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Whittaker functions, cluster algebras and Macdonald difference operators

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Whittaker functions are solutions of Toda-type differential/difference equations, originally built out of Whittaker vectors playing a central role in representation theory of Lie algebras. We give a new statistical weighted path formulation for these vectors, valid for simple and affine Lie algebras as well as the quantum algebra $U_q(s\ell_n)$.

We then consider graded tensor products of current algebra $g[t]$-modules, and show that their characters obey difference equations, generalizing the difference Toda equation, allowing for viewing graded characters as generalized Whittaker functions. This uses a constant term expression for the characters involving a solution of the quantum Q-system, a set of non-commuting integrable recursion relations that are particular mutations of a quantum cluster algebra attached to the Lie algebra g.

Finally, we obtain a new compact expression for graded $s\ell_n$ characters by constructing a representation of the quantum Q-system via generalized Macdonald– Ruijsenaars difference operators, expressed in terms of generators of the Double Affine Hecke algebra.

This is joint work with R. Kedem and B. Turmunkh.

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