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Integral points on modular curves

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The problem of determination of Q-rational points on modular curves reduces *grosso modo* to three types of curves of prime level, corresponding to three types of maximal subgroups of the linear group $\mathrm{GL}_2(F_p)$:

– the curve $X_0(p)$, corresponding to the Borel subgroup ;

– the curve $X^+_{\mathrm{sp}}(p)$, corresponding to the normalizer of a split Cartan subgroup ;

– the curve $X^+_{\mathrm{ns}}(p)$, corresponding to the normalizer of a non-split Cartan subgroup.

On the curves of the first two types the rational points are determined (almost) completely : Mazur (1978), B.-Parent–Rebolledo (2012). In particular, it is proved that for $p > 13$ the rational points are either cusps or the CM-points. Little is known however on rational points on $X^+_{\mathrm{ns}}(p)$.

I will speak on a recent progress in a simpler problem : classification of integral points on $X^+_{\mathrm{ns}}(p)$ (i.e. rational points P such that $j(P)$ is in Z). My students Bajolet and Sha obtained a rather sharp upper bound for the size of integral points. Also, *in a joint work with Bajolet* we proved that for $7 < p < 71$ there are no integral points on $X^+_{\mathrm{ns}}(p)$ other than the CM-points ; this improves on a recent work of Schoof and Tzanakis, who proved this for $p = 11$.

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