

Hadrons in AdS/QCD and other holographic models

Henrique Boschi Filho
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in "Applications of AdS/CFT to QCD and condensed matter physics",
CRM, Montreal, October 22, 2015

Work done in collaboration with:
Nelson Braga, Eduardo Capossoli, Hector Carrion,
Alex Miranda, Luis Mamani, Marcus Torres, Matthias Ihl,
and C. Alfonso Ballón-Bayona.

Summary of the talk:

- **Brief Review: AdS/CFT correspondence and AdS/QCD**
- **Glueballs in AdS/QCD and the Pomeron**
- **Glueballs in AdS/QCD and the Odderon**
- **Finite T AdS/QCD**
- **Other Results**

AdS/CFT correspondence, J. Maldacena, 1997

(simplified version of a particular useful case)

Exact equivalence between String Theory in a 10-dimensional space and a gauge theory on the 4-dimensional boundary.

Remarks.:

String theory space = $\text{AdS}_5 \times S^5$

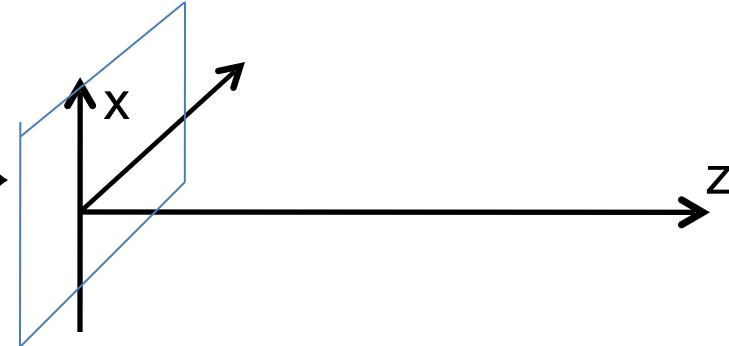
Gauge theory: $SU(N)$ with very large N
(supersymmetric and **conformal**).

At low energies string theory is represented by an effective
supergravity theory → **gauge / gravity duality**

Anti-de Sitter space (Poincaré patch)

$$ds^2 = \frac{R^2}{(z)^2} (dz^2 + (d\vec{x})^2 - dt^2)$$

The 4-dim boundary is at $z = 0$ →



Holographic bulk \leftrightarrow boundary mapping

E. Witten and L. Susskind (1998) ; A. Peet and J. Polchinski (1998)

States of the boundary
gauge theory with energy E



Region of AdS space
 $z \sim \frac{1}{E}$

Holographic relation suggests:

Cut off in AdS space: $0 < Z < Z_{max}$



infrared cut off in gauge theory.

This idea was introduced by Polchinski and Strassler (PRL 2002) to find the scaling of high energy Glueball scattering amplitudes at fixed angles.

Hadronic masses from AdS/CFT H.B.-F and N. Braga, JHEP2003, EPJC2004

Glueballs



Normalizable modes of a scalar field in an AdS slice with size $Z_{max} = 1/\Lambda_{QCD}$

$$\mathcal{O}_4 = F^2,$$

- Masses of scalar glueballs $J^{PC} = 0^{++}, 0^{++*}, 0^{++**}, \dots$ with good agreement with lattice results.

This kind of model was extended to other hadrons and then called the Hard wall AdS/QCD model.

Brodsky, Teramond PRL 2005, 2006; Erlich, Katz, Son, Stephanov PRL 2005.

Glueballs in the Hard-wall model and the Pomeron (J++) $P=C=+1$, $J=(0), 2, 4, \dots$

4d Glueball states are described in AdS(5) by Bessel functions which satisfy some boundary condition at $z=z_{\max}$.

For massive scalar fields in AdS_5:

Boundary operator:

$$\left[z^3 \partial_z \frac{1}{z^3} \partial_z + \eta^{\mu\nu} \partial_\mu \partial_\nu - \frac{(\mu R)^2}{z^2} \right] \phi = 0.$$

spin $\ell = J$.

$$\mathcal{O}_{4+\ell} = F D_{\{\mu_1 \dots \mu_\ell\}} F$$

$$\phi(x, z) = C_{\nu, k} e^{-i P_x z} z^\nu J_\nu(u_{\nu, k} z),$$

$$\Delta = 4 + \ell.$$

$$\Delta = 2 + \sqrt{4 + (\mu R)^2}. \quad (\mu R)^2 = \ell(\ell + 4). \quad \nu = 2 + \ell$$

Dirichlet boundary conditions

$$u_{\nu, k} = \frac{\chi_{\nu, k}}{z_{\max}} = \chi_{\nu, k} \Lambda_{\text{QCD}}; \quad J_\nu(\chi_{\nu, k}) = 0.$$

The zeros of the Bessel functions give the masses of the Glueballs

Dirichlet boundary conditions

TABLE I. Masses of glueball states J^{PC} with even J expressed in GeV, estimated using the sliced $\text{AdS}_5 \times S^5$ space with Dirichlet boundary conditions. The mass of 0^{++} is an input from lattice results [38,39].

Dirichlet glueballs	lightest state	1st excited state	2nd excited state
0^{++}	1.63	2.67	3.69
2^{++}	2.41	3.51	4.56
4^{++}	3.15	4.31	5.40
6^{++}	3.88	5.85	6.21
8^{++}	4.59	5.85	7.00
10^{++}	5.30	6.60	7.77

HBF, Braga, Carrion, PRD 2006

Our result for the ratio of masses $M_{2^{++}}/M_{0^{++}} = 1.48$ is
in good agreement with lattice |

Morningstar, Peardon, PRD 1997, 1999; Teper hep-lat/9711011.

Neumann boundary conditions

$$u_{\nu,k} = \frac{\xi_{\nu,k}}{z_{\max}} = \xi_{\nu,k} \Lambda_{\text{QCD}} \quad (2 - \nu) J_\nu(\xi_{\nu,k}) + \xi_{\nu,k} J_{\nu-1}(\xi_{\nu,k}) = 0.$$

TABLE II. Masses of glueball states J^{PC} with even J expressed in GeV, estimated using the sliced $\text{AdS}_5 \times S^5$ space with Neumann boundary conditions. The mass of 0^{++} is an input from lattice results [38,39].

Neumann glueballs	lightest state	1st excited state	2nd excited state
0^{++}	1.63	2.98	4.33
2^{++}	2.54	4.06	5.47
4^{++}	3.45	5.09	6.56
6^{++}	4.34	6.09	7.62
8^{++}	5.23	7.08	8.66
10^{++}	6.12	8.05	9.68

$$\frac{M_{2^{++}}}{M_{0^{++}}} = 1.56 \quad ; \quad \frac{M_{0^{++*}}}{M_{0^{++}}} = 1.83 \quad \text{very good agreement with lattice}$$

REGGE TRAJECTORIES

Dirichlet

$$J = \alpha(t = M^2) = \alpha_0 + \alpha' M^2.$$

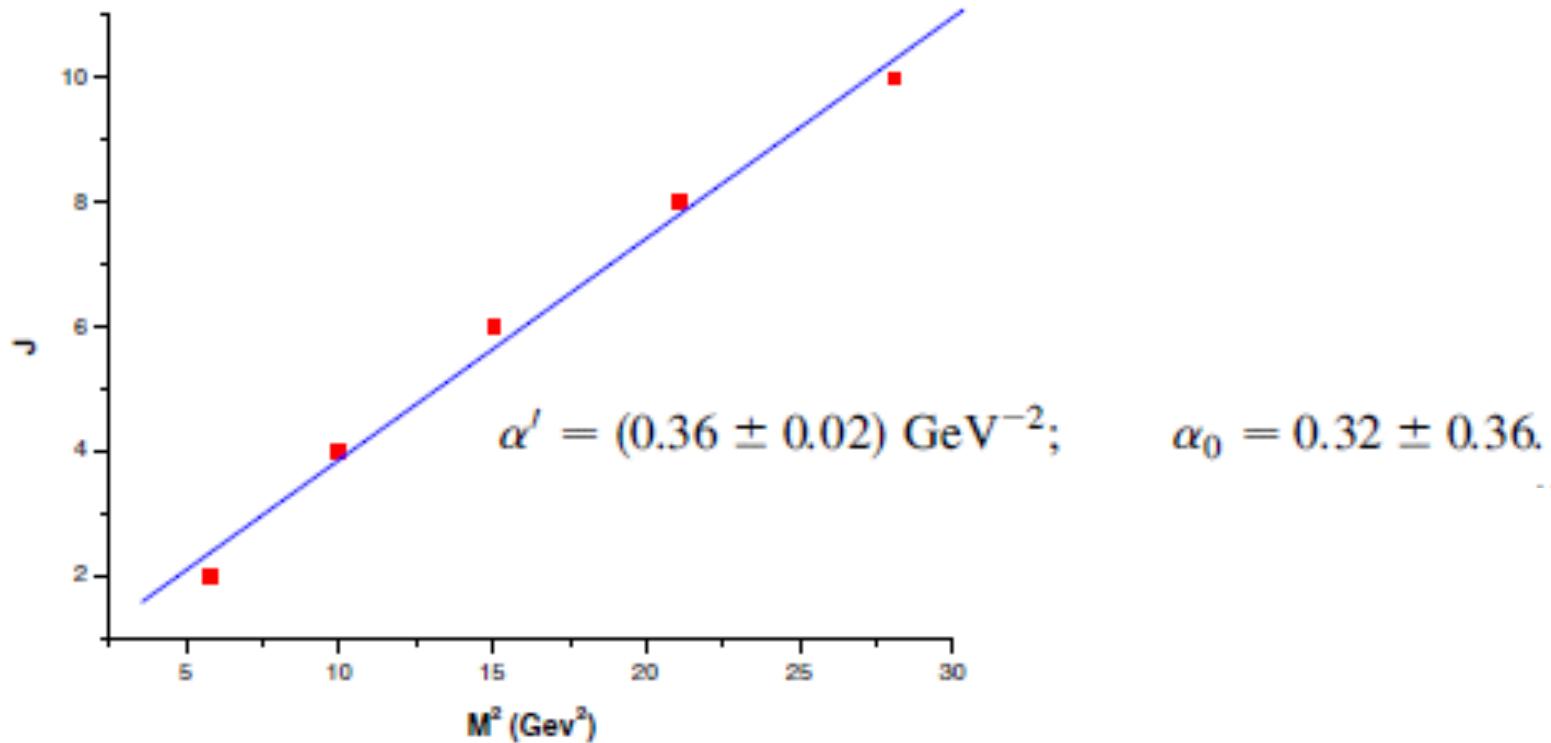


FIG. 2 (color online). Approximate linear Regge trajectory for Dirichlet Boundary condition for the states $2^{++}, 4^{++}, 6^{++}, 8^{++}, 10^{++}$.

REGGE TRAJECTORIES

Neumann

$$J = \alpha(t = M^2) = \alpha_0 + \alpha' M^2.$$

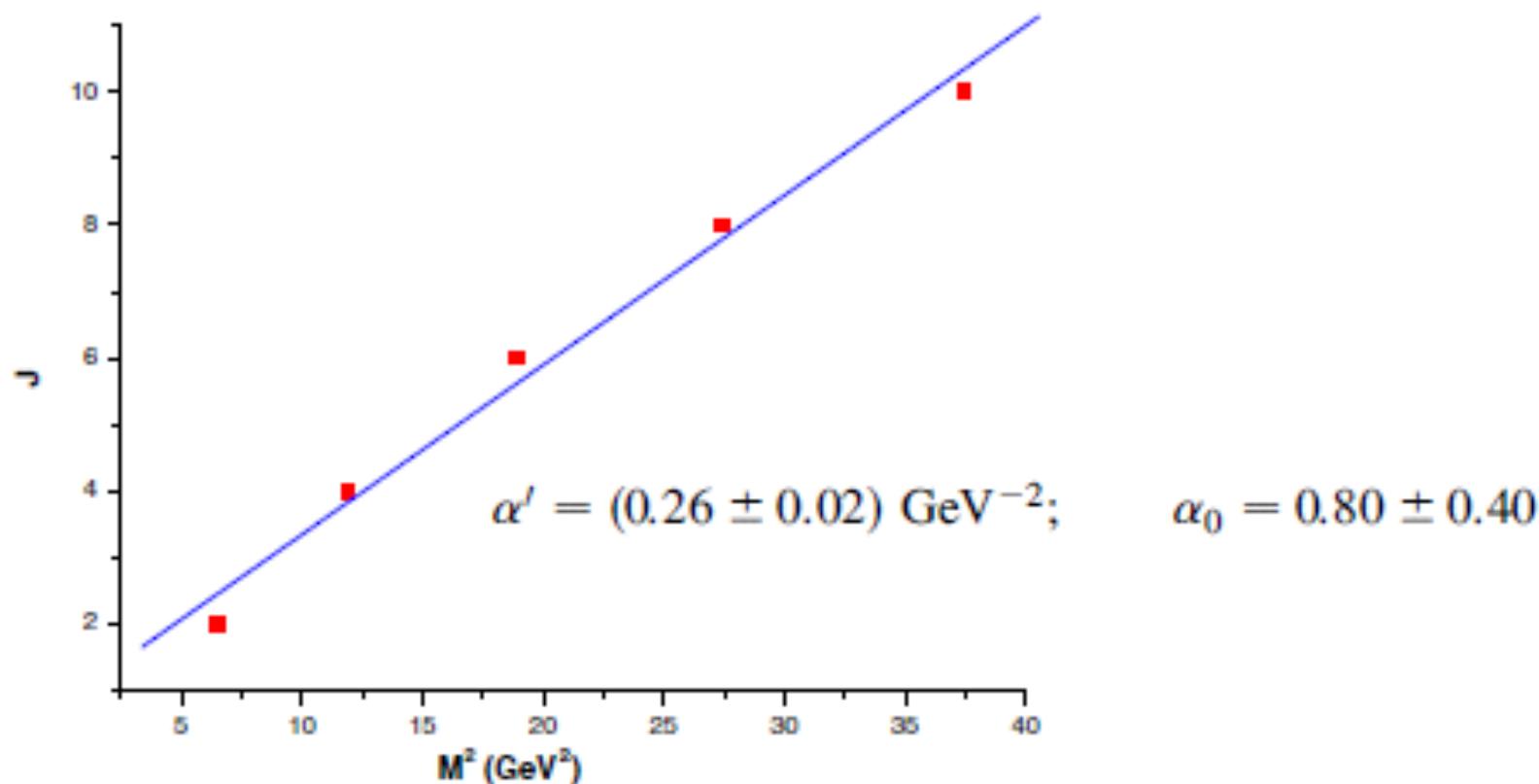


FIG. 1 (color online). Approximate linear Regge trajectory for Neumann Boundary condition for the states $2^{++}, 4^{++}, 6^{++}, 8^{++}, 10^{++}$.

Regge trajectory for the Pomeron (experimental)

$$\alpha(t = M^2) \approx 1.08 + 0.25M^2$$

The Hard-wall Regge trajectories for Glueballs with
Neumann boundary conditions

$$\alpha' = (0.26 \pm 0.02) \text{ GeV}^{-2}; \quad \alpha_0 = 0.80 \pm 0.40$$

are in good agreement.

Odderon Regge trajectories

F. J. Llanes-Estrada, P. Bicudo, and S. R. Cotanch, Phys. Rev. Lett. **96**, 081601 (2006).

Relativistic many-body model (RMB)

$$J_{\text{RMB}}(m^2) = -0.88 + 0.23m^2,$$

Non-relativistic constituent model (NRCM)

$$J_{\text{NRCM}}(m^2) = 0.25 + 0.18m^2.$$

Experimental signs of the Odderon

The best experimental evidence for the odderon occurred in 1985 at ISR CERN. A difference between differential cross sections for pp and $p\bar{p}$ in the dip-shoulder region $1.1 < |t| < 1.5 \text{ GeV}^2$ at $\sqrt{s} = 52.8 \text{ GeV}$ was measured, but these results were not confirmed [14].

There are two more evidences related to the nonperturbative odderon, that is, the change of shape in the polarization in $\pi^- p \rightarrow \pi^0 n$ from $p_L = 5 \text{ GeV}/c$ [16,17] to $p_L = 40 \text{ GeV}/c$ [18] and a strange structure seen in the UA4/2 dN/dt data for pp scattering at $\sqrt{s} = 541 \text{ GeV}$, namely a bump centered at $|t| = 2 \times 10^{-3} \text{ GeV}^2$ [19].

- [14] R. Avila, P. Gauronm, and B. Nicolescu, *Eur. Phys. J. C* **49**, 581 (2007).
- [15] Z.-H. Hu, L.-J. Zhou, and W.-X. Ma, *Commun. Theor. Phys.* **49**, 729 (2008).
- [16] D. Hill *et al.*, *Phys. Rev. Lett.* **30**, 239 (1973).
- [17] P. Bonamy *et al.*, *Nucl. Phys.* **B52**, 392 (1973).
- [18] V. D. Apokin *et al.*, *AIP Conf. Proc.* **95**, 118 (2008).
- [19] C. Augier *et al.* (UA4/2 Collaboration), *Phys. Lett. B* **316**, 448 (1993).

Experimental signs of the Odderon

LCH new results?

Some groups are looking for the Odderon...

Odd spin ($P=C=-1$) Glueballs and the Odderon

Eduardo Capossoli and H. Boschi PRD 2013

Massive scalar fields in AdS_5

$$\left[z^3 \partial_z \frac{1}{z^3} \partial_z + \eta^{\alpha\beta} \partial_\alpha \partial_\beta - \frac{m_5^2 R^2}{z^2} \right] \phi(x, z) = 0, \quad \text{Boundary operator}$$

$$m_5^2 R^2 = (\Delta - p)(\Delta + p - 4). \quad (p=0) \quad \begin{aligned} \mathcal{O}_{6+\ell} &= \text{Sym Tr}(\tilde{F}_{\mu\nu} F D_{\{\mu_1} \dots D_{\mu_\ell\}} F) \\ \text{conformal dimension } \Delta &= 6 + \ell \\ \text{spin } \ell &= J \geq 1 \end{aligned}$$

$$\phi(x, z) = A_{\nu, k} \exp^{-ip.x} z^2 J_\nu(u_{\nu, k} z),$$

$$\nu = \sqrt{4 + m_5^2 R^2}, \quad \nu = 4 + \ell.$$

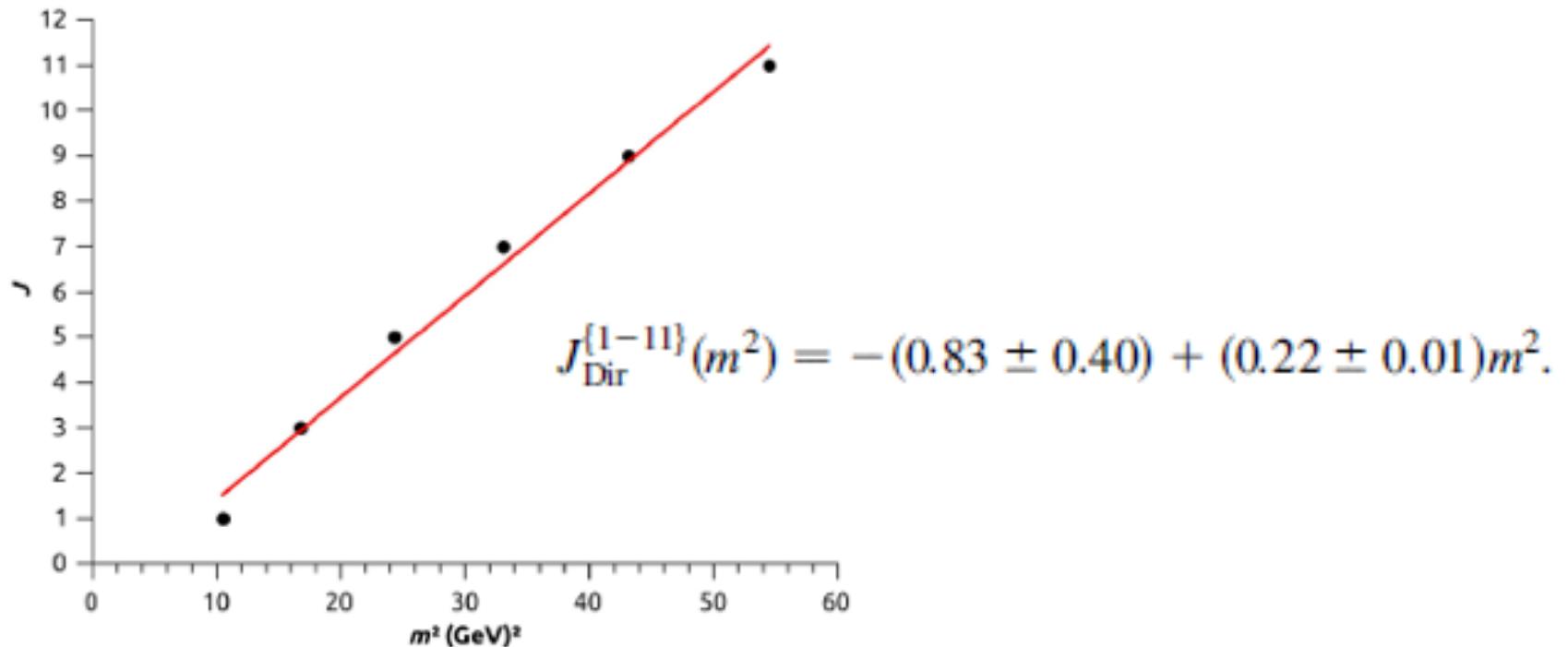
glueball states $1^{--}, 3^{--}, 5^{--}$, etc.

TABLE I. Glueball masses for states J^{PC} expressed in GeV, with odd J estimated using the hardwall model with Dirichlet and Neumann boundary conditions. The mass of 1^{--} is used as an input from the isotropic lattice [36,37]. We also show other results from the literature for comparison.

Models used	Glueball states J^{PC}					
	1^{--}	3^{--}	5^{--}	7^{--}	9^{--}	11^{--}
Hardwall with Dirichlet b.c.	3.24	4.09	4.93	5.75	6.57	7.38
Hardwall with Neumann b.c.	3.24	4.21	5.17	6.13	7.09	8.04
Relativistic many body [1]	3.95	4.15	5.05	5.90		
Nonrelativistic constituent [1]	3.49	3.92	5.15	6.14		
Wilson loop [38]	3.49	4.03				
Vacuum correlator [39]	3.02	3.49	4.18	4.96		
Vacuum correlator [39]	3.32	3.83	4.59	5.25		
Semirelativistic potential [40]	3.99	4.16	5.26			
Anisotropic lattice [41]	3.83	4.20				
Isotropic lattice [36,37]	3.24	4.33				

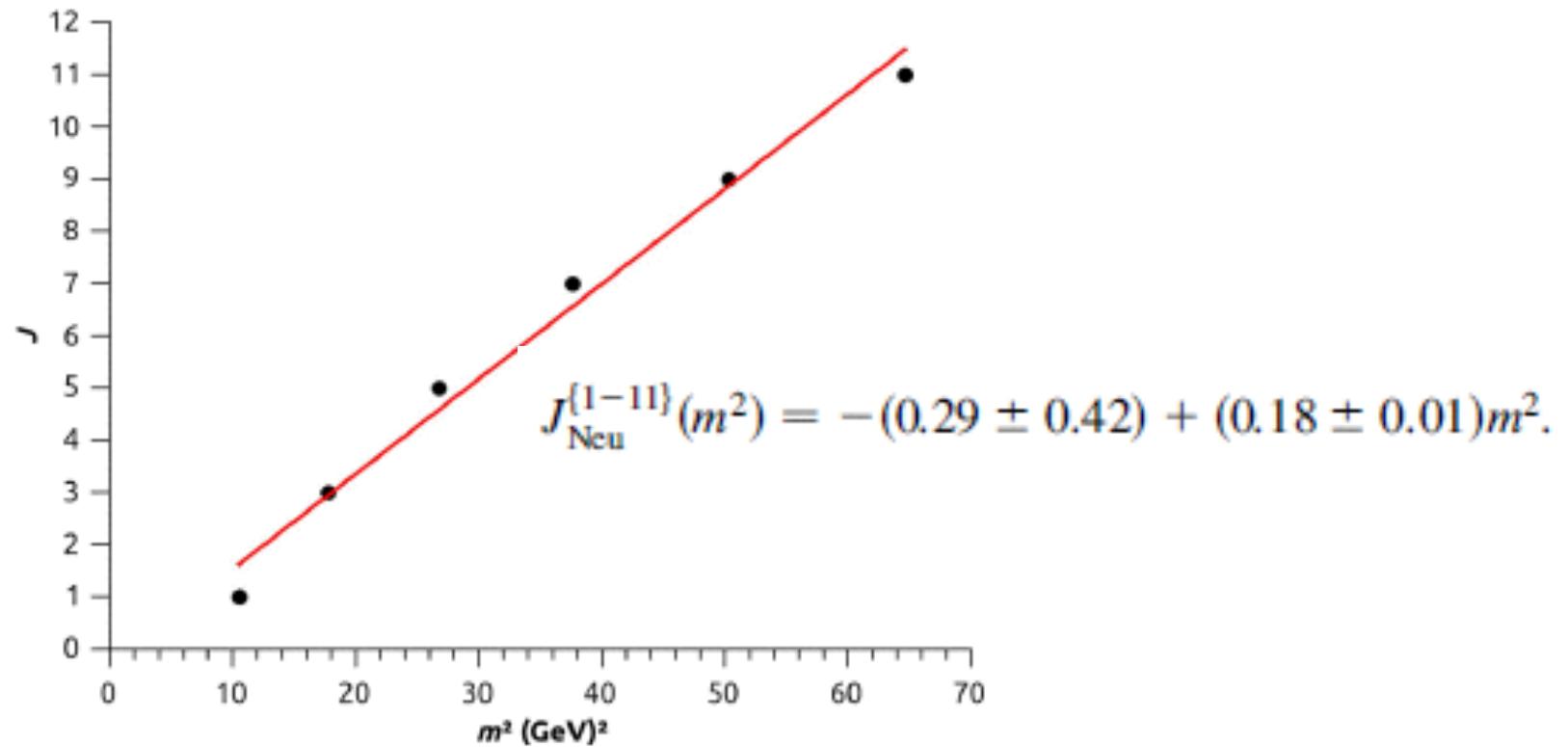
- [36] H. B. Meyer and M.J. Teper, Phys. Lett. B **605**, 344 (2005).
 → [37] H. B. Meyer, arXiv:hep-lat/0508002.

Odd Glueball states in the Hard-wall with Dirichlet Boundary condition



Good agreement with the Relativistic Many-body Model (RMB)

Odd Glueball states in the Hard-wall with **Neumann** Boundary condition



Good agreement with the Relativistic Many-body Model (RMB)

Open questions for the Odderon

Experimental confirmation?

The authors

F. J. Llanes-Estrada, P. Bicudo, and S. R. Cotanch, Phys. Rev. Lett. **96**, 081601 (2006).

suggest that the state 1^{--} does **NOT** belong to the Odderon trajectory

Our analysis with the Hard-wall is not conclusive in this regard

Finite Temperature AdS/CFT and AdS/QCD

Witten's proposal (1998)

Policastro, Son, Starinets, PRL 2001 (Shear viscosity...)

Finite temperature Yang-Mills theory in 4d dual to a modified AdS(5)×S(5) set up with a **Black Hole**
(Schwarzschild AdS (5) × S (5))

The temperature of the Yang-Mills theory is identified with the Hawking temperature of the Black Hole

Soft-wall AdS/QCD Model (T=0)

Soft cut off (Karch, Katz, Son, Stephanov PRD 2006)

$$\int d^5x \sqrt{-g} \mathcal{L} \quad \Rightarrow \quad \int d^5x \sqrt{-g} e^{-\Phi} \mathcal{L} . \quad ; \quad \Phi(z) = cz^2$$

spectrum of vector mesons $m_{V_n}^2 = 4c(n+1)$,

Glueballs in the soft-wall (T=0)
[Colangelo, De Fazio, Jugeau, Nicotri PLB(2007)]

The corresponding glueball spectrum is

$$m_{G_n}^2 = 4c(n+2) .$$

Soft-wall model at Finite Temperature

AdS black-hole spacetime

$$ds^2 = e^{2A(z)} \left[-f(z)dt^2 + \sum_{i=1}^3 (dx^i)^2 + f(z)^{-1}dz^2 \right],$$

$$A(z) = -\ln(z/L) \quad f(z) = 1 - (z/z_h)^4.$$

$$z_h = 1/\pi T.$$

Herzog PRL 2007;

Kajantie, Tahkokallio, Yee, JHEP 2007;

Ballon-Bayona, HBF, Braga, Pando Zayas, PRD 2008.

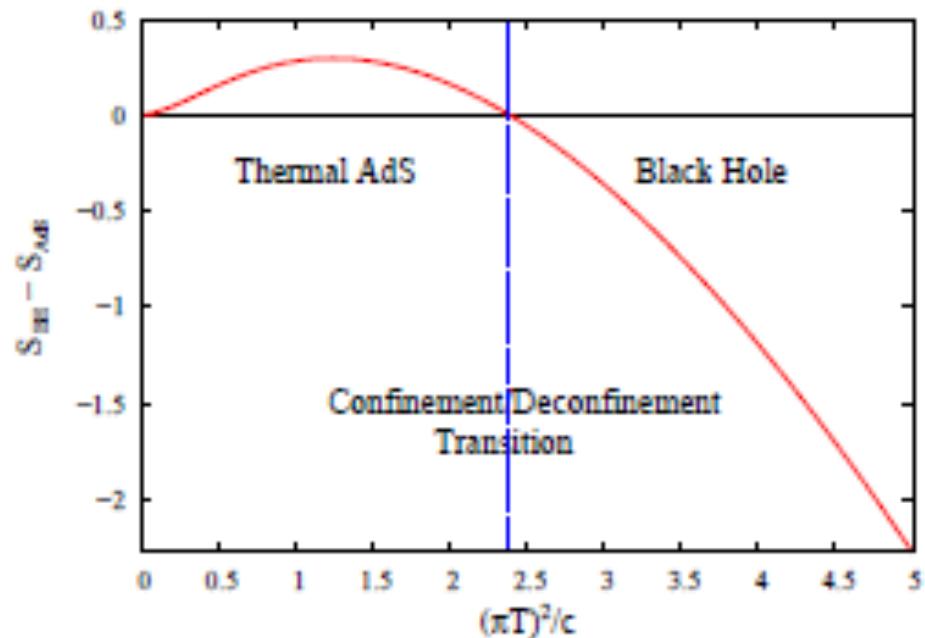
Hard-wall and Soft-wall at Finite Temperature: Confining/deconfining phase transition

Thermal AdS space (low temperature)
(confined phase)

Herzog, PRL 2007

AdS Black hole (high temperature)
(deconfined phase)

Hawking-Page phase transition



Quasinormal modes and scalar Glueballs in the Soft-wall at Finite Temperature

Quasinormal modes are formed when a particle/field falls onto a black hole horizon

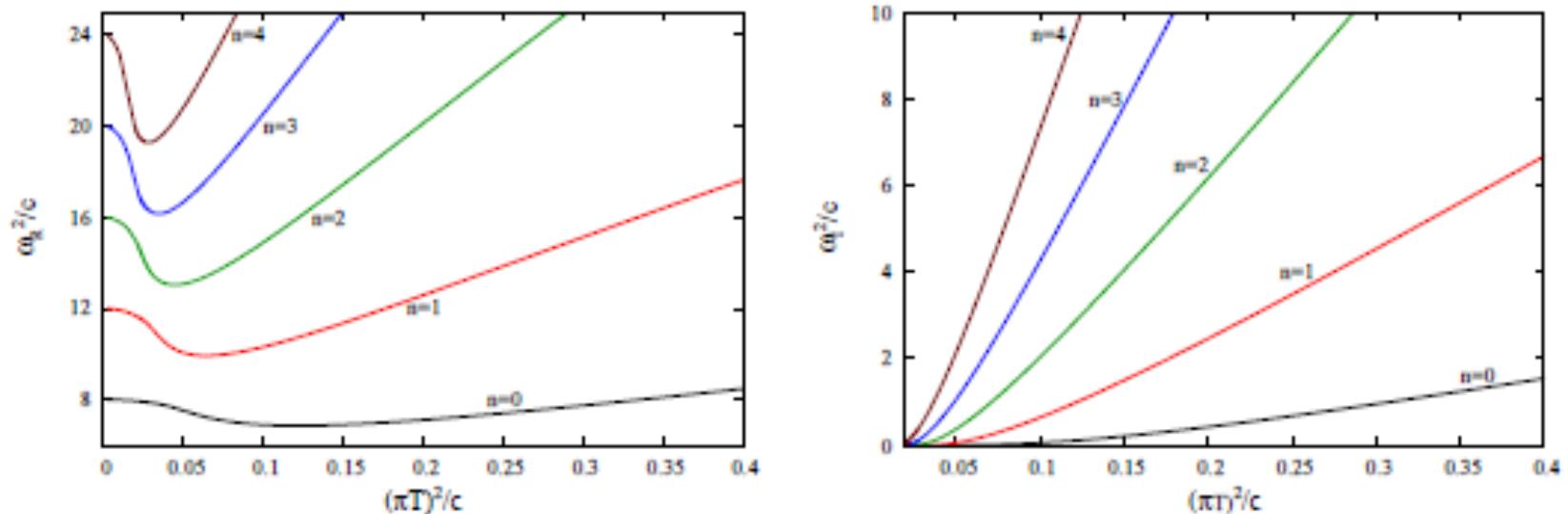


Figure 6. Numerical results for the square of the real and imaginary parts of the QN frequencies, ω_r^2/c and ω_i^2/c , for the first five quasinormal modes $n = 0, 1, \dots, 4$, with $q = 0$. (zero momentum)

Miranda, Ballon-Bayona, HBF, Braga, JHEP 2009

Quasinormal modes and Vector Mesons in the Soft-wall at Finite Temperature

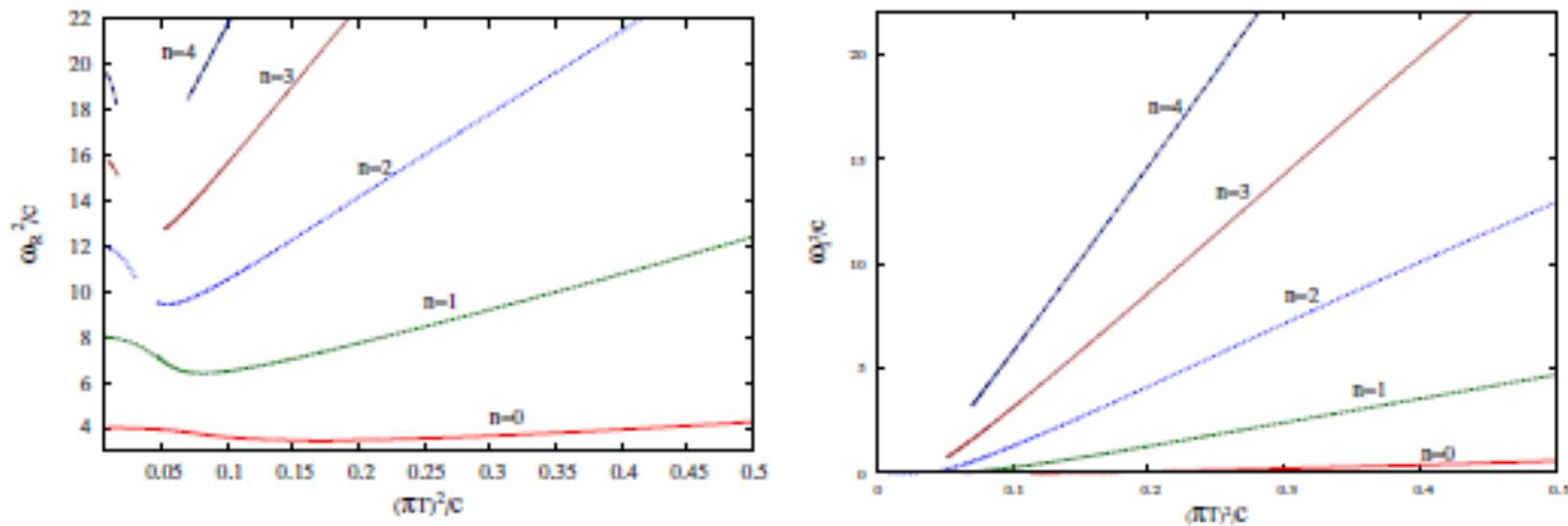
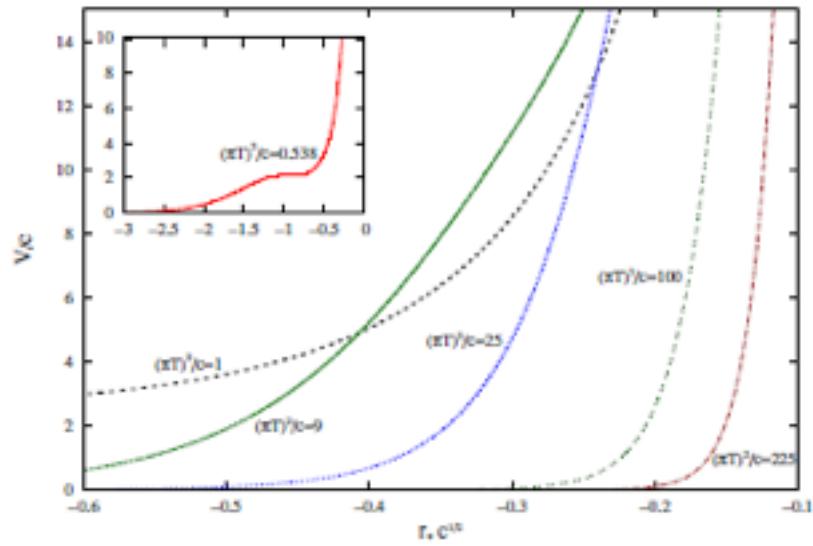


Figure 5. Numerical results for the quasinormal frequencies. On the left panel we show the real part, associated with mass of the vector mesons. On the right panel we show the imaginary part associated with the decay time of the quasiparticle states.

Mamani, Miranda, HBF, Braga, JHEP 2014

Vector Mesons at Finite T in the Soft-wall model

Figure 1. Potential at zero wave number for high temperatures.



the critical value $\tilde{T}_c^2 = 0.538$ in the detail.

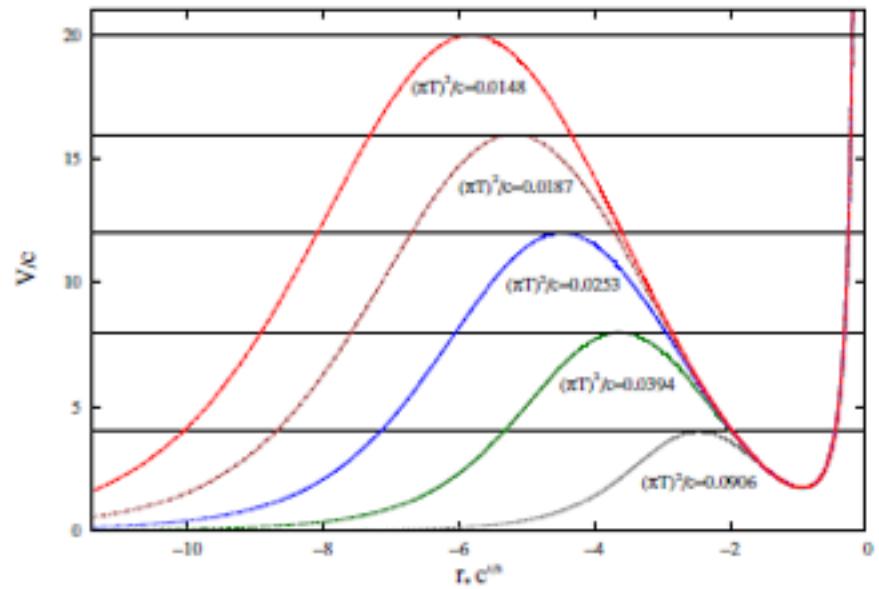


Figure 2. Potential at zero wave number for low temperatures.

Other Results:

Wilson loops in AdS/CFT and AdS/QCD (nonconfining/confining)

Vector mesons form factors in the D4-D8 model

Production of positive and negative parity Baryons in the D4-D8 model

Pion and vector mesons form factors from the Kuperstein-Sonnenschein model

D4-D8 brane model for hadrons

Sakai and Sugimoto (2005)

- D8 (probe) branes embedded in D4 brane space.
- Holographic model for (large N_c , strongly coupled) QCD.

D4 brane background:

$$ds^2 = \left(\frac{U}{R}\right)^{3/2} [\eta_{\mu\nu} dx^\mu dx^\nu + f(U) d\tau^2] + \left(\frac{R}{U}\right)^{3/2} \left[\frac{dU^2}{f(U)} + U^2 d\Omega_4^2 \right]$$

With: $f(U) = 1 - (U_{KK}/U)^3$, τ is a compact dimension.
Period of τ is related to minimum value of $U \rightarrow$ mass scale.

In this model **mesons** correspond to fluctuations of the D8 brane solutions in the D4 background.

Vector and axial vector mesons are described by $U(N_F)$ gauge field fluctuations.

4-dim effective action (after field redefinitions, ...)

$$\begin{aligned}\mathcal{L}_{eff}^{4d} = & \frac{1}{2} \text{tr} \left(\partial_\mu \tilde{v}_\nu^n - \partial_\nu \tilde{v}_\mu^n \right)^2 + \frac{1}{2} \text{tr} \left(\partial_\mu \tilde{a}_\nu^n - \partial_\nu \tilde{a}_\mu^n \right)^2 + \text{tr} (i \partial_\mu \Pi + f_\pi \mathcal{A}_\mu)^2 \\ & + M_{v^n}^2 \text{tr} \left(\tilde{v}_\mu^n - \frac{g_{v^n}}{M_{v^n}^2} \mathcal{V}_\mu \right)^2 + M_{a^n}^2 \text{tr} \left(\tilde{a}_\mu^n - \frac{g_{a^n}}{M_{a^n}^2} \mathcal{A}_\mu \right)^2 + \sum_{j \geq 3} \mathcal{L}_j\end{aligned}$$

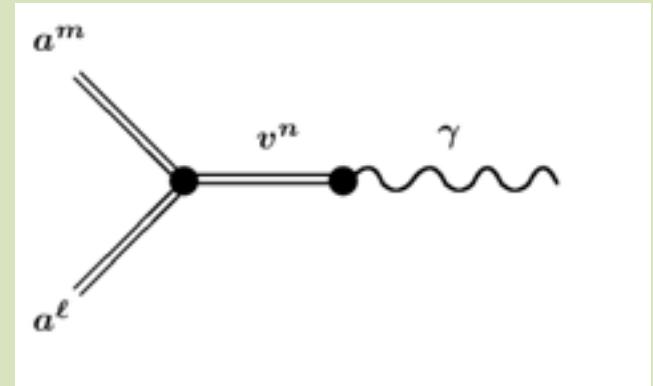
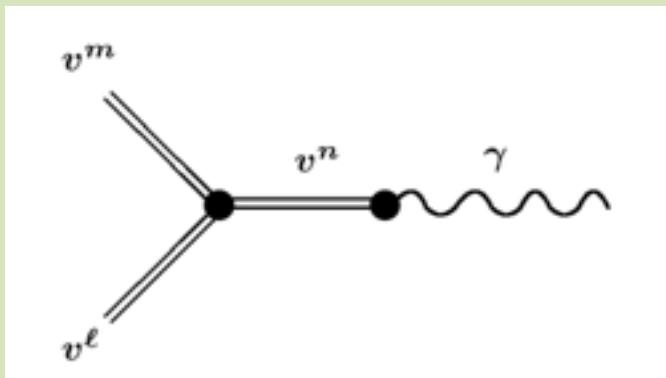
Vector and axial-vector mesons:

$$\tilde{v}_\mu^n, \quad \tilde{a}_\mu^n$$

- The effective actions show up with a set of prescriptions for calculating masses and couplings. (everything is solved numerically)

Vector Meson Dominance (VMD)

Interaction with a photon mediated by the exchange of vector mesons



D4-D8 model realizes naturally vector meson dominance (Sakai, Sugimoto, 2005);

VMD also realized naturally in Hardwall and Softwall models (Grigoryan, Radyushkin, PLB 2007, PRD 2007);

General discussion on VMD in holographic models (Son, Stephanov, PRD 2004)

Generalized form factors for vector (and axial vector) mesons in D4-D8 model: Ballon-Bayona, HBF, Braga, Torres, JHEP 2010

$$\langle v^{m a}(p), \epsilon | J^{\mu c}(0) | v^{\ell b}(p'), \epsilon' \rangle$$

$$= \epsilon^\nu \epsilon'^\rho f^{abc} [\eta_{\nu\rho} (2p + q)_\sigma + 2(\eta_{\sigma\nu} q_\rho - \eta_{\rho\sigma} q_\nu)] \left(\eta^{\mu\sigma} - \frac{q^\mu q^\sigma}{q^2} \right) F_{v^m v^\ell}(q^2)$$

Where, in the model:

$$F_{v^m v^\ell}(q^2) = \sum_{n=1}^{\infty} \frac{g_{v^n} g_{v^n v^m v^\ell}}{q^2 + M_{v^n}^2}$$

g_{v^n} is the coupling between the photon and the vector meson,

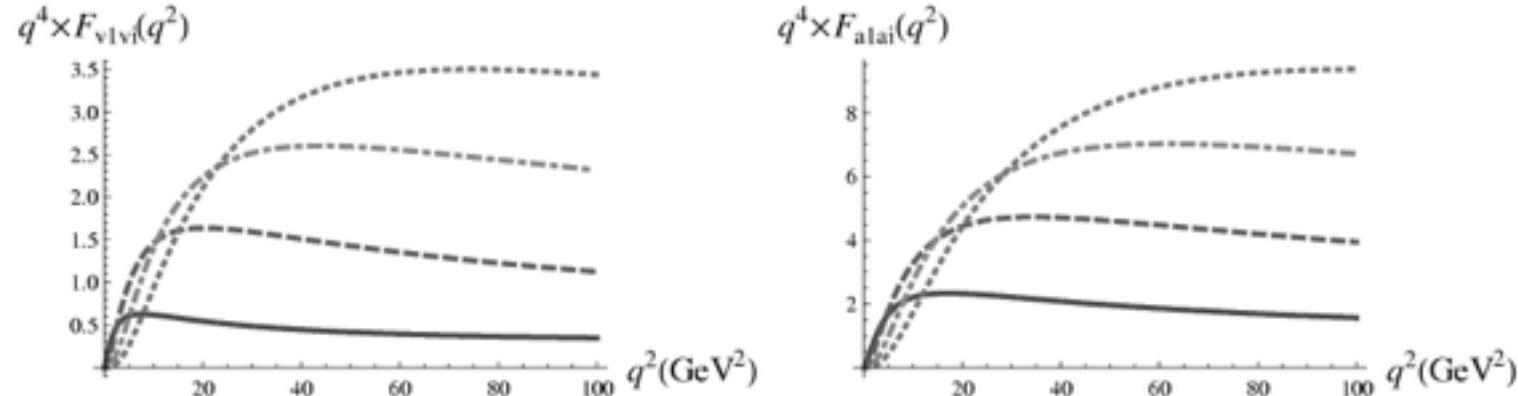
$g_{v^n v^\ell v^m}$ is the 3 vertex on vector mesons,

$$g_{v^n} = \kappa M_{v^n}^2 \int d\tilde{z} K(\tilde{z})^{-1/3} \psi_{2n-1}(\tilde{z}), \quad K(\tilde{z}) \equiv 1 + \tilde{z}^2$$

$$g_{v^n v^\ell v^m} = \kappa \int d\tilde{z} K(\tilde{z})^{-1/3} \psi_{2n-1}(\tilde{z}) \psi_{2\ell-1}(\tilde{z}) \psi_{2m-1}(\tilde{z}),$$

... and similar expressions for the axial-vector mesons.

Results: appropriate decrease with q^{-4} for large q .



q^4 times the form factors. Left panel: $F_{v^1 v^i}$, right panel: $F_{a^1 a^i}$, for $i = 1$ (solid line), 2 (dashed line), 3 (dot-dashed line), 4 (dotted line).

Interesting quantities in the elastic case:

- Form factors for vector mesons with transversal and longitudinal polarizations

$$F_{TT}(q^2) = \frac{\langle p, \epsilon_T | J_0(0) | p', \epsilon'_T \rangle}{2E} , \quad F_{LT}(q^2) = \frac{\langle p, \epsilon_T | J_x(0) | p', \epsilon'_L \rangle}{2E}$$
$$F_{LL}(q^2) = \frac{\langle p, \epsilon_L | J_0(0) | p', \epsilon'_L \rangle}{2E} .$$

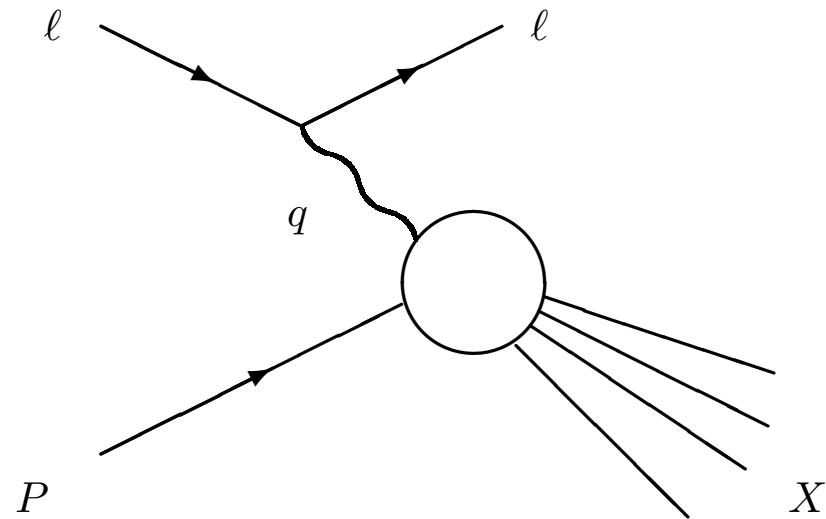
In the D4-D8 model we found:

$$F_{TT}^{(v^m)} = F_{v^m} , \quad F_{LT}^{(v^m)} = \frac{q}{M_{v^m}} F_{v^m} , \quad F_{LL}^{(v^m)} = \left(1 - \frac{q^2}{2M_{v^m}^2}\right) F_{v^m} .$$

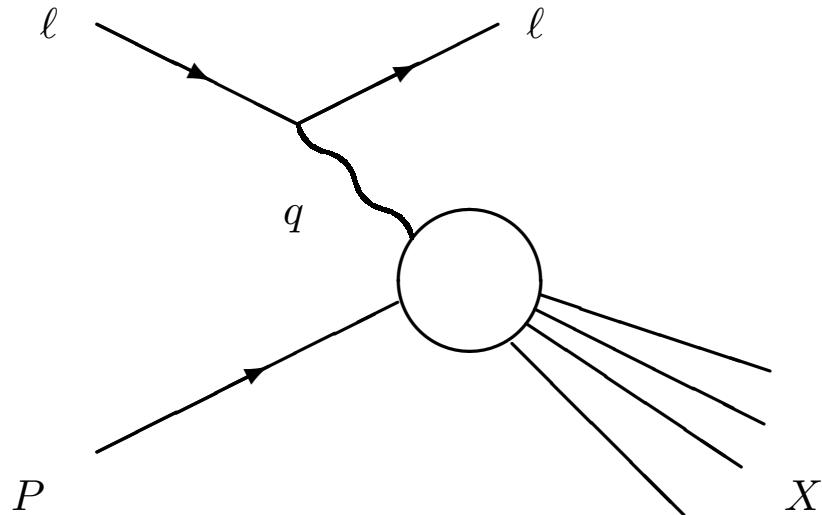
That imply the large q^2 behaviour expected from QCD:

$$F_{TT}^{(v^m)} \sim q^{-4} , \quad F_{LT}^{(v^m)} \sim q^{-3} , \quad F_{LL}^{(v^m)} \sim q^{-2}$$

Deep Inelastic Scattering (DIS)

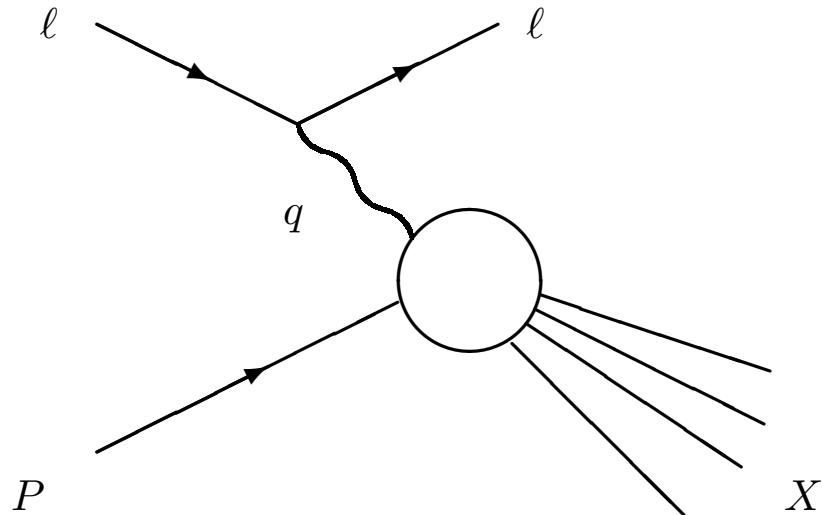


Deep Inelastic Scattering (DIS)



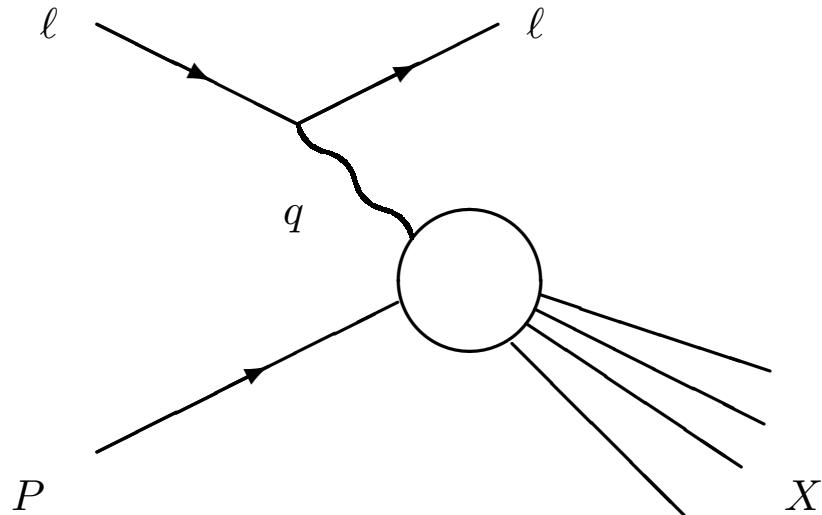
- Hardwall model (Polchinski, Strassler, JHEP 2003)

Deep Inelastic Scattering (DIS)



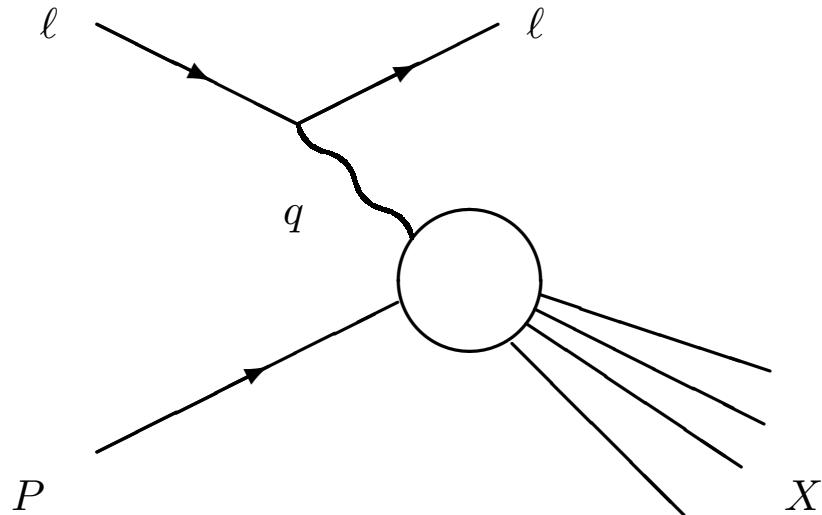
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- Softwall model (Ballon-Bayona, HBF, Braga, JHEP 2008)

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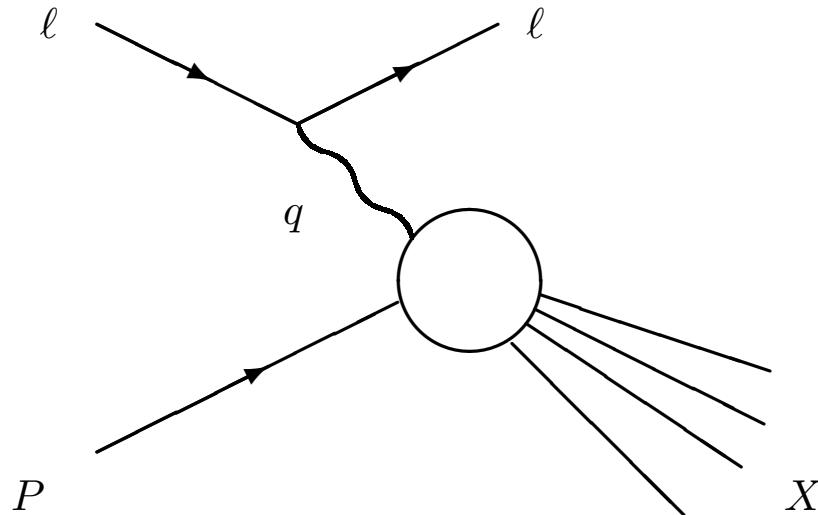
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- D3/D7 model (Ballon-Bayona, HBF, Braga, JHEP 2008)

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- D3/D7 model (Ballon-Bayona, HBF, Braga, JHEP 2008)
- D4/D8 model (Ballon-Bayona, HBF, Braga, Torres, JHEP 2010)

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- Softwall model (Ballon-Bayona, HBF, Braga, JHEP 2008)
- D3/D7 model (Ballon-Bayona, HBF, Braga, JHEP 2008)
- D4/D8 model (Ballon-Bayona, HBF, Braga, Torres, JHEP 2010)
=> Different technical details but similar results

Deep Inelastic Scattering: relation with experimental data

- Geometrical scaling from supergravity
(Ballon-Bayona, HBF, Braga, JHEP 2008)

$$\sigma(q^2, x) \sim (q^2 x^\lambda)^{\gamma_s - 1}, \quad \text{with } \lambda = 1 \text{ and } \gamma_s = 1/2.$$

Deep Inelastic Scattering: relation with experimental data

- Geometrical scaling from supergravity
(Ballon-Bayona, HBF, Braga, JHEP 2008)

$$\sigma(q^2, x) \sim (q^2 x^\lambda)^{\gamma_s - 1}, \quad \text{with } \lambda = 1 \text{ and } \gamma_s = 1/2.$$

- Hatta, Iancu, Mueller, JHEP 2008:

with $\lambda = 1$ and $\gamma_s = -1$.

Deep Inelastic Scattering: relation with experimental data

- Geometrical scaling from supergravity
(Ballon-Bayona, HBF, Braga, JHEP 2008)

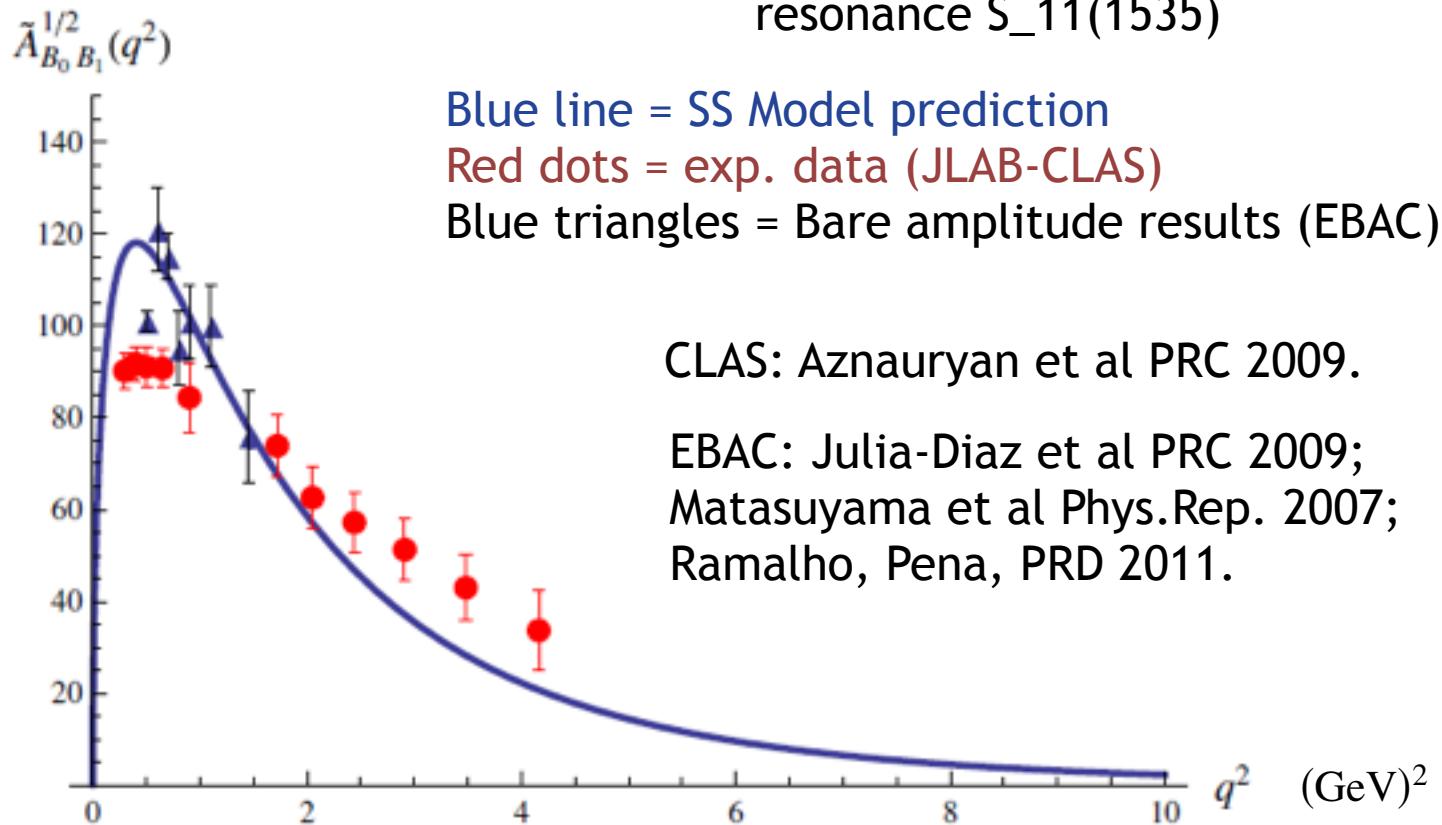
$$\sigma(q^2, x) \sim (q^2 x^\lambda)^{\gamma_s - 1}, \quad \text{with } \lambda = 1 \text{ and } \gamma_s = 1/2.$$

- Hatta, Iancu, Mueller, JHEP 2008:
with $\lambda = 1$ and $\gamma_s = -1$.
- Fit of HERA small x data (Brower, Djuric, Sarcevic, Tan, JHEP 2010)

Baryons Form Factors and Proton Structure in the Holographic Sakai-Sugimoto Model

Ballón-Bayona, HBF, Braga, Ihl, Torres, PRD 2012; NPB 2013

Helicity Amplitude $[10^{-3}(\text{GeV})^{-1/2}]$ for the observed negative parity resonance $S_{-1}(1535)$



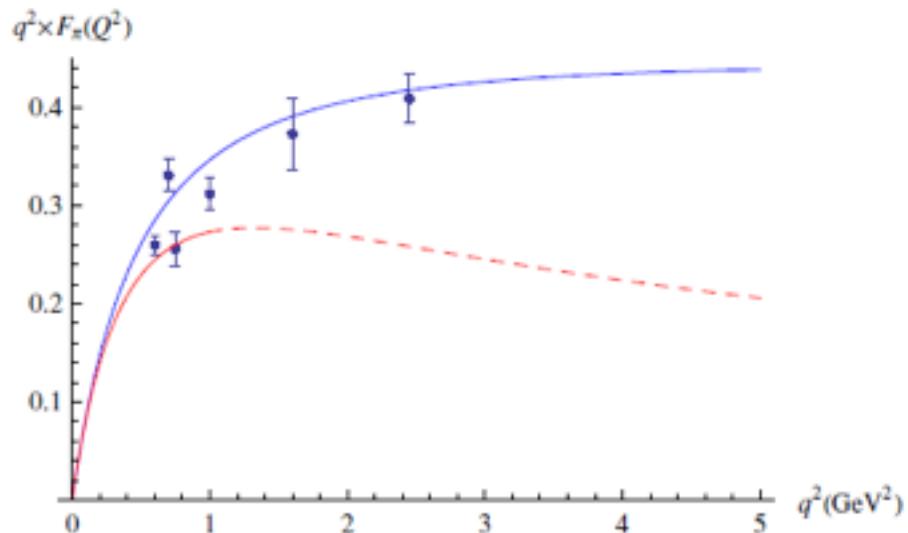
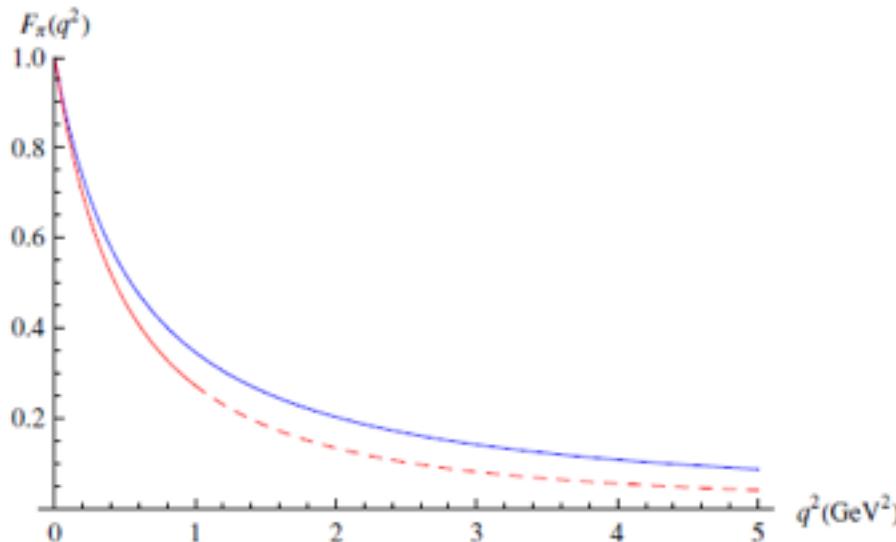
Pion (and vector meson) Form Factors in the Kuperstein-Sonnenschein Holographic model

Ballón-Bayona, HBF, Ihl, Torres, JHEP (2010)

D3-brane background
D7-brane profiles

The KS model is based on the *D3*-brane background with a conical singularity in type IIB superstring theory first studied by Klebanov and Witten

Stable, non-supersymmetric, but similar to D4-D8 with VMD



Red = SS model; Blue = KS model
Dots = experimental data (PDG)

Merci beaucoup!