

Open problems on the arithmetic of noncongruence modular forms

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Unlike their congruence counterpart, the arithmetic for noncongruence modular forms remains mysterious. A main reason is the lack of efficient Hecke operators. The progress in this area has been guided by numerical data.

Atkin and Swinnerton-Dyer made an amazing observation that at ordinary primes, the space of weight k cusp forms admits a basis whose Fourier coefficients satisfy 3-term congruence relations, called ASD congruences, analogous to the 3-term recursive relation satisfied by a Hecke eigenform. In his thesis Kibelbek exhibited examples to show that this no longer holds at non-ordinary primes.

Scholl attached ℓ -adic Galois representations to the space of noncongruence forms. These representations are motivic, and hence should correspond to automorphic representations according to the Langlands philosophy. The automorphy of Scholl representations is established only for special cases.

The area of noncongruence modular forms is a fertile ground to explore. In this talk we shall review the current status and discuss some open problems.

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