Atelier
« Nouvelles approches en théorie probabiliste et multiplicative des nombres »
08-12 décembre 2014

Workshop
"New approaches in probabilistic and multiplicative number theory"
December 08–12, 2014

# On the greatest prime factor of the twelfth cyclotomic polynomial

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Let $P^+(n)$ denote the greatest prime factor of the integer $n$ and $\Phi_{12}(X) = X^4 - X^2 + 1$ the twelfth cyclotomic polynomial. We prove that there exists $c > 0$ such that for $x$ large enough we have

$$P^+\left(\prod_{n\le x} \Phi_{12}(n)\right) \ge x^{1+c}.$$

The value $c = 10^{-26531}$ is admissible. The proof consists in adapting Heath—Brown's method concerning $P^+\left(\prod_{n\le x}(n^3+2)\right)$. The main ingredients are :

- an upper bound on some exponential sums involving the fractions $\frac{v}{m}$ with $\Phi_{12}(v) \equiv 0 \mod m$, $0 < v < m$, and the integers $m < M$ have a "good" factorization;
- a result on the joint distribution in arithmetic progressions of the values $\left(f_1(b,c), f_2(b,d)\right)$ where $f_1$ and $f_2$ are two irreducible binary forms of degree $\ge 2$.

This result for $\Phi_{12}$ has been extended recently by La Bretèche to the irreducible polynomials of degree four with Galois's group isomorphic to $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$.

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