From the twistor action to Correlahedra

Lionel Mason

The Mathematical Institute, Oxford
lmason@maths.ox.ac.uk

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review with Adamo, Bullimore & Skinner 1104.2890, & work with Lipstein arxiv:1212.6228, 1307.1443, more recently with Agarwala, Eden, Heslop, etc..

[Also. work by Alday, Arkani-Hamed, Cachazo, Caron-Huot, Drummond, Heslop, Korchemsky, Maldacena, Sokatchev, Trnka. (Annecy, Oxford, Perimeter and Princeton IAS).]

Twistors, amplitudes, Wilson loops, 'hedra & polylogs

Background [Penrose,Boels, M, Skinner, Adamo, Bullimore...]:

- Twistor space $\mathbb{CP}^{3|4} \leftrightarrow$ space-time; Klein correspondence.
- $N = 4$ Super Yang-Mills has twistor action in twistor space.
- Axial gauge Feynman diagrams $\rightsquigarrow$ 'MHV diagrams'.
- Also get MHV diagrams for Wilson loops & Correlators.
- Planar Wilson-loop/amplitude duality is planar duality for MHV diagrams.
- Super Amplitude/Correlator/Wilson-loop triality.

Focus of this talk [with Lipstein, Agarwala, Eden & Heslop]:

- Twistor Feynman rules do lead to 'hedra formulations.
- Diagrams give tilings of amplituhedra/correlahedra.
- give dlog form for loop integrands.
- Integration requires Feynman $i\epsilon$ in Lorentz signature.
- New formulae at 1-loop.

Twistor space

Super twistor space is $\mathbb{CP}^{3|4}$ with homogeneous coords:

$$Z = (\lambda_\alpha, \mu^{\dot\alpha}, \chi_a) \in \mathbb{T} := \mathbb{C}^2 \times \mathbb{C}^2 \times \mathbb{C}^{0|4}, \qquad Z \sim \zeta Z\,, \zeta \in \mathbb{C}^*\,.$$

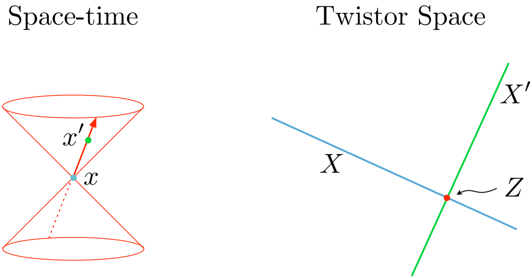
$\mathbb{T}$ = fund. repn of superconformal group $SU(2,2|4)$.

Super Minkowski space, $\mathbb{M} = \mathbb{R}^{4|8}$,

Incidence: a point $\mathbf{x} = (x, \theta) \leftrightarrow$ a line $X = \mathbb{CP}^1 \subset \mathbb{PT}$ via

$$\mu^{\dot\alpha} = i x^{\alpha\dot\alpha}\lambda_\alpha\,, \qquad \chi_i = \theta^\alpha_i \lambda_\alpha\,.$$

Two points x, x' are null separated iff X and X' intersect.



Supersymmetric Ward correspondence

Super Calabi-Yau: $\mathbb{CP}^{3|4}$ has weightless super volume form

$$D^{3|4}Z = D^3 Z \, \mathrm{d}\chi_1 \ldots \mathrm{d}\chi_4 \in \Omega_{Ber}.$$

'Super-Ward' for $\mathcal{N} = 4$ SYM:
A dbar-op $\bar{\partial}_{\mathcal{A}} = \bar{\partial}_0 + \mathcal{A}$ on bundle over $\mathbb{CP}^{3|4}$ has expansion

$$\mathcal{A} = a + \chi_a \psi^a + \chi_a \chi_b \phi^{ab} + \chi^{3a} \tilde{\psi}_a + \chi^4 b$$

and $\bar{\partial}^2_{\mathcal{A}} = 0 \; \leftrightarrow$ solns to self-dual $\mathcal{N} = 4$ SYM on space-time.

Action for fields with self-dual interactions:[Sokatchev, Witten]
SD interactions $\leftrightarrow$ holomorphic Chern-Simons action

$$S_{sd} = \int_{\mathbb{PT}} \mathrm{tr}(\mathcal{A} \wedge \bar{\partial}\mathcal{A} + \frac{2}{3}\mathcal{A}^3)_\wedge D^{3|4}Z \, .$$

Incorporating interactions of full $\mathcal{N} = 4$ SYM

Nair, M., Boels, Skinner, 2006

Extension to full SYM:

$$S_{full}[\mathcal{A}] = S_{sd}[\mathcal{A}] + S_{int}[\mathcal{A}]$$

includes non-local interaction term:

$$\begin{aligned} S_{int}[\mathcal{A}] &= g^2 \int_{\mathbb{M}} \mathrm{d}^{4|8}\mathbf{x} \ \log \ \det(\bar{\partial}_{\mathcal{A}}|_X) \\ &= g^2 \sum_{n=2}^{\infty} \frac{1}{n} \int_{\mathbb{M}\times X^n} \mathrm{d}^{4|8}\mathbf{x} \frac{\mathrm{tr}\,(\mathcal{A}_1\mathcal{A}_2 \ldots \mathcal{A}_n)\, D\sigma_1 \ldots D\sigma_n}{\langle\sigma_1\sigma_2\rangle\langle\sigma_2\sigma_3\rangle \ldots \langle\sigma_n\sigma_1\rangle} \end{aligned}$$

$X = \mathbb{CP}^1_{\mathbf{x}} \subset \mathbb{PT}$ for $\mathbf{x} \in \mathbb{M}^{4|8}$, $\sigma_i \in X_i$, i^{th} factor,

$$\mathcal{A}_i = \mathcal{A}(Z(\sigma_i))\,, \qquad \text{and} \qquad K_{ij} = \frac{D\sigma_j}{\langle\sigma_i\,\sigma_j\rangle}$$

is Cauchy kernel of $\bar{\partial}^{-1}$ on X at σ_i, σ_j.

Axial gauge Feynman rules

Choose 'reference twistor' Z_*, impose gauge: $\bar{Z}_* \cdot \frac{\partial}{\partial \bar{Z}} \lrcorner \mathcal{A} = 0$.

Payoff: Cubic Chern-Simons vertex $= 0 \rightsquigarrow$ Feynman rules:

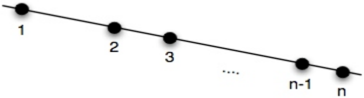
- Propagator = delta-function forcing Z, Z', Z_* to be collinear.

$$\Delta(Z, Z') = \frac{1}{2\pi i} \bar{\delta}^{2|4}(Z, Z_*, Z') := \frac{1}{2\pi i} \int \frac{\mathrm{d}c\,\mathrm{d}c'}{cc'} \bar{\delta}^{4|4}(Z_* + cZ + c'Z')$$

- log-det term gives 'MHV vertices':

$$V(Z_1, \ldots, Z_n) = \int_{\mathbb{M} \times X^n} \frac{\mathrm{d}^{4|4} Z_A \mathrm{d}^{4|4} Z_B}{Vol\, GL(2)} \prod_{r=1}^{n} \frac{\bar{\delta}^{3|4}(Z_r, Z_A + \sigma_r Z_B)}{(\sigma_{r-1} - \sigma_r)} d\sigma_r \,.$$

Vertices force $Z_1, \ldots Z_n$ to lie on $X = \{Z(\sigma) = Z_A + \sigma Z_B\}$



Simplicity: arises versus standard Feynman diagrams as number of propagators $=$ MHV degree $+2\times$# loops.

Amplitudes: Transform rules to momentum space $\rightsquigarrow$ 'MHV rules' [CSW 2005] where vertices $=$ off-shell MHV amplitudes.

Null polygons: Supermomentum conservation for colour ordered momenta $\rightsquigarrow$ null polygon $\{\mathbf{x}_i\} = \{(x_i, \theta_i)\} \subset \mathbb{M}$:

$$(p_i^{AA'}, \eta_i^a \lambda^A) = (x_i^{AA'} - x_{i+1}^{AA'}, \theta_i^A - \theta_{i+1}^A)\,.$$

Conjecture (Alday, Maldacena)

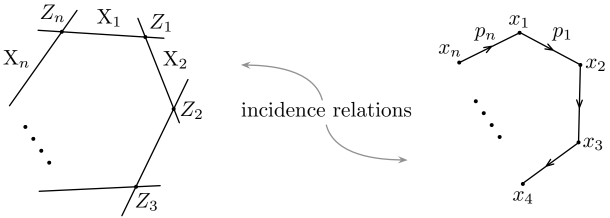
Let $W(x_1, \ldots, x_n) =$ Wilson-loop around momentum polygon.

$$\textit{All loop MHV amplitude} = \textit{MHV tree} \times \langle W(x_1, \ldots, x_n) \rangle.$$

New diagram formulation can also be used for correlation functions of Wilson loops etc..

Momentum polygons in twistor space

Generic polygon in $\mathbb{PT}$ [Hodges] $\leftrightarrow$ null polygon in space-time.



Change variables so that $(X_1, \ldots, X_n) = (Z_1, \ldots, Z_n)$.

Important simplification: $Z_i \in \mathbb{PT}$ are unconstrained.

What is Wilson loop in $\mathbb{PT}$?

For Wilson-loop, need holonomy around polygon in $\mathbb{PT}$.

- Vertices Z_i,
- Edges $X_i = \{Z(\sigma) = \sigma Z_{i-1} + Z_i, \sigma \in \mathbb{C} \cup \infty\}$.
- Global frame $F_i(\sigma)$ of $E|_{X_i}$ on X_i with

$$\bar\partial_{\mathcal{A}}|_{X_i} F_i(\sigma) = 0, \qquad F_i(\infty) = F_i|_{Z_{i-1}} = 1\,.$$

- Perturbatively iterate $F_i = 1 + \bar\partial^{-1}(\mathcal{A}F_i)$ to get

$$F_i = 1 + \sum_{r=1}^{\infty} \prod_{s=1}^{r} \bar\partial^{-1}_{s-1\,s} \mathcal{A}(\sigma_s)\,, \qquad (\bar\partial^{-1}_{r\,s} f)(\sigma_r) = \int_{L_{X_i}} \frac{f(\sigma_s)\,\mathrm{d}\sigma_s}{\sigma_r - \sigma_s}$$

- Define

$$W = \operatorname{tr} \prod_{i=1}^{n} F_i|_{Z_i} = \operatorname{tr} \prod_{i=1}^{n} F_i(0)\,.$$

Agrees with space-time Wilson loop on-shell.

The S-matrix as a holomorphic Wilson loop

Theorem (Bullimore, M., Skinner, 2010-11)

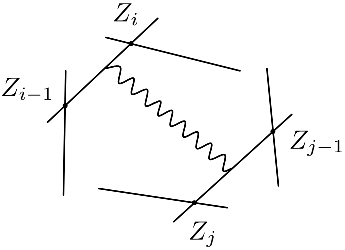
For planar $\mathcal{N} = 4$ SYM:
Amplitude loop-integrands = (holomorphic) Wilson loop integrand

$$A(1, \ldots, n) = \langle W(Z_1, \ldots, Z_n) \rangle A^{tree}_{MHV} .$$

- *Tree amplitudes $\leftrightarrow$ Wilson-loop in self-dual sector (g=0).*
- *Loop expansion for $\mathcal{A}$ = g-expansion for W.*
- *The Axial gauge twistor space diagrams for amplitude are planar duals of those for Wilson-loop correlator.*

Proof: comparison of Feynman rules on $\mathbb{PT}$ (or BCFW).

NMHV case: for $\mathcal{A}^2$ part of $\langle W \rangle$, $\langle \mathcal{A}(Z)\mathcal{A}(Z') \rangle = \Delta(Z, Z')$:
obtain $\langle W \rangle = \sum_{i<j} \Delta_{ij}$ where $\Delta_{ij} =$



$= [\, * \,, i-1, i, j-1, j] \,,$

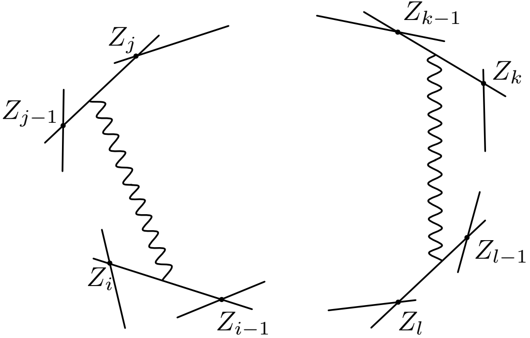
$$[1,2,3,4,5] \quad := \quad \int \frac{\mathrm{d}c_1 \mathrm{d}c_2 \mathrm{d}c_3 \mathrm{d}c_4 \mathrm{d}c_5}{c_1 c_2 c_3 c_4 c_5} \frac{1}{\mathrm{Vol\,Gl}(1)} \, \bar{\delta}^{4|4} \left(\sum_{i=1}^{5} c_i Z_i \right) \,,$$

$$= \quad \frac{\prod_{a=1}^{4} \left((1234)\chi_5^a + \text{cyclic} \right)}{(1234)(2345)(3451)(4512)(5123)}$$

is the ‘R-invariant’.

The N^kMHV tree

N^2MHV: quartic terms in $\mathcal{A}$ in W give Wick contractions



No crossed propagators for planarity.

N^kMHV tree amplitudes $\leftrightarrow$ k propagators $\rightsquigarrow$ k R-invariants.

Lagrangian insertions: $S_{int}[\mathcal{A}] = \int d^4x \mathcal{L}_{int}(x)$ where

$$\mathcal{L}_{int}(x) = \int d^8\theta \ \log\det(\bar\partial_{\mathcal{A}}|_X) \,.$$

Forming tree correlator with ℓ insertions gives loop integrand

$$\langle W(\mathbf{x}_1, \dots, \mathbf{x}_n) \mathcal{L}(x_1) \dots \mathcal{L}(x_l) \rangle_{tree} \,.$$

Super BPS operators: it is invariant to integrate over just 4 θs

$$\mathcal{O}(x, \theta, Y) := \operatorname{tr}(\Phi(\mathbf{x}) \cdot Y)^2 = \int (Y^{ab} d\theta_{a\alpha} d\theta^{\alpha}_{b})^2 \log\det(\bar\partial_{\mathcal{A}}|_X)$$

where $Y^{[ab}Y^{cd]} = 0$ (so depends on the 4 θs with $\theta_{a\alpha}Y^{ab} = 0$).

Proposition (Alday, Eden, Korchemsky, Maldacena, Sokachev, Heslop, Adamo, Bullimore, M., Skinner)

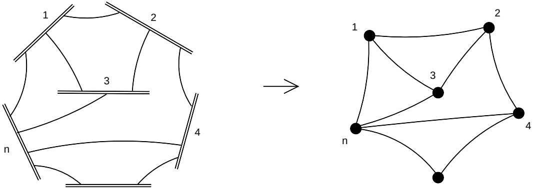
$$\lim_{(x_i - x_{i+1})^2 \to 0} \left\langle \prod_{i=1}^{n} \frac{(x_i - x_{i+1})^2}{Y_i \cdot Y_{i+1}} \mathcal{O}(\mathbf{x}_i, Y_i) \right\rangle = \langle W(\mathbf{x}_1, \dots, \mathbf{x}_n) \rangle^2 \,.$$

Gives 'triality' Amplitude $\leftrightarrow$ Wilson loop $\leftrightarrow$ BPS correlator.

MHV diagrams for correlators

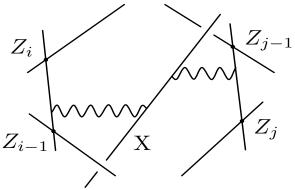
[Chicherin, Doobary, Eden, Heslop, Korchemsky, M., Sokatchev]

- Obtain diagrams in twistor space for $\langle \mathcal{O}(\mathbf{x}_1) \dots \mathcal{O}(\mathbf{x}_n) \rangle$.



- Double line $\leftrightarrow$ line $X_l = \langle Z_{l1}, Z_{l2} \rangle \subset \mathbb{PT} \leftrightarrow \mathbf{x}_i$.
- Solid lines $\leftrightarrow$ twistor propagators.
- As $(x_i - x_{i+1})^2 \to 0$, consecutive lines join.
- Diagram $\to 0$ unless consecutive lines are connected by propagator $\rightsquigarrow$ Wilson loop.

At MHV with one MHV vertex obtain $\sum_{i,j} K_{ij}$ with $K_{ij} =$



$$= \int_\Gamma \mathrm{D}^{3|4} Z_A \wedge \mathrm{D}^{3|4} Z_B \, [\,*\,, i-1, i, A, B'][\,*\,, j-1, j, A, B'']$$

Loop momenta $\leftrightarrow$ location of line $X = \langle Z_A \, Z_B \rangle$. Recall:

$$[*, i-1, i, A, B] := \int \frac{\mathrm{d}c_1 \mathrm{d}c_2 \mathrm{d}c_3 \mathrm{d}c_4}{c_1 \, c_2 \, c_3 \, c_4} \, \bar{\delta}^{4|4}(Z_* + c_1 Z_A + c_2 Z_B + c_3 Z_{i-1} + c_4 Z_i)$$

can integrate $\frac{D^{4|4} Z_A \wedge D^{4|4} Z_B}{vol \, GL_2}$ against delta functions

$$K_{ij} = \frac{1}{(2\pi i)^2} \int \frac{\mathrm{d}c_0 \mathrm{d}c_1 \mathrm{d}b_0 \mathrm{d}b_1}{c_0 c_1 \; b_0 b_1} \, .$$

External data encoded in integration contour (see later).

- A N^{k}MHV Wilson-loop diagram has k propagators gives

$$\prod_{r=1}^{k}\int_{\mathbb{CP}^4}\frac{\mathrm{d}c_{r\,0}\mathrm{d}c_{r\,i_1^r}\mathrm{d}c_{r\,i_2^r}\mathrm{d}c_{r\,i_3^r}\mathrm{d}c_{r\,i_4^r}}{Vol\,GL(1)\;c_{r\,0}c_{r\,i_1^r}c_{r\,i_2^r}c_{r\,i_3^r}c_{r\,i_4^r}}\bar{\delta}^{4|4}\left(Y_r\right)\,,$$

  where $Y_r = c_{r\,0}Z_* + \sum_{p=1}^{4} c_{r\,i_p^r}Z_{i_p^r}$
- For diagrams with propagators r aand $r+1$ on same edge $\langle Z_{i-1}, Z_i\rangle$, must replace $Z_{i_2^{r+1}} = Z_i$ by $c_{r\,i_1^r}Z_{i-1} + c_{r\,i_2^r}Z_i$ etc..
- Taking this into account we can write

$$Y_r = \sum_{i=1}^{n} c_{r0}Z_* + C_{ri}Z_i \qquad C_{ri} = C_{ri}\left(c_{r\,i_p^r}\right)$$

  and $C_{ri}(c_{ri_p^r})$ defines $4k$-cycle in $Gr(k,n)$.

Similarly $Gr(k+2L, n+2L)$ for loop integrands and correlators.

Bosonization, positivity and 'hedronizing

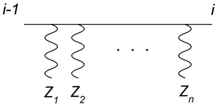
- Take real and bosonize fermionic parts of Y, Z by

$$\mathbb{C}^{4|4} \to \mathbb{R}^{4+k} \quad \text{with} \quad Z^r = \chi \cdot \phi^r, \quad r = 1, \ldots, k\,.$$

- The rth fermionic delta-function arises by

$$\delta^{0|4}(\chi) = \int (Y^r)^4 \, \mathrm{d}^4 \phi^r\,.$$

- Take data $\{Z_*, Z_1, \ldots, Z_n\}$ positive.
- Planarity $\Rightarrow$ positivity of $c_{r\, i^r_p} \Rightarrow C_{ri} \in Gr_+(k, n)$.
- Positivity respects 'Boundary diagrams' with



Tiling Amplituhedra/Wilsonohedra/Correlahedra

w/ Agarwala, Eden, Heslop, following Arkani-Hamed, Hodges, Trnka

- Positive $c_{r\, i_p^r}$ gives $4k$-dimensional tiles in $Gr(k, k+4)$

$$Y_r = c_{r0} Z_* + C_{ri} Z_i \,.$$

where $C = C\left(c_{r\, i_p^r}\right)$.

- Can we tile amplituhedra? Correlahedra?

$$\begin{aligned} 0 &< \langle Y_1 \dots Y_{k+2L} Z_{i-1} Z_i Z_{j-1} Z_j \rangle \\ 0 &< \langle Y_1 \dots Y_{k+2L} Z_{l1} Z_{l2} Z_{j-1} Z_j \rangle \\ 0 &< \langle Y_1 \dots Y_{k+2L} Z_{l1} Z_{l2} Z_{m1} Z_{m2} \rangle \end{aligned}$$

- Above gives $\langle W \rangle^2$ (cf correlator $\leftrightarrow$ Wilson-loop).
- Unlike BCFW, tiles lie both inside and out for $k \geq 2$.
- Spurious boundaries cancel (subtle for boundary diagrams).
- Gives definition for correlators.

[Still work in progress.]

Loop integrand: L loop 4-momenta p_l, $l = 1, \ldots, L$.

$$\langle W \rangle = \int_{(\mathbb{R}^4)^l} F(p_l, Z_i) \prod_{l=1}^{L} d^4 p_l \,.$$

Expect $\langle W \rangle =$ polylogs of degree $2L$ of invariants a_j of Z_i.

Definition

(Naive) A polylog of weight $2L$ is an iterated integral

$$Plog(a_1, a_2, \ldots) = \int_{[0,1]^{2l}} \prod_{m=1}^{2l} \frac{dR_m}{R_m}$$

where $R_m =$ rational fns of a_i and integration parameters s_m.

Conjecture: At MHV amplitude is polylog of degree $2l$.

$d\log$ form of loop integrand

Lipstein & M. arxiv:1212.6228.

- loop order $L = \#$ MHV vertices, $p_l \leftrightarrow (Z_{A_l}, Z_{B_l})$.
- MHV $\Leftrightarrow$ two propagators per vertex.
- integrate Z_{A_l}, Z_{B_l} against δ-functions in propagators

Theorem (Lipstein,M. (cf also Princeton+ group))

The MHV loop integrand can be explicitly expressed as

$$\langle W \rangle = \frac{1}{(2\pi)^{2L}} \int_C \prod_{m=1}^{4L} \frac{ds_m}{s_m}$$

$C =$ compact contour depending on Z_i.

- At N^{k}MHV have $(d\ \log s)^{4(L+k)} \times \left(\delta^{4|4}(Z + \dots)\right)^k$.
- Gives weight $2L = 4L - 2L$ with $(d \log)^{4L} \times (2\pi)^{-2L}$.
- **Strategy:** decompose graph into 'Kermits'=1-loop MHV's.

$$\text{1-loop MHV } = \sum_{i<j} K_{ij}\,, \qquad K_{ij} = \int_{C_{ij}} \frac{ds_0\, ds\, dt_0\, dt}{s_0\, t_0\, s\, t}\,.$$

Contour C_{ij}: $X_0 = Z_A \wedge Z_B$ must be real.

Support of δ-functions $\Rightarrow$

$$Z_A = is_0 Z_* + \frac{Z_{i-1} + sZ_i}{1+s}\,, \qquad Z_A = it_0 Z_* + \frac{Z_{i-1} + tZ_i}{1+t}$$

Set $Z_i \cdot \bar{Z}_* = 1$ and $a_{ij} = Z_i \cdot \bar{Z}_j$, then reality $\Rightarrow$

$$s_0 = \bar{s}_0, \;\; t_0 = \bar{t}_0\,, \text{ and } s = -\frac{(a_{i-1\,j} - v)\bar{t} + a_{i-1\,j-1} - v}{(a_{ij} - v)\bar{t} + a_{ij-1} - v}\,,$$

with $v = s_0 - t_0$.

The real s_0, t_0 integrals need $i\epsilon$: with $Z_* = (0, \eta)$

$$s_0 = \frac{x_{oi}^2}{\langle \bar{\eta} | x_{0i} | \eta]} \to s_0 + i\epsilon f_i := \frac{x_{oi}^2 + i\epsilon}{\langle \bar{\eta} | x_{0i} | \eta]}\,, \qquad \text{etc.}$$

Integration without Feynman parameters

- $s_o + t_0$ decouples $\rightsquigarrow$ contour integral incorporating $i\epsilon$'s.
- Outputs step function $\theta(-f_i f_j)$.
- $\rightsquigarrow$ definite integral in $v \in [v_*, \infty]$, $v_* = x_{ij}^2/2\langle\bar{\eta}|x_{ij}|\eta]$.
- Stokes reduces compact complex integrals $\rightsquigarrow$ logs [Nima].

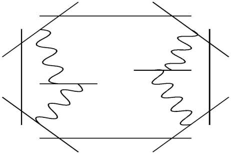
$$K_{ij} = \mathrm{Li}_2\left(\frac{a_{i\,j}}{v_*}\right) + \mathrm{Li}_2\left(\frac{a_{i-1\,j-1}}{v_*}\right) - \mathrm{Li}_2\left(\frac{a_{i-1\,j}}{v_*}\right) - \mathrm{Li}_2\left(\frac{a_{i\,j-1}}{v_*}\right) + c.c.$$

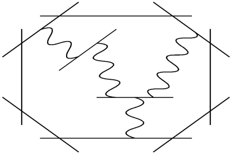
For $j = i + 1$ integrals diverge. Mass regularization $\rightsquigarrow$:

$$K_{ii+1} = -\frac{1}{4}\left(\ln^2\left(\frac{m^2}{x_{ii+2}^2}\right) + \ln\left(\frac{x_{i-1i+1}^2}{x_{ii+2}^2}\right)\ln\left(x_{ii+2}^2\right)\right) - \frac{2\pi^2}{3} + O(m).$$

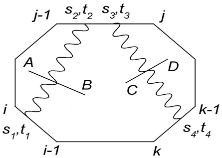
Two loops

- At two loops many diagrams are products of 1-loop terms.





- Nontrivial cases when diagram just connects three edges.



- Diagrams are concatenation of 'kermits'

$$\int_{v_*}^{\infty} \mathrm{d}\ln v \int_{\Gamma} \mathrm{d}\ln s \, \mathrm{d}\ln t \, \mathrm{Li}_2\left(\frac{at+b}{t+1}\right), \qquad s = \frac{\bar{t} - c(v)}{\bar{t} - d(v)},$$

a, b, invariants, c, d mobius transformations.

$$
\int_{v_*}^{\infty} \mathrm{d} \ln v \int_{\Gamma} \mathrm{d} \ln s \, \mathrm{d} \ln t \, \mathrm{Li}_2\left(\frac{at+b}{t+1}\right) =
$$

$$
2\pi i \int_{v_*}^{\infty} \mathrm{d} \ln v \left[\ln\left(\frac{c(v)}{d(v)}\right) \mathrm{Li}_2(b) + \int_{\bar{c}(v)}^{\bar{d}(v)} \mathrm{d} \ln t \, \mathrm{Li}_2\left(\frac{at+b}{t+1}\right) \right.
$$

$$
\left. +(b-a) \int_1^{\infty} \frac{\mathrm{d}z}{(a-z)\,(b-z)} \ln\left(\frac{(c(v)+1)\,z - c(v)a - b}{(d(v)+1)\,z - d(v)a - b}\right) \ln z \right] .
$$

General structure:

- Diagrams still decompose into Kermits.
- Products of 1-loop or more generally lower loop terms when Kermits widely separated.
- At L loops, first new cases when diagram connects $L + 1$ edges. Remaining terms are degenerations.

Integration:

- At L loops, $2L$ integrals are real; must insert $i\epsilon$s for each.
- L real integrals decouple. Contour Integration incorporates $i\epsilon$'s to get L step functions $\theta(-f_i f_j)$ (surprisingly DCI).
- Remaining real integrals are indefinite.
- Havent succeeded in decoupling these from complex integrals for hard 2-loop diagram ($\mathrm{Li}_{2,2}$).

Summary & conclusions

- Geometry of amplituhedra and Grassmannians is built into Feynman rules of twistor action in axial gauge.
- Framework extends to more general correlahedra.
- MHV diagram tiling is imperfect with tiles crossing in and out of correlahedra.
- Need to turn it into a better oiled machine (unitarity, motives, symbols, clusters, Fuchsian differential equations, integrability, regularization....)
- Some progress towards pure weight/transcendentality conjecture (down to 3l rational integrals).

Thank You!