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Bipartite graphs and microlocal geometry

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The combinatorics of bipartite graphs on surfaces plays a major role in the theory of cluster algebras, for example in the context of positroid cells in the Grassmannian, character varieties of open surfaces, and linear systems on toric surfaces. We revisit these examples from the point of view of microlocal geometry, the interplay between the sheaf theory of manifolds and the symplectic geometry of their cotangent bundles. The constructible sheaves we discuss will be implicitly familiar to anyone who has worked with the relevant combinatorics, but the change in perspective brings to bear a variety of new tools and ideas to cluster theory. In this language, the above examples can be thought of as modular invariants of Legendrian knots, and their cluster structures reflect the relations among Lagrangian fillings of these knots. In particular, each bipartite graph on a surface encodes a canonical exact Lagrangian in its cotangent bundle, and the geometry of these Lagrangians completely determines the structure of the associated cluster transformations as well as the analogue of the boundary measurement map for arbitrary spaces of local systems. Conversely, this construction lets us leverage cluster combinatorics to obtain new results about Lagrangian fillings of Legendrian knots.

This is joint work with Vivek Shende, David Treumann, and Eric Zaslow.

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