From categories to logic, linguistics and physics: a tribute for the 90th birthday of Joachim Lambek

Abstracts of talks

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Full Lambek Calculus in logic and linguistics

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Full Lambek Calculus (**FL**) is the (associative) Lambek calculus **L*** (admitting sequents with the empty antecedent) augmented with lattice connectives $\vee, \wedge$ and, optionally, constants $1, 0, \top, \bot$. It is a basic substructural logic; substructural logics are often defined as axiomatic and rule extensions of **FL** (Galatos *et al.* 2007). The family of substructural logics includes almost all significant nonclassical logics, *e.g.* many-valued and fuzzy logics, relevant logics, linear logics, constructive logic with strong negation, up to the limit cases of intuitionistic logic and classical logic.

In formal linguistics one considers different variants of **L***, less popular among logicians, as *e.g.* the calculus **L** (not admitting the empty antecedents), due to Lambek (1958), the nonassociative systems **NL**, **NL*** and others. In the last decade the Poznań group (W.B., M. Farulewski, Z. Lin) and other authors also studied Full Nonassociative Lambek Calculus **FNL** and its extensions as well as type grammars based on these systems; a variant of **FNL** is called Groupoid Logic **GL** in (Galatos *et al.* 2007).

In this talk I plan to discuss some selected associative and nonassociative systems of this kind and their models, to explain their role in logic and linguistics, and to suggest some directions of further research.

Claudia Casadio *and* Mehrnoosh Sadrzadeh

Cyclic Properties In Linear Logic Vs. Pregroups—Theoretical insights and linguistic analysis

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The calculus of pregroups [Lambek 2001, 2008] is a type of categorial grammar introduced by Lambek for the analysis and computation of grammatical structures in natural languages: similarly to Syntactic Calculus (Lambek 1958), pregroups are non-commutative structures, but the syntax of natural languages admits certain cyclic patterns, in particular those exhibited by the - so called - movement rules or transformations. In this paper we make this observation precise by showing that the logic of pregroups admits two cyclic rules analogous to those holding in the systems of non-commutative multiplicative linear logic studied by Yetter [1991] and Abrusci [1991, 1998, 2002].

First, we discuss the connections between the cyclic properties of non-commutative linear logic and pregroups and we define appropriate formulations of these properties. Then we prove that, by introducing guarded versions of cyclicity to avoid overgeneration, interesting applications to different languages can be obtained allowing for a straightforward and computationally easy analysis of different kinds of movement. The linguistic justification of this paper comes from previous work [Casadio, Sadrzadeh 2009, 2011], where guarded formulations of cyclic rules were added as meta-rules to pregroups and used to analyze clitic movement in Persian, French, and Italian, and word movement in Hungarian [Sadrzadeh 2010].

Keywords: Pregroup calculus, Cyclic Rules, Word Order, Movement

Bob Coecke

The truth unveiled by quantum robots: Lambek does not like Lambic!

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This talk has two parts. The 1st one will be given by Bob Coecke and the 2nd one, which involves a demo, by Aleks Kissinger.

Part I: Quantum linguistics. Categorical quantum mechanics, due to its common structure with Lambek's pregroups, resulted in a compositional distributional model of meaning in which meanings of words can be computed from meanings of sentences. The observation of the common structure was made by Lambek himself, at a seminar at McGill by the current speaker. As an example, we compute the meaning of: "The man who does not like Lambic".

Part II: Quantum robotics. Lambek's work on deductive systems showed us a beautiful connection between coherence of categories and syntactic proofs: when a category is nice enough, it comes with a syntax and a proof theory so easy that even a dumb machine could handle it. When the categories are compact, the nicest syntax is graphical, and the dumb machine is called Quantomatic, who will beyond any doubt establish that Lambek does not like Lambic.

Michael Makkai

Multicategories

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Jim Lambek has done fundamental work on multicategories, both in the general theory of the subject and in its applications. The talk will survey Lambek's work related to multicategories. I will also try to give an idea about the ways multicategories remain important in current problems.

Michael Moortgat

Multidimensional Dyck languages

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Typelogical grammar, ever since the rediscovery of Lambek's Syntactic Calculus in the 1980s, has been struggling to come to grips with *discontinuity*: information flow between physically detached parts of an utterance. Emmon Bach's MIX language is an extreme example: the words of this language are strings over a three-letter alphabet with an equal number of occurrences of each letter but no constraints on their order. A natural generalization allows variation in the size (or: dimension) of the alphabet, keeping the multiplicity constraint on the letters.

I consider a similar multidimensional generalization for Dyck languages. These are d-dimensional MIX languages with an extra prefix constraint: reading a word from left to right, there are always at least as many letters a_i as $a_{(i+1)}$. In the two-dimensional case, we have the familiar language of well-balanced brackets.

I discuss a bijection between multidimensional Dyck languages and standard Young tableaux of rectangular shape, obtained via the Yamanouchi words of these tableaux.

Generalized Dyck languages and the corresponding rectangular tableaux have fascinating connections outside the field of mathematical linguistics. For the two-dimensional case, Abramsky 2009 already discussed the connections between Temperley-Lieb algebras, mathematical physics, planar λ-calculus and Lambek's Syntactic Calculus. In the three-dimensional case, there is the bijection (due to Khovanov-Kuperberg) between $3 \times n$ tableaux and the combinatorial graphs ('webs') for the $\mathfrak{sl}(3)$ spider, *cf.* Tymoczko for a recent contribution. Surprisingly, these webs are *planar* directed graphs (with certain additional properties) embedded in a disk, suggesting that the discontinuity of 3-dimensional Dyck languages is illusory.

I discuss the implications of these bijections for a typelogical understanding of non-local dependencies in language.

References: E. Bach, Discontinuous Constituents in Generalized Categorial Grammar. Proceedings NELS 11:1-12, 1981 • J. Tymoczko. A simple bijection between standard $3 \times n$ tableaux and irreducible webs for $\mathfrak{sl}(3)$. Journal of Algebraic Combinatorics, 35:611–632, 2012 • S. Abramsky. Temperley-Lieb Algebra: From Knot Theory to Logic and Computation via Quantum Mechanics, ArXiv 0910.2737, 2009.

Philip Scott

From Gödel to Lambek: studies in the foundations of mathematics

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For many years, Jim Lambek has studied logic and the foundations of mathematics from a novel, independent viewpoint. His interests have ranged from computability theory to higher-order logic to a reinterpretation of Gödel's Incompleteness theorems, as well as various approaches to reconciling the traditional philosophies of mathematics. In this talk I will examine some of Lambek's seminal theories, some recent developments, as well as some of our still ongoing work.

Peter Selinger

Control categories and duality

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The well-known Curry-Howard-Lambek correspondence is between intuitionistic propositional logic, simply-typed lambda calculus, and cartesian-closed categories. It was long believed that such a correspondence could not be extended to classical logic without making all the structure trivial. However, in the 1990's, a Curry-Howard-Lambek-type correspondence was found between classical propositional logic, the simply-typed lambda-mu calculus (Parigot 1992), and control categories (Selinger 1998). In fact, there are two such correspondences, mutually dual, which are called "call-by-name" and "call-by-value". On the categorical side, this gives rise to interesting concepts such as premonoidal categories; on the lambda calculus side, it gives rise to continuations. In this talk, I will explain and examine these concepts in retrospective.