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Symmetric polynomials, generalized Jacobi—Trudi identities and τ-functions

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An element $[\Phi]$, of the Grassmannian of n-dimensional subspaces of the Hardy space H_2, extended over the field $F = C(x_1, \dots, x_n)$, may be associated to any polynomial basis. The Plucker coordinates, labelled by partitions λ, provide an analog of Jacobi's bialternant formula, defining a generalization of Schur polynomials. Applying the recursion relations satisfied by the polynomial system to the analog of the complete symmetric functions generates a doubly infinite matrix of symmetric polynomials that determine an element of the Grassmannian. This may be shown to coincide with $[\Phi]$, implying a set of generalized Jacobi—Trudy identities, extending a result obtained by Sergeev and Veselov for the case of orthogonal polynomials. The symmetric polynomials are shown to be Kadomtsev—Petviashvili τ-functions in terms of the power sums $[x]$ of the x_a's, viewed as KP flow variables. A fermionic operator representation is given for these, as well as for the infinite sums over Schur functions associated to any pair of polynomial bases (φ, θ). These are shown to be 2D Toda lattice τ-functions. A number of applications are given, including classical group character expansions, matrix model partition functions and generators for random processes.

Based on joint work with Eunghyun Lee.

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