PROGRAMME THÉMATIQUE
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On Calabi families of hyperkähler manifolds

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Let (M, g) be a Riemannian hyperkähler manifold and $T \to P^1$the corresponding Calabi family parametrising deformations of holomorphic symplectic structures on the underlying manifold M. Like twistor spaces, the total space T comes equipped with a natural complex structure and a natural hermitian metric h, that is usually not Kähler. We compute the complex hessian of the natural metric form $Im(h)$ on T.

It is conjectured that every compact holomorphic symplectic manifold (X, η) contains an entire curve. The choice of a Kähler class ω on X gives, thanks to Yau's theorem, a hyperkähler metric g on X_{diff} and hence a Calabi family $T = T(X, \eta, \omega)$. The space T is fibered by rational curve transversal to $T \to P^1$, called twistor lines. Frédéric Campana used the deformations of the twistor lines to produce an entire curve in one member of every Calabi family $T(X, \eta, \omega) \to P^1$. Due to the non-Kähler property of T, the component C of the cycles space of T that contains the twistor lines is not compact. We derive from our computations a convexity property of C.

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