The extended degree zero subalgebra of the Ext algebra of a linear module

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Let $\mathbb{k}$ be a field and let $R$ be a Koszul $\mathbb{k}$-algebra. Let $M$ be a linear $\mathbb{k}$-module and let $\Gamma$ be the Ext-algebra of $M$. View $\Gamma$ as a bigraded algebra with the bigrading induced by the homological degree and by the internal grading of $M$, that is

$$\Gamma = \text{Ext}^*_R(M, M) = \bigoplus_{n \geq 0} \bigoplus_{i \in \mathbb{Z}} \text{Ext}^n_R(M, M)_i.$$ 

We consider next the extended degree zero subalgebra $\Delta_M$ of $\Gamma$,

$$\Delta_M = \bigoplus_{n \geq 0} \text{Ext}^n_R(M, M)_0.$$ 

So $\Delta_M$ is generated by all the homogeneous elements of $\Gamma$ having internal degree zero. It turns out that the extended degree zero subalgebra can be used to obtain a characterization of the graded center of a Koszul algebra. I will also present some other applications of the ideas involved.

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