R-operator and knot invariant via cluster algebra

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The cluster algebra was introduced by Fomin and Zelevinsky around 2000. A characteristic operation of the algebra called 'mutation' has been related to various notions in mathematics. In this talk, based on joint work with Kazuhiro Hikami (Kyushu University), I introduce our recent research on the R-operator constructed from the mutations.

First we define the $R$-operator and apply it to study the complex volume, (hyperbolic volume) $+ i$ (Chern—Simons invariant), of knot complements in $S^3$. In three-dimensional hyperbolic geometry it is known that a mutation produces an ideal tetrahedron, and the $R$-operator is thought as a hyperbolic octahedron. We show that the cluster variables are interpreted as Zickert’s edge parameters used to compute complex volume. Second we introduce the $q$-deformation of the $R$-operator by using quantum cluster algebra a la Fock and Goncharov. We construct a braiding operator in terms of quantum dilogarithm function, which realizes the quantum $R$-operator as adjoint action. When we take a limit that $q$ goes to a root of unity, the braiding operator reduces to the Kashaev $R$-matrix up to a simple gauge-transformation.


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