

Combinatorics of toric cascades: Aztec castles and beyond

Gregg Musiker^{*}

musiker@math.umn.edu

Mutation sequences involving mutations at vertices with two in-coming arrows and two out-going arrows are often referred to as toric cascades in the string theory literature. We study such toric cascades for the dP3 (del Pezzo 3) quiver and related examples. This includes quiver gauge theories associated to cones over other del Pezzo surfaces as well as quivers associated to Gale—Robinson sequences.

Such toric cascades give rise to interesting discrete integrable systems on the level of cluster variable dynamics. *Previous work with REU students I. Jeong and S. Zhang*, and *later with REU students M. Leoni, S. Neel, and P. Turner*, led to combinatorial interpretations for cluster variables arising from periodic toric cascades in the case of Gale—Robinson sequences and a two-dimensional subspace of toric cascades in the dP3 case. *More recent work with T. Lai* extends this combinatorial interpretation to a three-dimensional subspace of cascades, and motivates further study of the integrability of such mutation sequences.

^{*}Department of Mathematics, University of Minnesota, Vincent Hall, 206 Church Street, Minneapolis, MN 55455, USA.