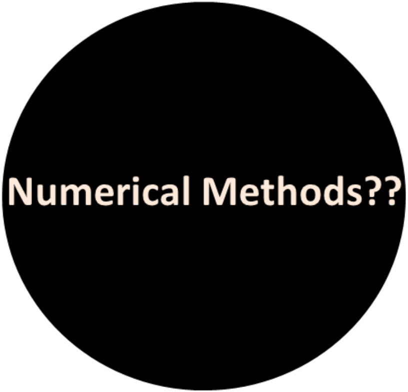
Part 1: Background Solution

The model:

$$S = \frac{1}{2\kappa_D^2}\int d^D x\left(R - \frac{4}{D-2}(\partial\Phi(r))^2 - V(\Phi(r)) + \beta G(\Phi(r)) R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho}\right)$$

- $V(\Phi(r))$: **Dilaton Potential.**
- $G(\Phi(r))$: **Higher derivative matter coupling** $= e^{\gamma\Phi(r)}$

- $SO(D-2)$ **symmetry in boundary spatial coordinates implies** 4 **coupled differential equations.**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Unavailability of initial data

Analytic solution

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Hints:

- **Limit** 1**: No matter,** $\Phi(r) \to 0$
- **Limit** 2**: No higher derivatives,** $\beta \to 0$

Background in no matter limit:

Einstein gravity with higher derivatives

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$$ds^2 = \frac{1}{r^2}\left(-f(r)dt^2 + d\vec{x}^2 + \frac{dr^2}{f(r)}\right)$$

with $$\Lambda = -\frac{(D-1)(D-2)}{2}$$

$$f(r) = 1 - r^{D-1} + \beta\left(\delta + \kappa \times r^{2(D-1)}\right)$$

- $\delta(r)$= 2(D-4)/(D-2)
- $\kappa(r)$=(D-3)(D-4)

[Kats and Petrov, arXiv:0712.0743]

The horizon shifts as $r_h = 1 + \beta r_1$

Background without higher derivatives:

scalar matter coupled 2-derivative action

$$\begin{aligned} ds^2 &= -r^{-2a}(1-r^c)dt^2 + \frac{dr^2}{r^{-2a}(1-r^c)r^4} + r^{-2a}d\vec{x}^2 \\ \Phi(r) &= m\mathrm{Log}(r) \end{aligned}$$

[arXiv:1211.5972]

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When $V(\Phi) = 2\Lambda e^{\alpha\Phi(r)}$,

$$\Lambda = \frac{8(D-2)\left(-16(D-1)+(D-2)^2\alpha^2\right)}{\left(16+(D-2)^2\alpha^2\right)^2}$$
$$a = \frac{16}{(D-2)^2\alpha^2+16} \; ; \; m = \frac{2(D-2)^2\alpha}{(D-2)^2\alpha^2+16}$$
$$c = \frac{16(D-1)-(D-2)^2\alpha^2}{(D-2)^2\alpha^2+16}$$

The full action

$$S = \frac{1}{2\kappa_D^2}\int d^D x(R - \frac{4}{D-2}(\partial\Phi(r))^2 - V(\Phi(r)) + \beta G(\Phi(r))R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho})$$

$$V(\Phi(r)) = 2\Lambda e^{\alpha\Phi} \; ; \; \alpha > 0.$$

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- **To find the solution up to $\mathcal{O}(\beta)$, we choose the generalization as:**

$$\begin{aligned} ds^2 &= -r^{-2a}(1 - r^{c(\beta,r)})dt^2 + \frac{dr^2}{r^{-2a}(1 - r^{c(\beta,r)})r^4} + r^{-2a} d\vec{x}^2 \\ \Phi(r) &= m\mathrm{Log}(r) + \beta\Phi_1(r) \end{aligned}$$

where $c(\beta, r)$ is taken as:

$$c(\beta, r) = c + \frac{Log(1 - \beta\kappa(r))}{Log(r)}$$

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$$\begin{aligned} ds^2 &= -r^{-2a}(1 - r^{c_+(\beta,r)})dt^2 + \frac{dr^2}{r^{-2a}(1 - r^{c_-(\beta,r)})r^4} + r^{-2a}d\vec{x}^2 \\ \Phi(r) &= m\text{Log}(r) + \beta\Phi_1(r) \end{aligned}$$

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Backreactions

The differential equations provide:

$\kappa(r) =$

$$\frac{2pr^{-q-1}}{D-2}\left[-b(D-2)r^{2q}\left\{2^{11}(D-4)(D-1)(D-3)+\gamma^2(D-2)^2\times\right.\right.$$
$$\left.\left(\gamma^4(D-2)^4-8\gamma^2(D-2)^2(3D-10)+2^8(D-5)\right)\right\}+64\gamma^2(D-2)^3r^2\times$$
$$\left(\gamma^2(D-2)^2-8\right)\left(\gamma^2(D-2)^2-16(D-1)\right)\log(r)-\left\{2^7(D-4)(D-1)\right.$$
$$\left.\left.-2^5\gamma^2(D-2)^2\left(D^2-3D+1\right)+\gamma^4(D-2)^4(3D-7)\right\}32br^2\right]$$

where, $$q=\frac{16D}{\gamma^2(D-2)^2+16}=a.D;\quad b=\gamma^2(D-2)^2+16;$$

and $$p=\frac{1}{\left(\gamma^2(D-2)^2-16(D-1)\right)\left(\gamma^2(D-2)^2+16\right)^2}. \tag{1}$$

Backreactions

$\delta(r) =$

$$\frac{4\gamma^2(D-2)^2 p\left(1-r^{1-q}\right)}{r}\Big[b(D-2)\left\{\gamma^4(D-2)^3-16\gamma^2(D-2)^2+2^8\right\}r^q$$
$$+br\left\{\gamma^4(D-2)^4-16\gamma^2(D-8)(D-2)^2+2^8(D-5)\right\}\log\left(r^q-r\right)$$
$$+\left\{-\gamma^2(D-2)^2\left(\gamma^2(D-2)^2\left(\gamma^2(D-2)^2-16(D-7)\right)+2^9(D+1)\right)\right.$$
$$\left.+2^{14}\right\}r\log(r)\Big]$$

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$\Phi_1(r) =$

$$\frac{4\gamma(D-2)^2 p}{r}\left[-br\left\{\gamma^4(D-2)^4 - 16\gamma^2(D-8)(D-2)^2 + 2^8(D-5)\right\}\times \log\left(r^q - r\right) - \left\{\gamma^4(D-2)^4 - 16\gamma^2(D-4)(D-2)^2 + 2^8(D-3)\right\} r^q b + \left\{\gamma^2(D-2)^2\left(\gamma^4(D-2)^4 - 16\gamma^2(D-5)(D-2)^2 + 2^8(4D+1)\right) - 2^{12}(D+3)\right\} r\log(r)\right]$$

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Rewrite $log(r^q - r)$ as,

$$log(r^q - r) = q\ log(r) + log(1 - e^{2\pi i(1-q)}(1 + (r-1)e^{-2\pi i})^{1-q})$$

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and choose,

$$q = \frac{1}{2}(2l+1)\ ;\ l \in \mathbb{Z}_+.$$

at $r = 1$ $\rightarrow$log(2)

$$\gamma^2 = \frac{16(D-q)}{(D-2)^2 q}$$

Closer look at $V(\Phi)$ and γ (or α)

1. Improved holographic theories of QCD
   U. Gursoy and E. Kiritsis, arXiv:0707.1324 hep-th and arXiv:0707.1349,
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- $q = \frac{5}{2}$

Very well backed up by theory and regularity of solution.

Part 2

Kubo formula for viscosity Low energy and low momentum limit of retarded Green's function of stress tensor,

$$\eta = -\lim_{\omega \to 0} \frac{1}{\omega} Im G^R_{xy;xy}(\omega, \mathbf{k} = 0)$$

- Translate the calculation of the correlator to a holographic one
- perturbation in the xy components of the metric.

$$g_{xy} = g^0_{xy} + \epsilon h_{xy}(r,x) = g^0_{xy} + \epsilon g_{ii}\phi(r,x), \qquad \left[x = (t, \overrightarrow{x})\right]$$

with

$$\phi(r,x) = \frac{1}{(2\pi)^4} \int d^4k e^{-ik.x} \phi(r,k),$$

$$\left[k = (\omega, \overrightarrow{k}), \qquad k.x = k_\mu x^\mu\right]$$

- Effective action up to order two for any power of curvature tensor under this perturbation:

$$S \sim \int \frac{d\omega d^{D-2}k}{(2\pi)^{D-1}} dr \left[A(r,k)\phi''(r,k)\phi(r,-k) + B(r,k)\phi'(r,k)\phi'(r,-k) \right.$$
$$+C(r,k)\phi'(r,k)\phi(r,-k) + D(r,k)\phi(r,k)\phi(r,-k)$$
$$\left. +E(r,k)\phi''(r,k)\phi''(r,-k) + F(r,k)\phi''\phi'(r,-k)\right] + S_{GH}S_{GH}s_{GH}s_{GH}$$

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- Corresponding to this metric,

$$\eta = \left[\sqrt{\frac{-g_{rr}}{g_{tt}}} \left(A - B + \frac{F'}{2} \right) + \left(E \left(\sqrt{\frac{-g_{rr}}{g_{tt}}} \right)' \right)' \right] \Bigg|_{r=r_H}$$

[Myers, Poulos and Sinha: arXiv 0903.2834]

Entropy density

Wald formula

$$S = -2\pi \int_{\Sigma} d^{D-2}x\sqrt{-h}\frac{\delta L}{\delta R_{\mu\nu\rho\sigma}}\epsilon_{\mu\nu}\epsilon_{\rho\sigma}$$

1. $\epsilon_{\mu\nu}\epsilon^{\mu\nu} = -2$
2. $\epsilon_{\mu\nu} = \xi_\mu \eta_\nu - \xi_\nu \eta_\mu$

The L above is chosen by writing the action as

$$S_{action} \sim \int d^5x\sqrt{-g}L$$

- $r_h = 1 + \beta r_1 + \beta^2 r_2,$
- $\eta = \eta^{(0)} + \beta\eta^{(1)}(r_1) + \beta^2\eta^{(2)}(..., r_1, r_2),$
- $s = s^{(0)} + \beta s^{(1)}(r_1) + \beta^2 s^{(2)}(..., r_1, r_2)$

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- $s = s^{(0)} + \beta s^{(1)}(r_1) + \beta^2 s^{(2)}(..., r_1, r_2)$
- $\frac{\eta}{s} =$

$$\frac{1}{4\pi} + \beta\frac{\left(81\gamma^4 - 504\gamma^2 - 512\right)}{\pi\left(9\gamma^2+16\right)^2} + \beta^2\frac{1}{2\pi\left(9\gamma^2+16\right)^4} \times$$
$$\left(-6561\gamma^9\xi'(1) + 13122\gamma^9\xi(1) - 13122\gamma^8\delta'(1) + 13122\gamma^8\kappa'(1)\right.$$
$$-13122\gamma^8\kappa(1) + 11664\gamma^7\xi'(1) - 34992\gamma^7\xi(1) - 58320\gamma^6\delta'(1)$$
$$+81648\gamma^6\kappa'(1) + 81648\gamma^6\kappa(1) + 46656\gamma^6 + 186624\gamma^5\xi'(1)$$
$$-331776\gamma^5\xi(1) - 62208\gamma^4\delta'(1) + 186624\gamma^4\kappa'(1) + 456192\gamma^4\kappa(1)$$
$$-622080\gamma^4 + 405504\gamma^3\xi'(1) - 552960\gamma^3\xi(1) + 36864\gamma^2\delta'(1)$$
$$+184320\gamma^2\kappa'(1) + 552960\gamma^2\kappa(1) + 1769472\gamma^2 + 262144\gamma\xi'(1)$$
$$-262144\gamma\xi(1) + 65536\delta'(1) + 65536\kappa'(1) + 131072\kappa(1) + 2097152$$
$$\left.-13122\gamma^8 r_1 + 151632\gamma^6 r_1 - 311040\gamma^4 r_1 - 811008\gamma^2 r_1 + 524288 r_1\right)$$

Qualitative numbers

$\gamma^2 = \frac{8}{9}$, $q = \frac{5}{2}$ and for $\beta = .01$

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- $\frac{\eta}{s}^{(0)} = 0.7853981634$
- $\frac{\eta}{s}^{(1)} = -0.00495149$
- $\frac{\eta}{s}^{(2)} = 0.000257798$

Summary and outlook

- Found the first order correction to the background metric for matter coupled higher derivative AdS gravity.
- Horizon gets a linear order correction due to matter presence.
- Shear viscosity and Entropy density get linear order correction via the correction in horizon
- However, the ratio η/s remains unaffected due to the backreaction in the metric.

A lot more to go: need the analysis with QCD-like phenomenological potentials.

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Thank You