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Symmetry and geometric structure for the Worpitzky identity

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The classical Worpitzky theorem in combinatorics expresses powers in terms of Eulerian numbers. It corresponds to the volume decomposition of a simplex into hypersimplices having volumes given by the Eulerian polynomials $A_{n-1}(q)$. We replace the simplex and hypersimplex volumes with S_n-modules coming from simplicial decompositions. Let $\sigma \in S_n$ have cycle lengths $\lambda_1, \dots, \lambda_k$. Characters have number theoretic properties.

THEOREM: *The character of σ on the module of the simplex Δ_r^n is $\delta_{g,1} r^{k-1}$, where* $g = \gcd(\lambda_1, \dots \lambda_k, r)$.

THEOREM: *The character of σ on the module of the hypersimplex $B_{a,b}$ is the coefficient of q^a in $\delta_{g,1} q A_{k-1}(q).[\lambda_1] \cdots [\lambda_k]$ with $A_{k-1}(q)$ the Eulerian polynomial and* $g = \gcd(\lambda_1, \dots, \lambda_k, a, n-a)$. *The two characters are connected by an equivariant Worpitzky identity $\chi_{\Delta_r^n} \simeq \chi_{\text{poly}} \otimes B_{a,b}$. We construct the character values geometrically as volumes and as linear module dimensions of new generalized hypersimplices.*

This is joint work with Adrian Ocneanu.

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