A local-global principle for power maps

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Let $f$ be a function from the set of integers into itself. We call $f$ a global power map if there exists a non-negative integer $k$ so that $f(x) = x^k$ for every integer $x$. We call $f$ a local power map at the prime number $p$ if $f$ induces a well-defined group homomorphism on the multiplicative group of integers modulo $p$. It has been conjectured that, if $f$ is a local power map at infinitely many primes $p$, then $f$ is a global power map. In this talk, I will discuss a theorem implying that, if $f$ is a local power map at all primes $p$ in a set with positive upper density relative to the set of all primes, then $f$ must be a global power map.