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On the powerful and squarefree parts of an integer

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Every integer $n \geq 2$ can be written in a unique way as the product of its powerful part and its squarefree part, that is as $n = mr$ where m is a powerful number and r a squarefree number, with $(m, r) = 1$. We denote these two parts of an integer n by $pow(n)$ and $sq(n)$ respectively, setting for convenience $pow(1) = sq(1) = 1$. We first examine the behavior of the counting functions $\sum_{n \leq x,\, sq(n) \leq y} 1$ and $\sum_{n \leq x,\, pow(n) \leq y} 1$. Letting $P(n)$ stand for the largest prime factor of n, we then provide asymptotic values for $A_y(x) := \sum_{n \leq x,\, P(n) \leq y} pow(n)$ and $B_y(x) := \sum_{n \leq x,\, P(n) \leq y} sq(n)$ when $y = x^{1/u}$ with u fixed. We also examine the size of $A_y(x)$ and $B_y(x)$ when $y = (\log x)^\eta$ for some $\eta > 1$. Finally, we prove that $A_y(x)$ will coincide with $B_y(x)$ in the sense that $\log(A_y(x)/x) = (1 + o(1)) \log(B_y(x)/x)$ as $x \to \infty$ if we choose $y = 2 \log x$.

This is joint work with Maurice-Étienne Cloutier and Nicolas Doyon

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