Atelier
"Arbres aléatoires"
12–16 août, 2013

Workshop
"Random Trees"
August 12–16, 2013

Convergence of voter model densities

J. Theodore Cox*

jtcox@syr.edu

URL: http://as-cascade.syr.edu/profiles/pages/cox-theodore.html

---

We consider a sequence of voter models ξ_t^n on finite sets $\mathcal{S}_n$, $n = 1, 2, \ldots$, with $|\mathcal{S}_n| \to \infty$. For each n let q_n be an irreducible transition matrix on $\mathcal{S}_n$ with stationary probability distribution π_n. The voter model ξ_t^n is a Markov process taking values in $\{0,1\}^{\mathcal{S}_n}$ such that $\xi_t^n(x)$ flips to $1 - \xi_t^n(x)$ at rate $\sum_{y \in \mathcal{S}_n} q_n(x,y) 1\{\xi_t^n(y) \neq \xi_t^n(x)\}$. Given a sequence of positive constants γ_n, the corresponding time-scaled voter model density process is $Y_t^n = \sum_{x \in \mathcal{S}_n} \xi_{t\gamma_n}^n(x)/|\mathcal{S}_n|$.

It is well known that in the mean-field case $q_n(x,y) = 1/(|\mathcal{S}_n| - 1)$ for $x \neq y$, if $\gamma_n = |\mathcal{S}_n|$ then Y^n converges as $n \to \infty$ to the Wright–Fisher diffusion Y, the diffusion on $[0,1]$ which has generator $\frac{1}{2}u(1-u)\frac{d^2}{du^2}$. This convergence also takes place if $\mathcal{S}_n$ is the d-dimensional torus $[0,n)^d \cap \mathbb{Z}_d$, $q_n(x,y) = 1/2d$ if $|x-y| = 1$, $\gamma_n = c_2|\mathcal{S}_n| \log |\mathcal{S}_n|$ for $d = 2$ and $\gamma_n = c_d|\mathcal{S}_n|$ for $d \geq 3$. We give generals condition under which this convergence holds when γ_n is the expected first meeting time of two independent Markov chains with transition matrix $q^{(n)}$, each started with initial distribution π_n. One condition is that the associated Markov chain mixing time $\mathbf{t}_{\text{mix}}^n$ be small relative to γ_n, i.e., $\mathbf{t}_{\text{mix}}^n/\gamma_n \to 0$ as $n \to \infty$. By duality, this result is closely related to recent results of Oliveira on the coalescing times of Markov chains on finite sets.

*Department of Mathematics, Syracuse University, 215 Carnegie Bldg., Syracuse, NY 13244–1150, USA.