

Cluster duality and mirror symmetry for Grassmannians

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We consider the Grassmannian $X = \text{Gr}(n - k, n)$ and its mirror dual Landau—Ginzburg model (X', W) as constructed by Marsh—Rietsch. In this Landau—Ginzburg model, X' is the complement of a particular anti-canonical divisor in a Langlands dual Grassmannian, and the superpotential W is expressed very naturally in terms of Plucker coordinates. We describe a natural and explicit duality correspondence between the positive chart Φ_G from $(\mathbb{C}^*)^N \rightarrow X$, and the cluster chart Φ'_G from $(\mathbb{C}^*)^N \rightarrow X'$, where $N = k(n - k)$ and G is a plabic graph. For any projective embedding of X and positive chart Δ there is an associated Newton—Okounkov polytope which is determined by giving its integer lattice points. Therefore to each chart Φ_G as above and any positive integer r , we obtain a Newton—Okounkov polytope NO_G^r . On the other hand, using the cluster chart Φ'_G for the mirror X' , and the same positive integer r , we can obtain a polytope Q_G^r —described in terms of inequalities—by “tropicalizing” the superpotential W . Our main result is that the polytopes NO_G^r and Q_G^r coincide.

This is joint work with Konstanze Rietsch.

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