Selfinjective algebras of finite representation type with maximal almost split sequences

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Let $A$ be a finite dimensional $K$-algebra over an arbitrary field $K$ and $\text{mod } A$ the category of finite dimensional right $A$-modules. For a nonprojective indecomposable module $X$ in $\text{mod } A$, there is an almost split sequence

$$0 \rightarrow \tau_A X \rightarrow Y \rightarrow X \rightarrow 0,$$

where $\tau_A X$ is the Auslander—Reiten translation of $X$. Then we may associate to $X$ the numerical invariant $\alpha(X)$ being the number of summands in a decomposition $Y = Y_1 \oplus \cdots \oplus Y_r$ of $Y$ into a direct sum of indecomposable modules in $\text{mod } A$, which measures the complexity of homomorphisms in $\text{mod } A$ with domain $\tau_A X$ and codomain $X$. It has been proved by R. Bautista and S. Brenner (1981) that, if $A$ is of finite representation type and $X$ is a nonprojective indecomposable module in $\text{mod } A$, then $\alpha(X) \leq 4$, and if $\alpha(X) = 4$, then the middle $Y$ of an almost split sequence in $\text{mod } A$ with the right term $X$ admits an indecomposable projective-injective direct summand. An almost split sequence in the module category $\text{mod } A$ of an algebra $A$ of finite representation type with the middle term being a direct sum of four indecomposable modules is called a maximal almost split sequence in $\text{mod } A$.

We will discuss the structure of basic, indecomposable, finite dimensional selfinjective $K$-algebras $A$ of finite representation type over a field $K$ for which $\text{mod } A$ admits a maximal almost split sequence.

Joint work with A. Skowroński.