# Minimum energy configurations in the $N$-body problem and the celestial mechanics of granular systems

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Celestial Mechanics systems have two fundamental conservation principles : conservation of momentum and conservation of (mechanical) energy. Of the two, conservation of momentum provides the most constraints on a general system, with three translational symmetries (which can be trivially removed) and three rotational symmetries. If no external force acts on the system, these quantities are always conserved independent of the internal interactions of the system. In contrast, conservation of energy involves assumptions on both the lack of exogenous forces and on the nature of internal interactions within the system. For this reason energy is often not conserved for "real" systems that involve internal interactions, such as tidal deformations or impacts, even though such systems conserve total momentum. Thus, mechanical energy generally decays through dissipation until the system has found a local or global minimum energy configuration that corresponds to its constant level of angular momentum.

This observation motivates a fundamental question for celestial mechanics :

What is the minimum energy configuration of a $N$-body system with a fixed level of angular momentum ?

It can be shown that this is an ill-defined question and has no answer for traditional point-mass celestial mechanics systems. If, instead, the system and problem are formulated accounting for finite density distributions this question becomes well posed and we can prove the existence of minimum energy configurations for all suitably formulated celestial mechanics systems (Scheeres, CMDA 113(3) : 291–320, 2012). This small change also leads to fundamental changes in the nature and stability properties of relative equilibria and, ultimately, the dynamics of these systems.

Finally, we can also show that this naturally leads to a "granular mechanics" extension of celestial mechanics, with fundamental links between this topic and the science of small solar system bodies.

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