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Inclusive prime number races

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Let $\pi(x;q,a)$ denote the number of primes up to x that are congruent to $a \pmod q$. A "prime number race", for fixed modulus q and residue classes $a_1, \dots, a_r$, investigates the system of inequalities $\pi(x;q,a_1) > \pi(x;q,a_2) > \cdots > \pi(x;q,a_r)$. We expect that this system should have arbitrarily large solutions x, and moreover we expect the same to be true no matter how we permute the residue classes a_j; if this is the case, the prime number race is called "inclusive". Rubinstein and Sarnak proved, conditionally, that every prime number race is inclusive; they assumed not only the generalized Riemann hypothesis but also a strong statement about the linear independence of the zeros of Dirichlet L-functions. *In joint work with Nathan Ng*, we show that the same conclusion can be reached with a substantially weaker linear independence hypothesis. The methods involve interpreting a limiting distribution associated with the prime number race as the distribution of a complicated sum of infinitely many random variables, whose dependencies are governed by the $\mathbb{Q}$-linear relations among the zeros of L-functions.

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