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Approximation numbers of composition operators on a Hilbert space of Dirichlet series

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The rate of decay of the approximation numbers of a compact operator with symbol ϕ is a quantitative way to measure its degree of compactness in terms of its symbol ϕ. This study was achieved (1995) for Hankel operators by Megretskii, Peller and Treil, and undertaken for composition operators on Hardy, Bergman, or Dirichlet spaces by Lefevre, Li, Queffélec, and Rodríguez-Piazza (2012–2013). In a recent joint work (2013, under progress) with K. Seip, I began a similar study for composition operators on the space $\mathscr{H}^2$ formed by Dirichlet series $f(s) = \sum_{n=1}^{\infty} b_n n^{-s}$ with square-summable coefficients. One more Hilbert space of analytic functions on the half-plane $\mathbb{C}_{1/2} = \left\{s : \operatorname{Re} s > \frac{1}{2}\right\}$, introduced by Beurling (about 1945), and revisited by Hedenmalm, Lindqvist, and Seip (1997), to study completeness or Rieszness problems. The situation is yet significantly different in several respects:

(1) There are few composition operators on $\mathscr{H}^2$, exactly described by a theorem of Gordon and Hedenmalm (1999), and depending on a non-negative integer c_0.

(2) The space $\mathscr{H}^\infty$ of multipliers of $\mathscr{H}^2$ does not live on the same half-plane $\mathbb{C}_{1/2}$ as $\mathscr{H}^2$.

(3) The study of composition operators on $\mathscr{H}^2$ quite often leads to the study of composition operators in several variables, namely on the polydisk of $\mathbb{C}^d$, a study still full of mysteries.

In this presentation of the work with K. Seip, I will focus on our main tools (alternative definitions of approximation numbers, spectrum, reproducing kernel equation), as well as on our main results, namely:

(1) A general lower and upper estimate, in terms of Blaschke products, Carleson measures, interpolation sequences.

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(2) A general device to exploit those general estimates without help of the space $\mathscr{H}^\infty$.

(3) The fact that, if $c_0 \geq 1$, the approximation numbers decay quite slowly.

(4) A rather sharp estimate of those approximation numbers for polynomials symbols, in terms of the so-called “dimension” of the symbol. This latter result is a nearly optimal improvement of a previous result of the author with C. Finet and S. Volberg (2004).