

The ranges of some familiar functions

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Consider those integers which appear as values of $\sigma(n)$, of $\varphi(n)$, or of $\lambda(n)$. Here σ is the sum-of-divisors function, φ is Euler's function (returning the order of $(\mathbb{Z}/n\mathbb{Z})^\times$), and λ is the Gauss-Carmichael function (returning the exponent of $(\mathbb{Z}/n\mathbb{Z})^\times$). That most numbers are never values of these functions is not immediately obvious (especially so for λ). Studying the problem for φ 80 years ago, Erdős was led to discover the normal number of prime factors of a shifted prime. This talk will survey the subject and discuss some very new results.

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