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Vector-valued wave functions on the torus and Jack polynomials

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The theory of symmetric and nonsymmetric Jack polynomials on $\mathbb{C}^N$ can be extended to polynomials taking values in irreducible modules V_τ of the symmetric group $\mathscr{S}_N$ (C.D. 2010, SLC B64a). Such modules correspond to partitions τ of N. The group action on V_τ-valued polynomials is given by $wp(x) := \tau(w)p(xw)$ where $w \mapsto \tau(w)$ denotes the representation of $\mathscr{S}_N$ on V_τ. Just as products of scalar Jack polynomials with the base state can appear as wave functions for the Calogero—Moser—Sutherland model of N identical particles on the circle with a $1/r^2$ interaction, the vector-valued Jack polynomials have interpretations as eigenfunctions of the same Hamiltonian but with a modified group action. This produces a completely integrable system. The states have the form $L(x)J(x)$, where $J(x)$ is a symmetric V_τ-valued Jack polynomial (i.e. $wJ(x) = J(x)\ \forall w$) and $L(x)$ is a linear operator (matrix) acting on V_τ satisfying a first-order differential system on the region $\mathbb{C}^N_{\mathrm{reg}} := (\mathbb{C}\backslash\{0\})^N \backslash \bigcup_{i<j}\{x : x_i = x_j\}$. Additionally $L(x)$ has the transformation property $L(xw) = M(w;x)L(x)\tau(w)$ where $M(w;x)$ is a locally constant unitary matrix related to the monodromy of L on the connected non-simply connected region $\mathbb{C}^N_{\mathrm{reg}}$. The result is that $x \mapsto J(x)^* L(x)^* L(x) J(x)$ (suitably normalized) is a symmetric probability distribution on the N-torus $\mathbb{T}^N \cap \mathbb{C}^N_{\mathrm{reg}}$, a region consisting of $(N-1)!$ connected components, each of which is homotopic to the circle. The coupling parameter κ for this structure is restricted to $1/h_\tau < \kappa < 1/h_\tau$, where h_τ is the maximum hook-length of the Young diagram of the partition τ.

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