Near invariance of the hypercube

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We will give an almost complete description of matrices $M$ of order $n$ under which the hypercube $-1,1^n$ is nearly invariant; that is $\Pr_x(Mx \in -1,1^n) > n^{-O(1)}$, where $x$ is sampled uniformly from $-1,1^n$. As an application, we show that if $M$ is orthogonal then it must be very close to products of permutation and reflection matrices.

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