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Generalized reciprocity in spatially structured populations

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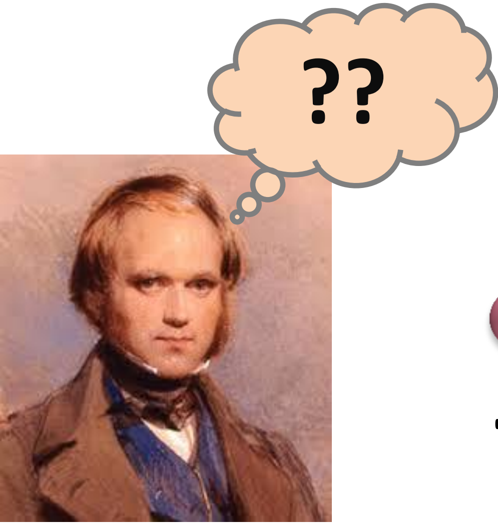
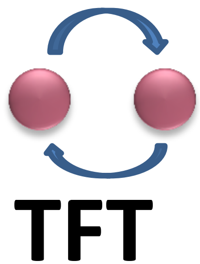
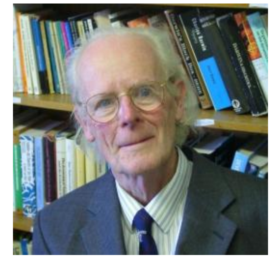
*“A **hydrogen bomb** is an example of mankind’s enormous capacity for **friendly cooperation**. Its construction requires an intricate network of human **teams**, all working with a single minded devotion towards a **common goal**. Let us pause and savor in the glow of self-congratulation we deserve for belonging to such an **intelligent** and **sociable** species.”*

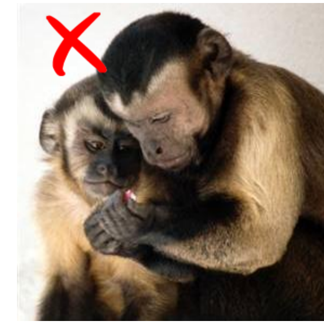
R. Bigelow, *The Dawn Warriors*

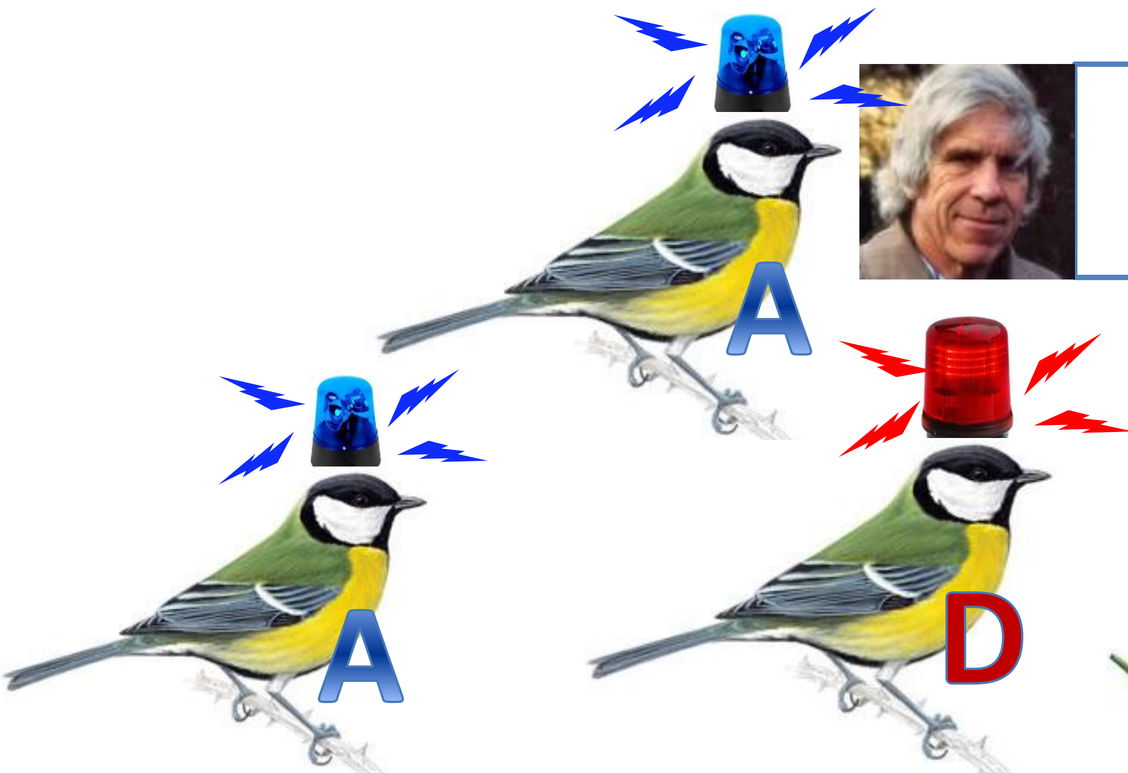
Altruism





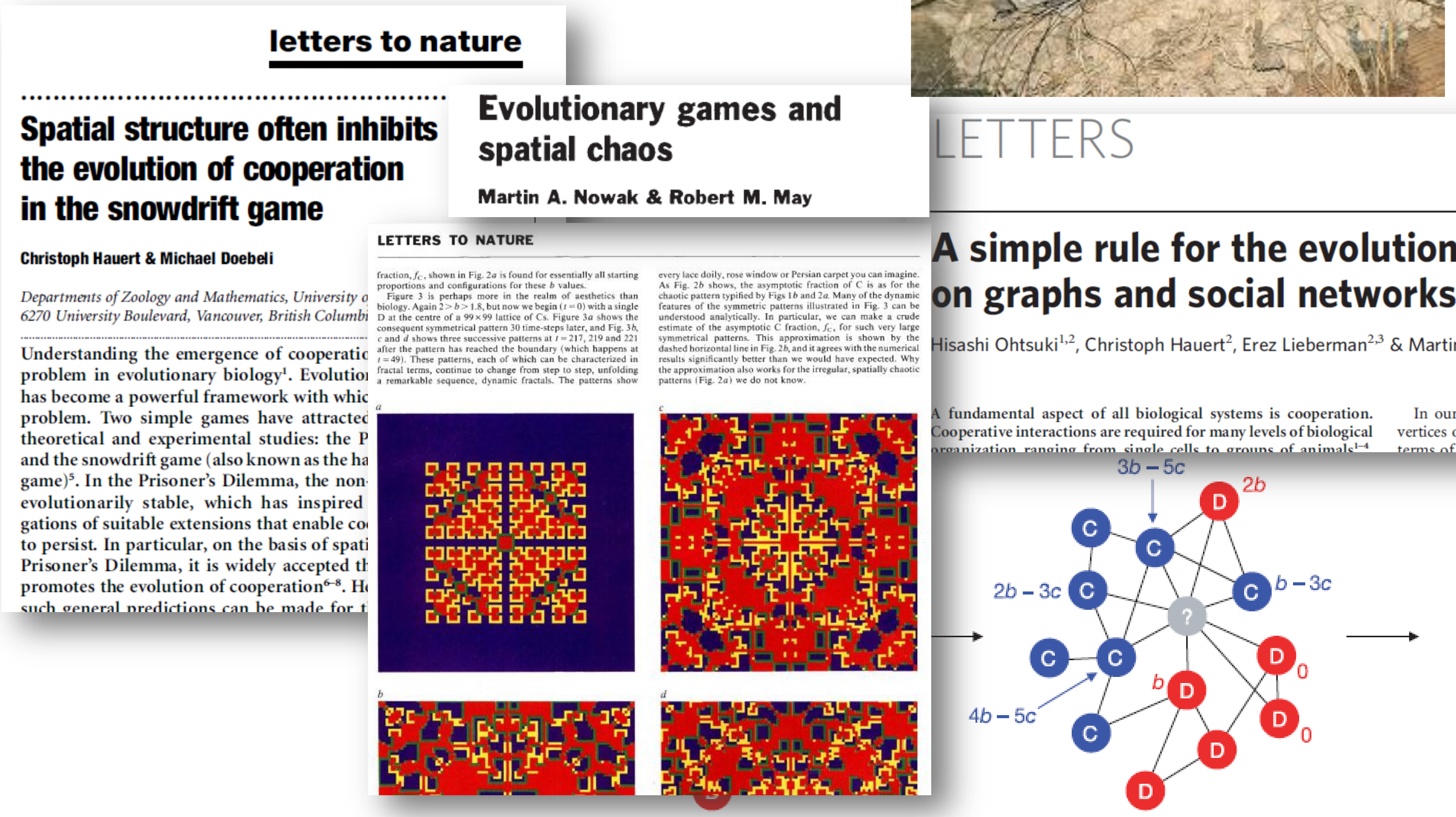






$c < r b$

Spatial reciprocity
Network reciprocity



Indirect reciprocity

Large-scale cooperation among humans can be understood as resulting from networks of indirect reciprocity.

[R. Alexander, 1987]

"what comes around goes around"

Boyd & Richerson (1989):

Groups of size n are sampled from a large population and interact repeatedly. Probability w that the group persists for one more generation.
Upstream TFT: cooperate if individual 'upstream' cooperated before.
Downstream TFT: cooperate if individual 'downstream' cooperated before.
ALLD: Always defect.

Indirect reciprocity is unlikely to be important unless interacting groups are fairly small.

Indirect reciprocity

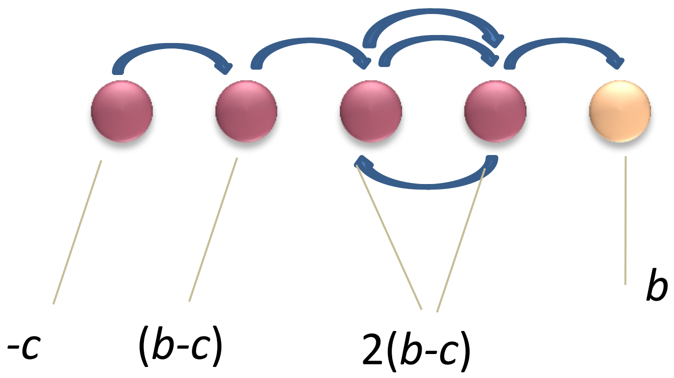
Nowak & Roche (2007):

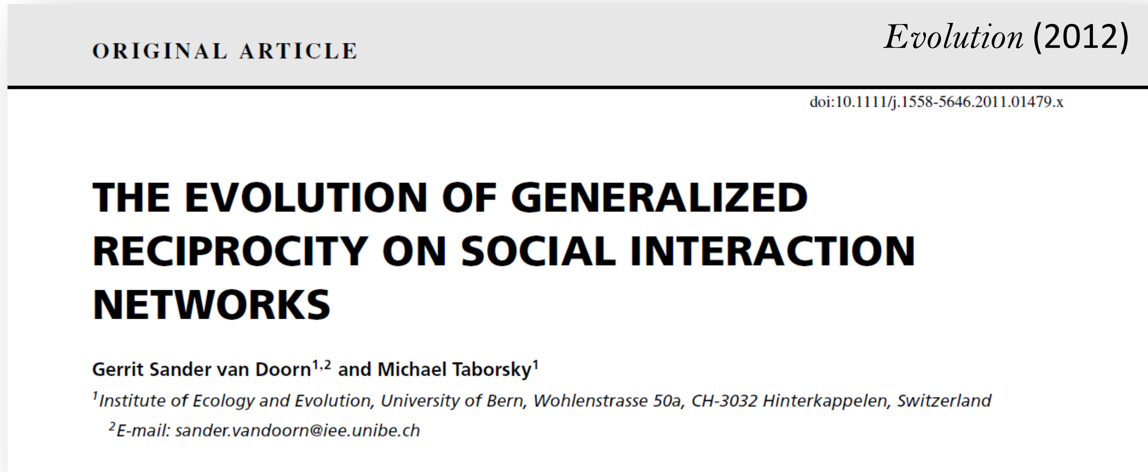
Continuous strategies:
q: probability to initiate
p: probability to pass on

*“Upstream reciprocity alone does **not** lead to the evolution of cooperation, but it can evolve and increase the level of cooperation if it is linked to [..] spatial reciprocity.”*

imitation updating

Generalized reciprocity





ORIGINAL ARTICLE

Evolution (2012)

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THE EVOLUTION OF GENERALIZED RECIPROCITY ON SOCIAL INTERACTION NETWORKS

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Evolutionary dynamics:

+ discrete strategies: ***A D***
+ fitness dependent reproduction
+ offspring randomly placed on graph

Cognitively simple ✓

Without kin assortment ✓

Assumption 1: perfect genetic linkage

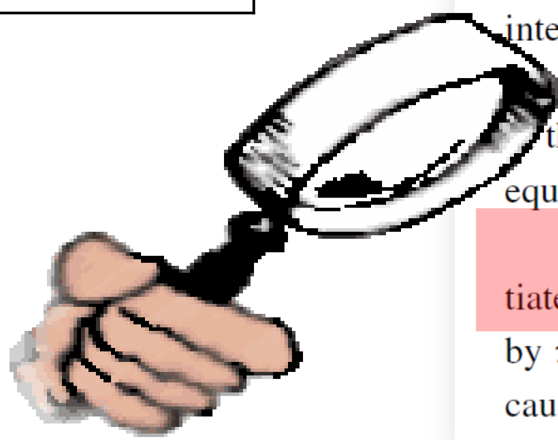
	Initiate: Yes	Initiate: No
Reciprocate: Yes	A	L
Reciprocate: No	S	D

accumulated by reciprocal altruists is therefore given by

$$\pi_A = \mathrm{E}[(n + n_i')(b - c)(k_i + 1) - n\,b], \qquad (2)$$

where n is the expected number of times that the focal individual initiates cooperation, and n_i' denotes the expected number of times that individual i is hit for the first time by a sequence of altruistic interactions that originated elsewhere on the network. Without [illegible] of generality, we choose $n = 1$ in the analytical treatment [illegible] the model. As elsewhere in this manuscript, the expectation in equation (2) is taken over all positions in the network.

If we conservatively assume that only reciprocal altruists initiate cooperation, then the expected payoff of defectors is given by $\pi_D = b\,\bar{n}'$, where $\bar{n}' = \mathrm{E}[n_i'] = f_t/(1 - f_t)$. This follows because every sequence that is initiated by a reciprocal altruist must



Assumption 2: discrete strategies

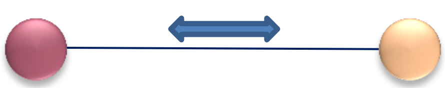
	Yes	No
Reciprocate	A	D

$\rightarrow$

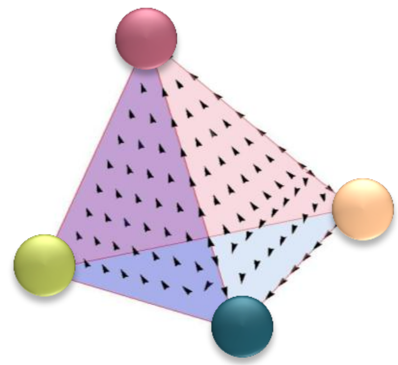
P (reciprocate) r

Assumption 1: perfect genetic linkage

	Initiate	
Reciprocate	**Yes**	**No**
Yes	A	L
No	S	D



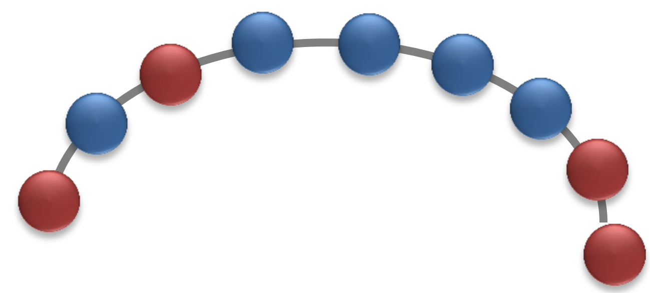
S(2)



S(4)

Generalized reciprocity on the cycle

		Initiate: Yes	Initiate: No
Reciprocate	Yes	A	L
	No	S	D



Average payoff of strategy X

$$\Pi_{\bar{X}} = \left(\frac{s}{N} \Pi_{X|S} + \frac{a}{N} \Pi_{X|A} \right) \frac{1}{x}$$

$$\Pi_{A|S} = \frac{a+l}{N-1} \sum_{j=1}^{a+l} j \varphi_j \frac{a}{a+l} (b-c)$$

$$\Pi_{A|A} = \sum_{j=1}^{a+l} \left(\frac{j-1}{3} + \frac{j^2+j-2}{6} \frac{a-1}{a-1+l} \right) \tilde{\varphi}_j (b-c) - c$$

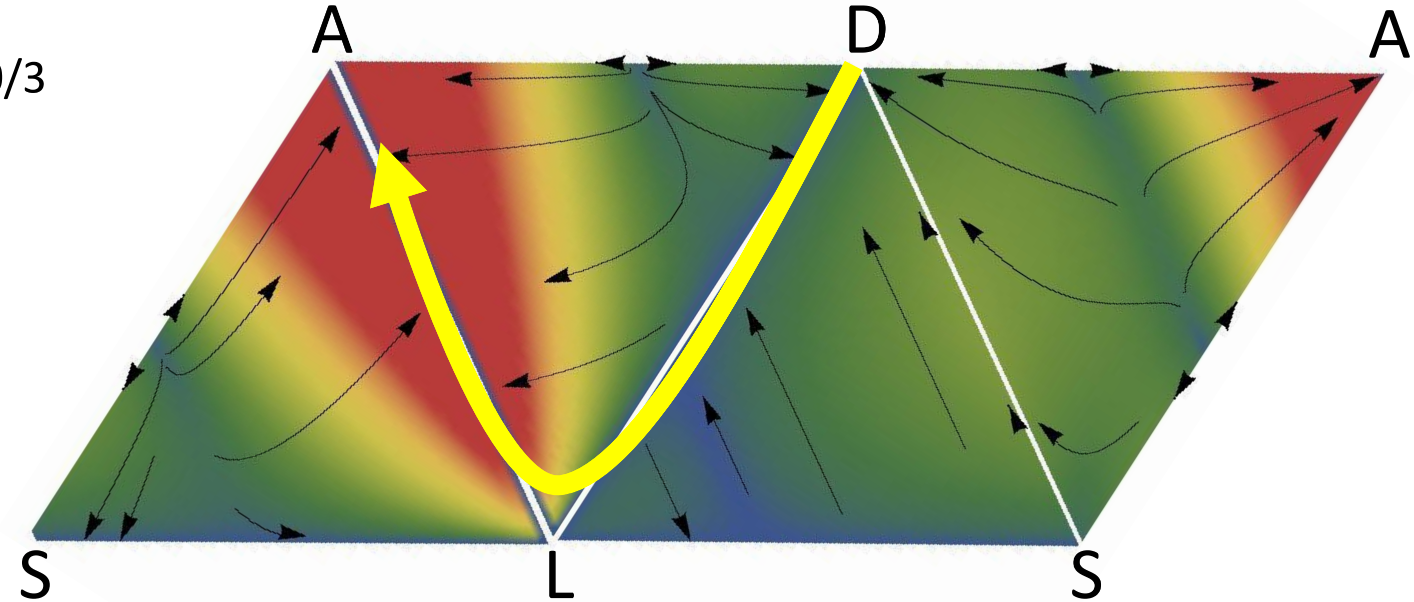
$$\vdots$$

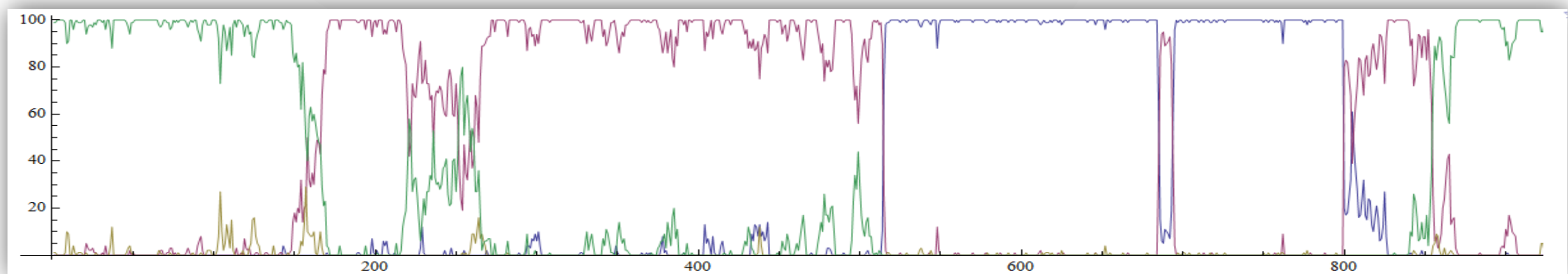
How frequently do we get segments with j individuals in a row?

$$\varphi_j = \frac{\sigma_j}{\sum_{i=1}^{a+l} \sigma_i}, \quad \sigma_j = \frac{N}{(a+l-j)!} \prod_{k=1}^{a+l-j} (N-j-k-1)$$

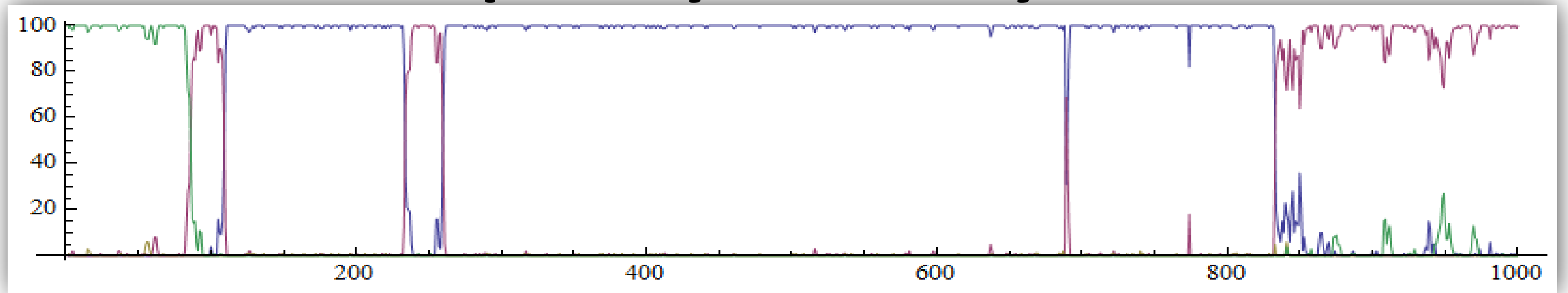
Generalized reciprocity on the cycle

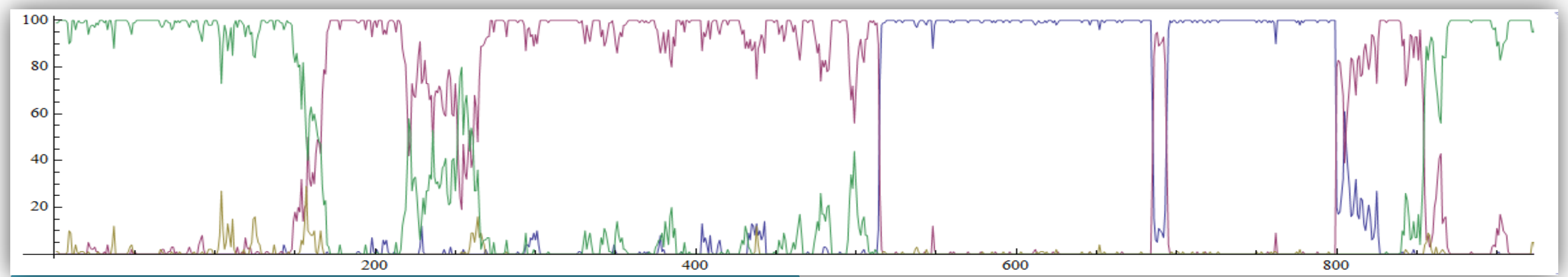
b/c=10/3

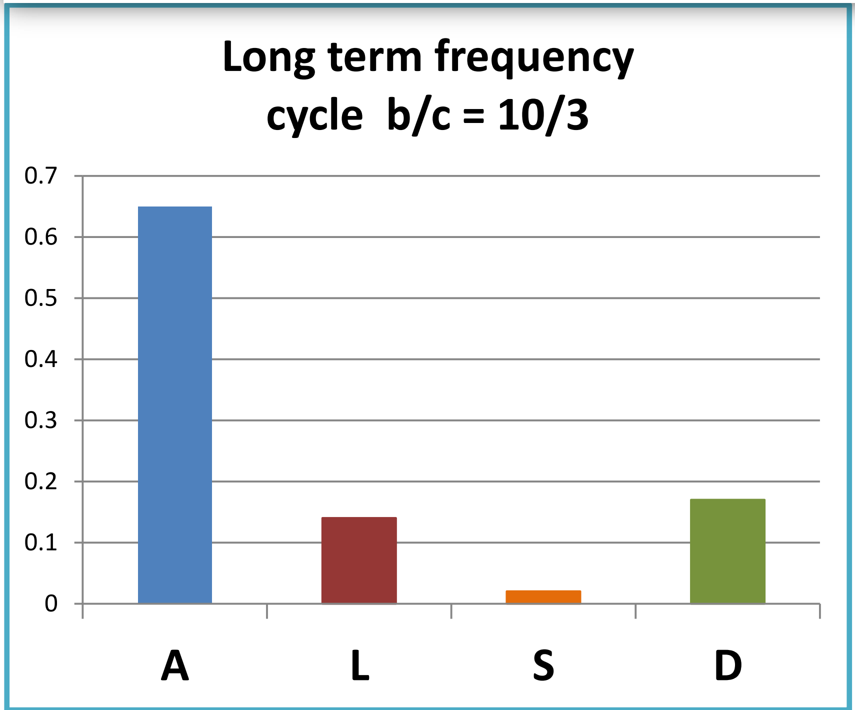




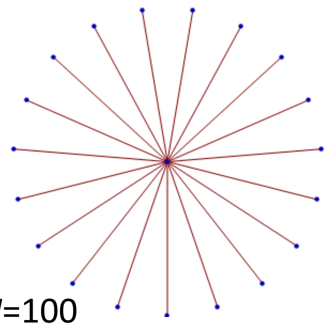
Generalized reciprocity on the cycle



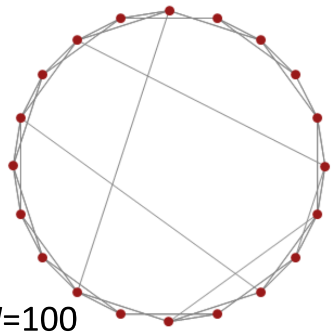




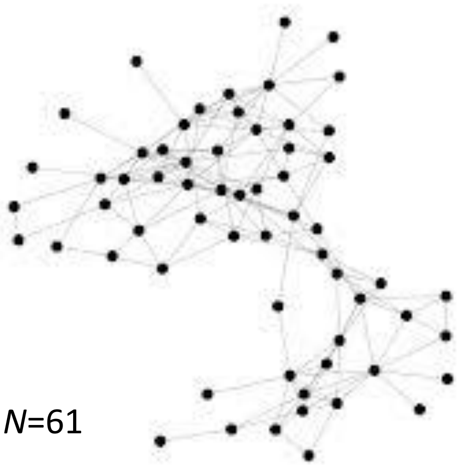
.. and on other graphs



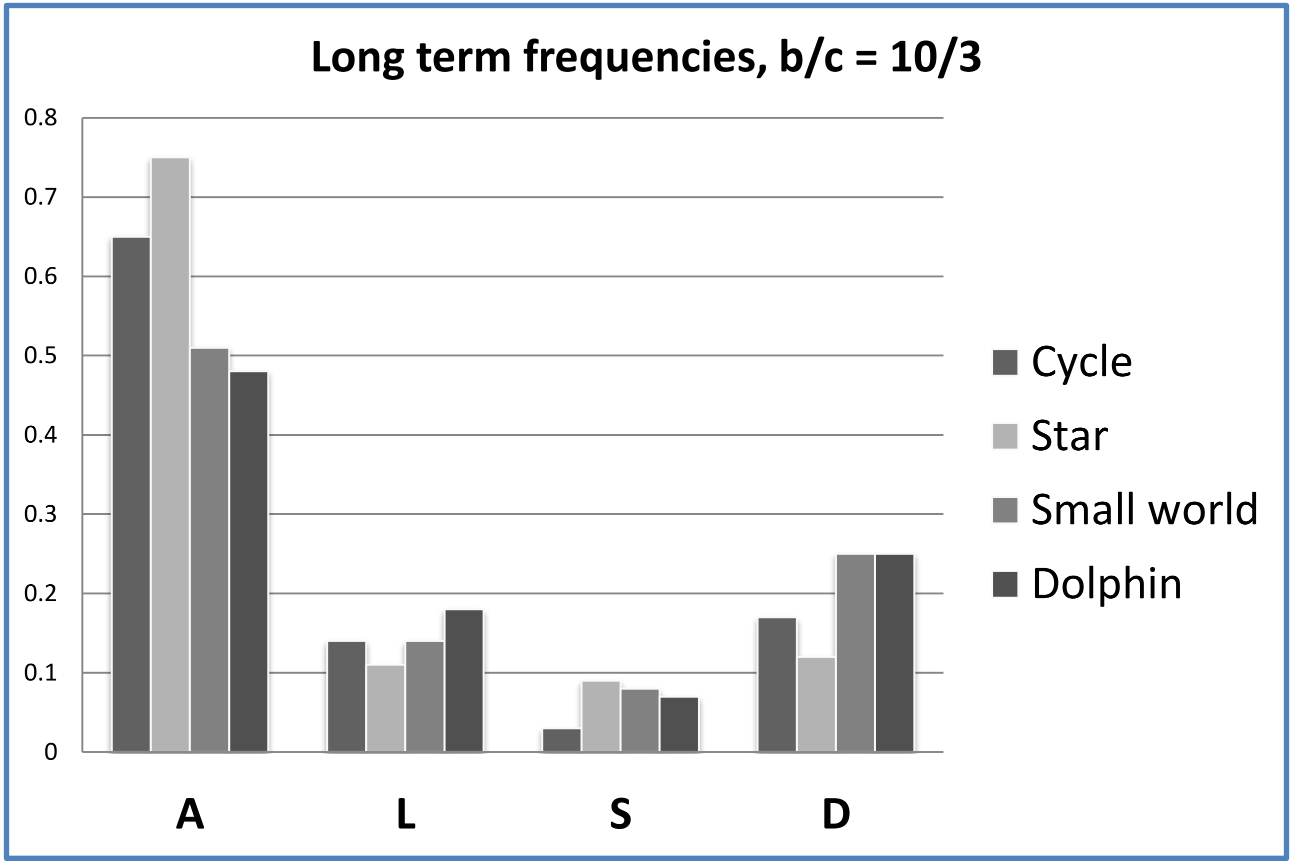
N=100



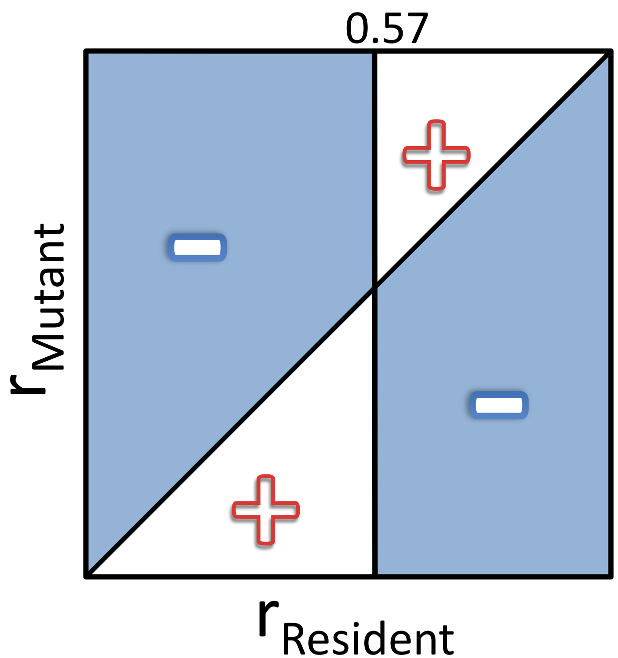
N=100



N=61



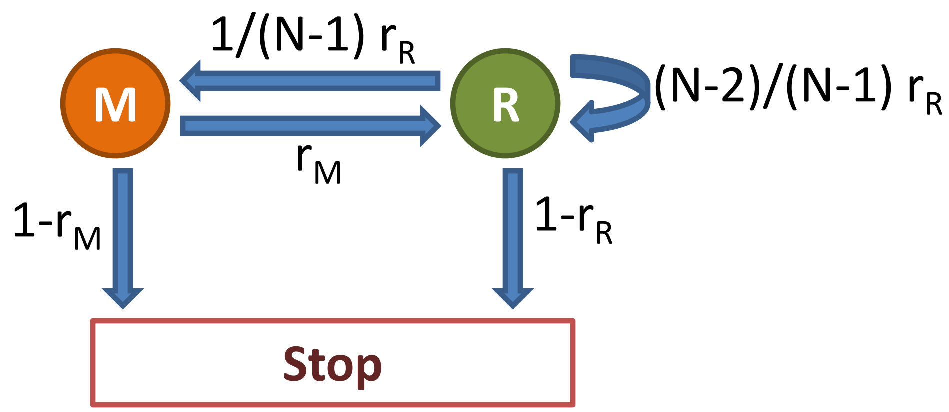
Continuous case



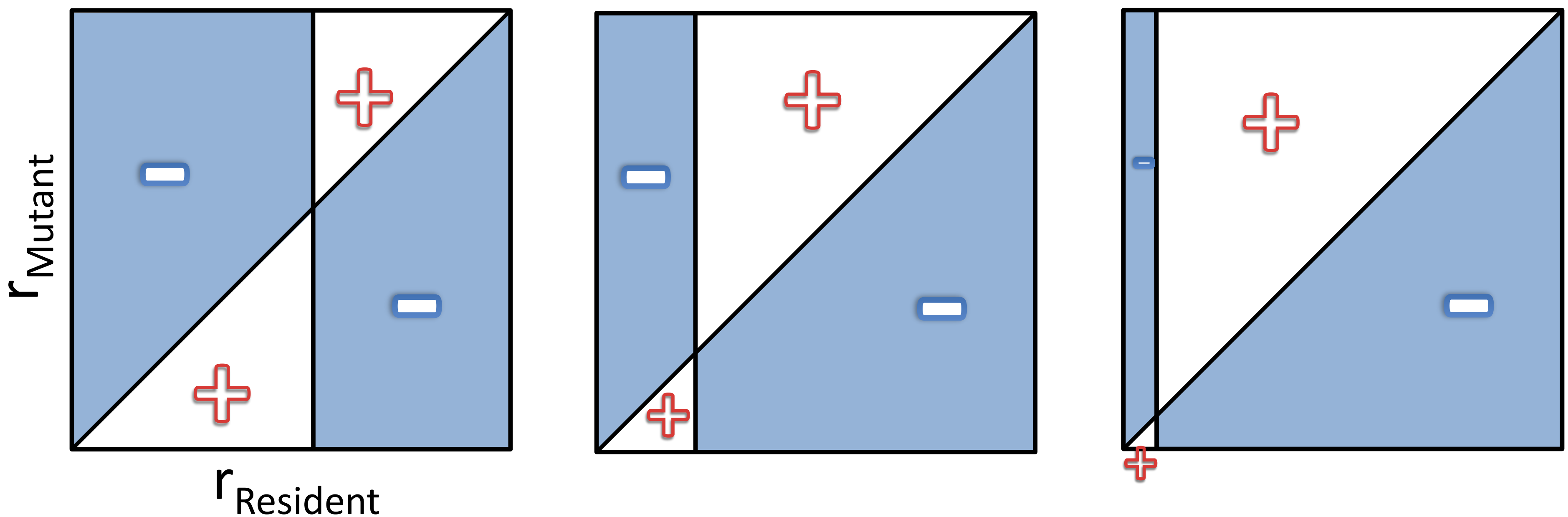
Cycle: b/c = 10/3

r: probability to reciprocate

Initial growth rate starting with a single mutant?

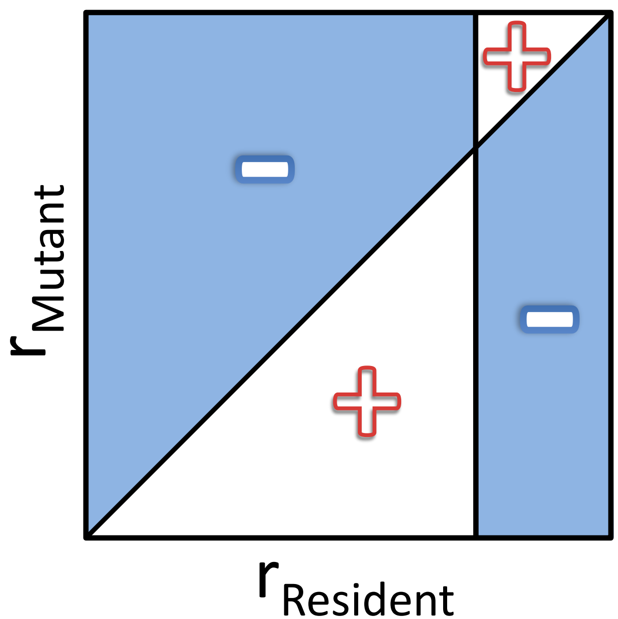


Continuous case

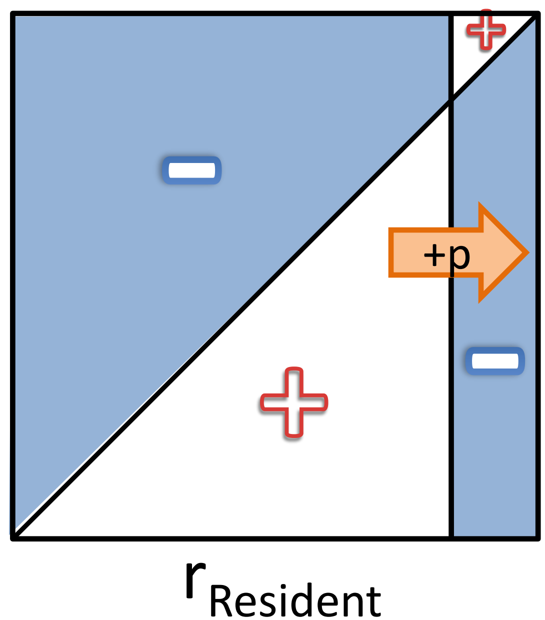


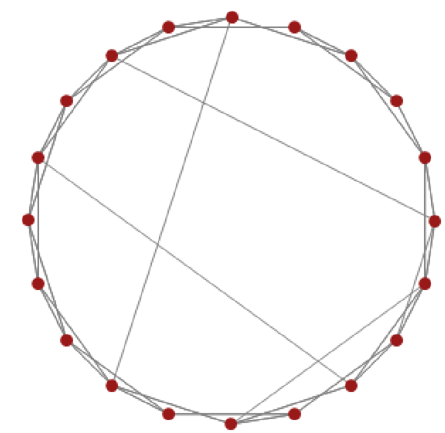
Cycle: b/c = 10/3 b/c = 10/1 b/c = 100/1

Continuous case



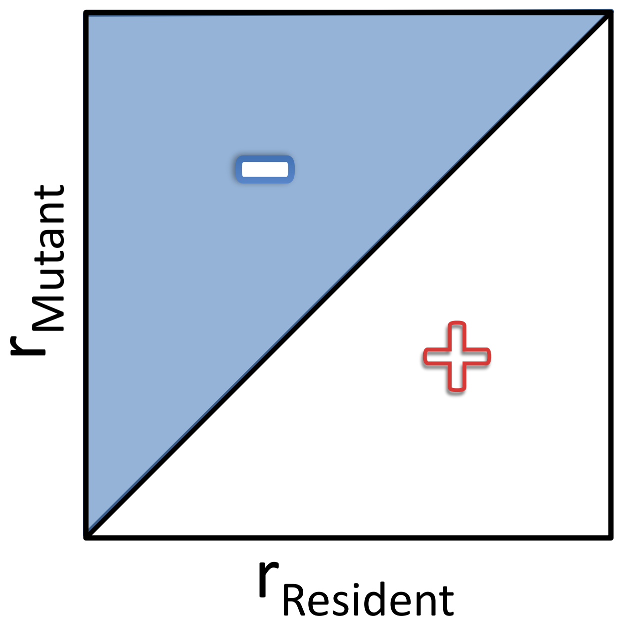
1D lattice: $b/c = 10/3$
$k=4$, $p=0$

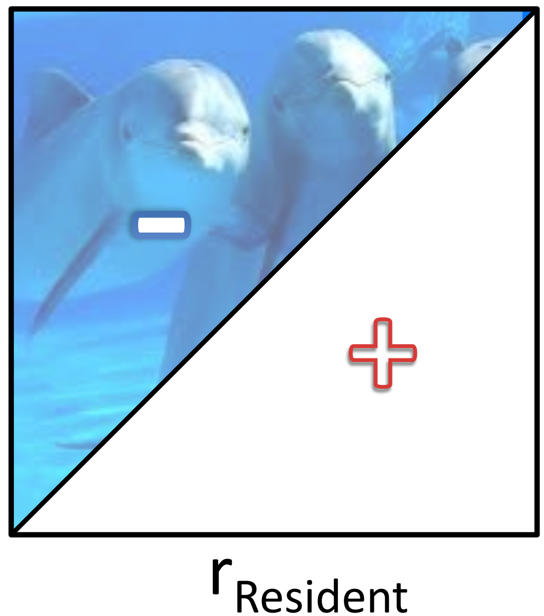




Small world: $b/c = 10/3$
$k=4$, **$p=0.1$**

Continuous case

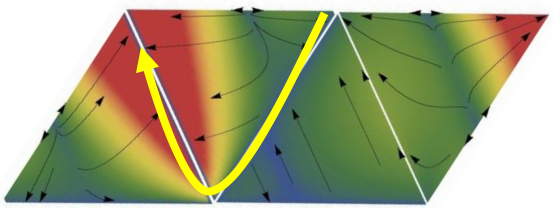




Star: $b/c = 10/3$
N=100

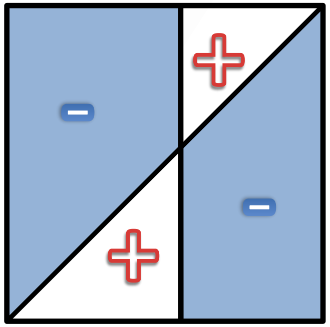
Dolphin: $b/c = 10/3$
N=61

Generalized Reciprocity



Discrete strategies without genetic linkage

- sparse graphs
- neutral path to altruism
- high rates of reciprocation



Continuous strategies

- evolution of cooperation requires
  - extreme population structure
  - extreme b/c ratio
  - perturbations

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