

Sharp benefit-to-cost rules for the evolution of cooperation on regular graphs

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Some simple rules for the evolution of cooperation on finite graphs were discovered by insightful, but non-rigorous, methods in Ohtsuki, Hauert, Lieberman, and Nowak [*Nature* 441 (2006) 502–505]. Although spatial structures inherently complicate the underlying evolutionary dynamics, it is surprising that only average degrees of graphs and payoffs defining interactions enter these rules. Prior to our work, the simple rules were rigorously verified on very few graphs. In particular, a recent work by Cox, Durrett, and Perkins [*Astérisque* 349 (2013)] proved one of the rules, adapted to \mathbb{Z}^d for $d \geq 3$, from the viewpoint of voter model perturbations. It inspires us to derive a general machinery which makes possible a rigorous study of several evolutionary games on any finite spatial structure. I will review the setup by Ohtsuki *et al.* and discuss related methods in interacting particle systems. I will then present our rigorous result for one rule. Its proof is based on the general machinery, and in fact is very different from the original non-rigorous argument. In the context of regular graphs, we obtain the universality of the rule, and moreover, reinforce the associated original discovery.

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