On a homological problem for module categories with infinite radical cube zero

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Let $A$ be an artin algebra over a commutative artin ring $K$. We denote by $\text{mod } A$ the category of finitely generated right $A$-modules, by $\text{ind } A$ the full subcategory of $\text{mod } A$ consisting of indecomposable modules, and by $\text{rad}^{\infty}_A$ the infinite Jacobson radical of $\text{mod } A$ (being the intersection of all powers $\text{rad}^i_A$, $i \geq 1$, of the Jacobson radical $\text{rad}_A$ of $\text{mod } A$). It has been proved by M. Auslander that an artin algebra $A$ is of finite representation type if and only if $\text{rad}^{\infty}_A = 0$. In fact, by a result proved by F. U. Coelho, E. M. Marcos, H. A. Merklen and A. Skowroński, $(\text{rad}^{\infty}_A)^2 = 0$ implies that $A$ is of finite representation type. Moreover, they investigated also the structure of module categories $\text{mod } A$ with $(\text{rad}^{\infty}_A)^3 = 0$.

About 12 years ago A. Skowroński conjectured that the following two conditions for an artin algebra $A$ are equivalent:

1. For all but finitely many isomorphism classes of modules $X$ in $\text{ind } A$, we have $\text{pd}_A X \leq 1$ or $\text{id}_A X \leq 1$.

2. $A$ is a generalized double tilted algebra or a quasitilted algebra.

The aim of this talk is to confirm the above conjecture for module categories with infinite radical cube zero.