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# Spectral methods for solving nonlinear Schrödinger equations with singular defect terms

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Singular source terms play an important role in various nonlinear PDEs such as the nonlinear Schrödinger equation, the sine--Gordon equation, and the forced Korteweg--de Vries equation [1, 5, 6]. It is well known that the presence of such singular source terms yields the critical phenomenon of the solution to those PDEs [3, 7, 9]. The singular source term in the equation mimics the local defect. The defect is highly localized and is represented mathematically as a Dirac delta function that can be understood in the distribution sense. High-order approximation, such as the spectral approximation, of the solution with the singular source term is challenging. Moreover, if the singular source term is sensitive to the associated parameter values due to its locality and those values are stochastic, the high-order approximation needs a special treatment particularly to reduce the computational complexity. In this talk, we will explain several developments that deal with the PDEs perturbed by the singular source terms including the consistent method and the generalized polynomial chaos (gPC) method. The consistent method is based on the Schwartz duality of the delta distribution and uses the consistent derivative operator to the numerical scheme. The consistent method provides the cancellation of higher-order error terms and improves the convergence order [4, 8]. We also use the gPC method [10] for the nonlinear Schrödinger equation under the assumption that the singular source term is parameterized by the random variables. For the solution in the random space, we adopt the orthogonal polynomials associated with the distribution that construct the random space. We, then, seek a numerical solution in the polynomial space by the Galerkin procedure. Using the gPC method, we show that we can find the critical values efficiently and determine the statistical quantities with high accuracy [2].

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