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Integrable hierarchies, enumerative geometry and the combinatorics of Hurwitz numbers

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A family of 2-Toda τ functions is introduced which, when expanded in a basis of products of power sum symmetric functions $p_\lambda p'_\mu$, the coefficients are shown to have both a geometrical and a combinatorial meaning. These extend the well-known case of one and two point Hurwitz numbers studied by Okounkov. On the geometrical side, they extend the class of generalized multi-point Hurwitz numbers studied previously, which count sums over ramification cycle types of specified weights. The newer classes considered involve both ``coloured'' and ``uncoloured'' ramification points, with the ``coloured'' ones consisting of groups for which the sums of the weights are specified.

On the combinatorial side, these coefficients also count paths in the Cayley graph of S_n, generated by transpositions, consisting of sequences that are grouped into three successive types; the first are unordered; the second consist of successive groups having specified lengths, in which the subsequences are strictly monotonically increasing; the third consists of groups, with specified lengths, that are weakly monotonically increasing.

A new viewpoint on this subject is introduced consisting of adding a weighting for the enumeration of branched covers, paths in the Cayley graph. This leads to the notion of ``weighted Hurwitz numbers'', associated to all 2D Toda τ functions of generalized hypergeometric type. In particular, we introduce two different q-deformations of the weighting, associated to strictly and weakly monotonic paths, using the quantum dilogarithm function to generate the weights. The resulting weighted enumeration problem turns out to be closely linked to the statistical mechanics of Bose and Fermi gases.

Based in part on joint work with Mathaieu Guay-Paquet and Alexander Orlov.

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