Viability kernels for by-catch fisheries

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Victoria University of Wellington

Based on ``Computation of viability kernels: a case study of by-catch fisheries"
a 2013 *Computational Management Science* paper coauthored by Pharo, Serea & Sinclair

Workshop on Biodiversity and Environment:
Viability and Dynamic Games Perspectives

Montréal, 4 - 8 November, 2013

My aim is

to entice you to read the paper, in which

- some specific questions concerning a by-catch fishery are answered using viability theory.
- It is argued that viability theory is an important mathematical tool for the solution of economic problems, and that
- it can be more robust than optimisation or stability analysis.
- An introduction to viability theory (for $T = \infty$) is provided.
- An account of its principal analytical tool -- viability kernel $\mathcal{V}$ -- is given.

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- Given a dynamic system with state constraints, $\mathcal{V}$ is the set of all state-space points, starting from which it is possible to remain within the constraints indefinitely.
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Viability ≈ Sustainability

Sustainability problems arise in situations where (human) interaction with a dynamic, changing environment can potentially lead to catastrophic outcomes.

- In ecology, not understanding the dynamics of a fish population compounded with economic pressures (for employment preservation) can lead to extinction.
- In macroeconomics, low interest rates can lead to ``bubbles"; high interest rates - to unemployment.
- The sustainable``solution" to these problems is to find a way to avert catastrophe; i.e. maintain the system within the realms of safety or acceptability.

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Which questions can *vt* answer?

Viability theory *(vt)* has been explicitly developed to analyse invariant sets in which a dynamic system will remain, making it a perfect fit for considering problems of sustainability. In particular, *vt* can ascertain

1. whether a system will be able to sustain itself according to the given sustainability criteria over some time-frame; and also
2. which system states afford the possibility of the system sustaining itself, and which do not.

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Where systems are susceptible to control by a regulator, *vt* can also determine normative rules:

3. what policies can be pursued to guarantee the sustainability of the system;
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What is viable and what is not?

- If from a given system state there is an evolution which is possible according to what is known about the system's dynamics and which sustains the system within the imposed bounds, then that system state is viable.
- Conversely, where there is no conceivable way for the system to remain within those bounds when starting from a given state, then this state is said to be non-viable.
- Identification of states as viable or non-viable is achieved in *vt* by computing the viability kernel $\mathcal{V}$ -- the largest closed subset of points in the constraint set for which all points are viable.
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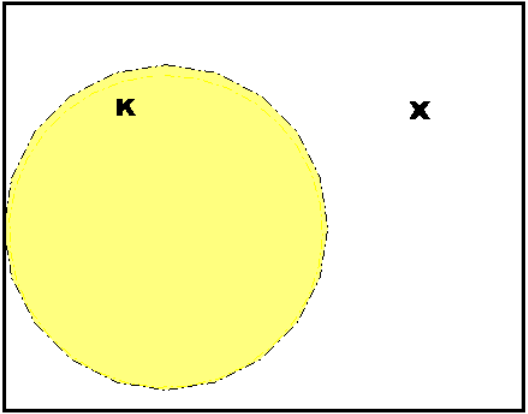

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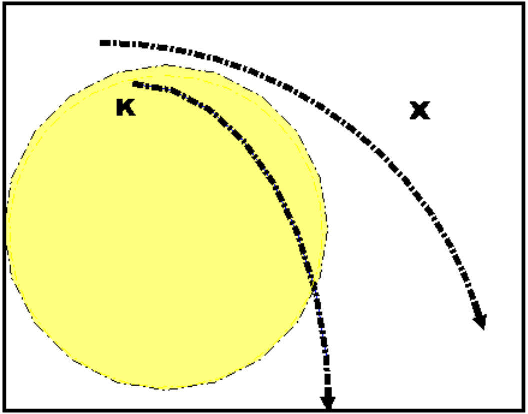

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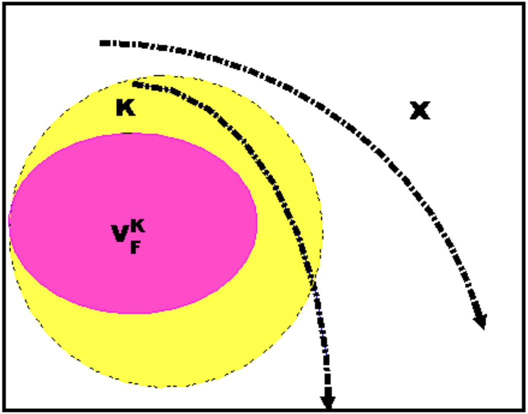

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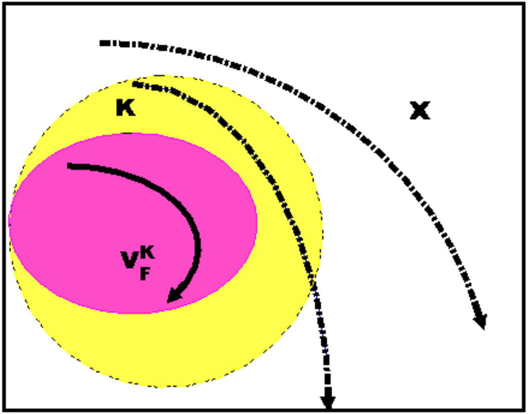

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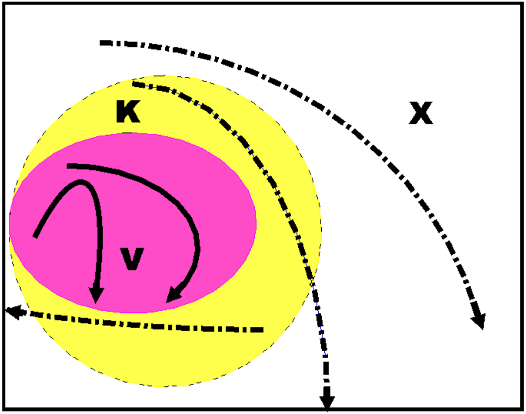

Is *vt* ``better" than optimal control?

If a regulator knows what needs be optimised, applying the optimal strategy is the unique way to control the dynamic system.

However, if agents are H. Simon's agents *i.e.*, believed to employ strategies that are ``good enough" in that they satisfy normative and modal (imposed by reality) constraints then *vt* is useful.

In particular, *vt* introduces additional ``viable" control strategies, based around the concept of the viability kernel: unless the system is in danger of travelling from a viable to a non-viable state any control will be viable.

Under this view, *vt* provides (arguably) a better fit for the real concerns of regulators than optimal control does (constraints may be more ``objective" than a loss function, potentially complicated).

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Behaviourists' delight?

In essence,

- *vt* has the potential to respond to the behaviourists' challenges and to provide insights into compatibility between the system's dynamics and the constraints' geometry;
- *vt* is an appropriate analytical tool for the analysis of sustainability problems which can be solved if the above compatibility has been understood.

In this paper, *vt* will be used to solve the twin ecological-economic problem of sustaining fish population at a safe level whilst at the same time maintaining the profitability of fishing operations that impact that population.

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The differential inclusion

$\dot{x}(t) \in F(x(t)) \quad (*)$ -- says that $\dot{x}$ will be drawn from $F(x)$, the set of all possible velocities at $x(t)$ (F-correspondence). Exactly which element from $F(x(t))$ will eventuate is subject to uncertainty which may come from any of the following sources:

1. the system may be controllable by a regulator. In this case, we write $\dot{x}(t) = f(x(t), u(t)), \quad u(t) \in U(x(t))$;
2. there may be uncertainty about the underlying model dynamics *i.e.*, there may be a number (j) of proposed rhs $\{f_1, f_2, \ldots f_j\}$ describing the system's evolution. So, $\dot{x}(t) \in \{f_1(x(t)), f_2(x(t)), \ldots f_j(x(t))\}$. If there is uncertainty about model parameters then $\dot{x}(t) = f(x; \gamma)$ where $\gamma \in \Gamma$ is drawn from a range of hypothesised values.

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3. The system may be truly non-deterministic (tychastic *i.e.*, not subject to any identifiable regularities). This means that any evolution which satisfies $(*)$ may eventuate.
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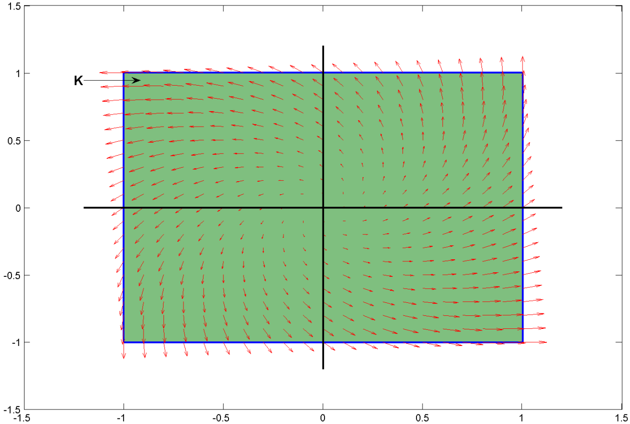
There is a point of *no return*

A vector field $\frac{dx}{dt}$, $\frac{dy}{dt}$ for an uncontrolled system's dynamics.



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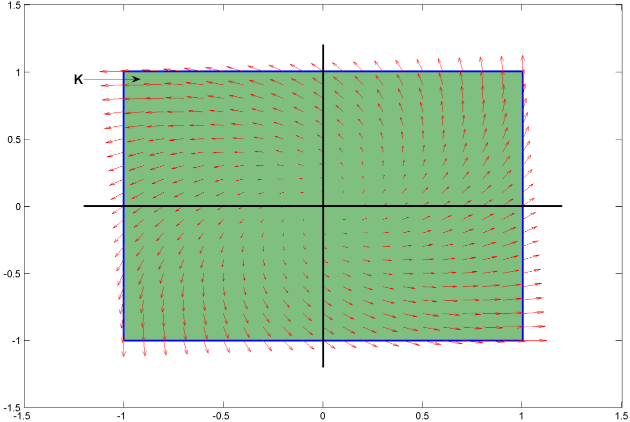

Viable points

Given a differential inclusion F over some set X, $x_0 \in K \subset X$ is viable in K under F if, starting from $x(0) = x_0$ $\exists x(\cdot) : \Theta \mapsto X$

$$\forall t \in \Theta \begin{cases} x(t) \in K, \\ \dot{\mathrm{x}}(t) \in F(x(t)), \quad \forall\, t \in \Theta \equiv [0, \infty), \end{cases}$$

K is the constraint set imposed on the system evolving under F.

The above formulation has a philosophical interpretation: an evolution that starts at a viable point follows a path that satisfies fate F and desire K (``kraving").

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Viable areas

Theorem

Assume D is a closed set in $\mathbb{R}^n$. Suppose that the set valued map $F : \mathbb{R}^n \rightsquigarrow \mathbb{R}^n$ is Lipschitz continuous with convex, compact, nonempty values. Then the two assertions are equivalent :

a. $\forall x_0 \in D$, *there exists a solution* $x(\cdot) : \Theta \mapsto \mathbb{R}^n$ *of*

$$\begin{cases} \dot{x}(s) = F(x(s)) \text{ for almost every } s \\ x(0) = x_0 \end{cases}$$

which remains in D ;

b. $$\forall x \in D, \quad \forall p \in \mathcal{NP}_D(x), \quad \min_{v \in F(x)} \langle v, p \rangle \leq 0 \,.$$

Viable areas *cont.*

- The viability theorem states that wherever the directions available in $F(x)$ and a proximal normal form an obtuse angle, then x will be viable.

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Definition

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Let K be a closed set in $\mathbb{R}^n$. The problem's *viability kernel* for dynamics F, denoted: $\mathcal{V}_F(K)$, is the *largest* possible viability domain under F that is also a subset of K.

- Therefore $\mathcal{V}_F(K)$ is the set of *all* points that are viable in K under F.
- Establishing the viability kernel $\mathcal{V}_F(K) \neq \emptyset$ solves the viability problem. ``Good" -- viable -- states $x(t) \in \mathcal{V}_F(K)$ are separated from ``bad" $x(t) \notin \mathcal{V}_F(K)$.

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Policy

- For control problems, the existence of $\mathcal{V}_F(K)$ indicates an area for which sufficient control exists to maintain the system within $\mathcal{V}_F(K) \in K$ from any point in $\mathcal{V}_F(K)$.
- I.e., $\forall\ x_0 \in K$, there exists a feedback rule $g : X \mapsto Y$ that takes an element $x \in X$ and returns a control policy u such that $x(t) \in K$.

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Satisficing policy

This generic policy can be decomposed into two normative directives: at x

1. use any admissible control for x in the *interior* of the viability kernel $\mathcal{V}_F(K) \setminus \text{fr}\, \mathcal{V}_F(K)$;
2. when one gets ``near" to the boundary of the kernel $\text{fr}\, \mathcal{V}_F(K)$, an extreme instrument, or a specific path, must be followed (unless a steady state has been reached).

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Differences

- Problems modelled using a viability approach do not need to determine utility or loss functions in order to formulate policy rules, and therefore there is no need to calibrate such functions, hence no subjective appraisal of which constraints are more important is needed.
- Determining the bounds of the set K is a potentially much simpler task, given that such bounds x(e they normative or modal) are often trivially observable.
- This contrasts with the optimisation approach where the constraints that define K are usually implicit in the loss function.
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Viability generalises stability

- The kernel is a closed set and it can be characterised by some measure, which the distance between two states in the kernel will never exceed.
- Knowing $\mathcal{V}_F(K)$, makes the regulator aware of the locus of states in which the dynamic system can continue to exist, for a given ``strength" of implementable controls.
- If the system is in $\mathcal{V}_F(K)$ (``stable") and when more than one control is viable, the regulator may strive to achieve other goals (e.g., political or ``wants" rather than ``needs").

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Algorithms

- An analytical characterisation of the viability kernel is rarely possible and most models using *vt* rely on numerical methods to compute their viability kernels.
- Classical algorithms were proposed by Frankowska, Quincampoix, Saint-Pierre. They work by ``whittling away'' points that exit the set after one discrete-time step. Chapel and Deffuant have implemented Saint-Pierre's algorithm in a software package Kaviar.
- We propose two simple algorithms:
  1. inclusion algorithm where points are included in *D* if there are controls that slow the system to an approximate steady state, and
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- VIKAASA = Viability Kernel Approximation, Analysis and Simulation Application. (The Sanskrit word vikaasa, विकास, means ``progress" or ``development".)
- Vikaasa is a tool which can be used to create approximate viability kernels (actually, domains) for the classes of viability problems considered here (rectangular constrains, infinite horizon).
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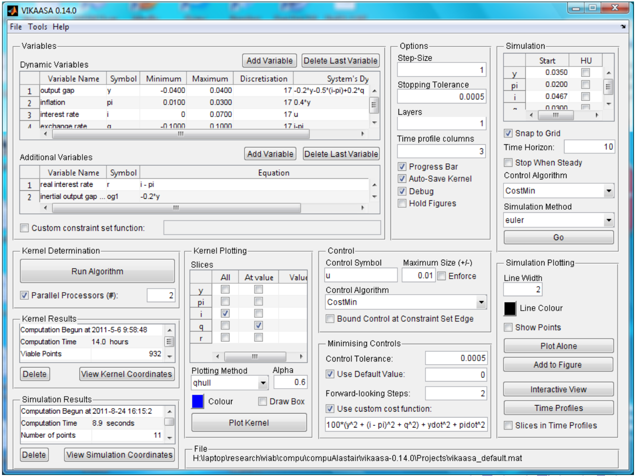
The main window



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One-species fishery

Viability has proven popular with ecologists seeking to model resource use. Béné et al have solved the following problem:

- Profits are given by $R(x(t), e(t)) = pq_x e(t)x(t) - ce(t) - C$, p - price, $q_x e(t)x(t)$ is the catch size, $ce(t)$, C - variable and fixed costs, respectively.
- $K = \{(x, e) : x \geq x_{\min} \wedge pq_x eb - ce - C \geq 0 \wedge e \in [0, e_{\max}]\}$, $U = [u^-, u^+]$; harvest rate $h_x(t) = q_x e(t)x(t)$.
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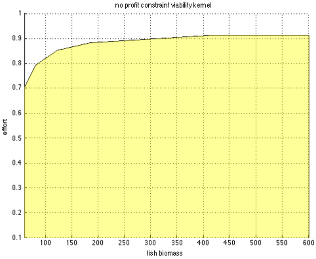
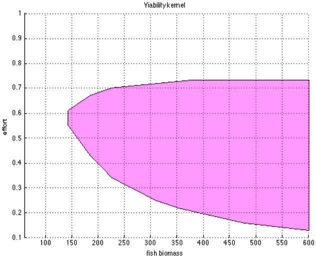
Kernels without and with cost constraint

This problem is now fed into VIKAASA.





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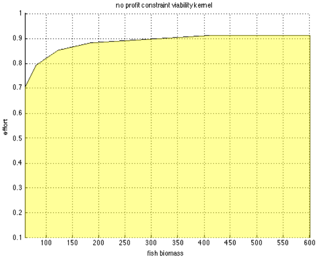

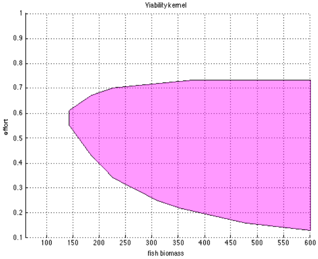

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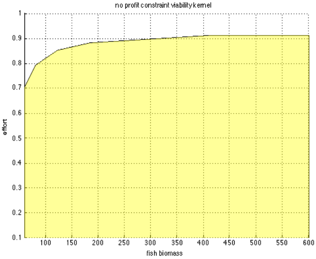

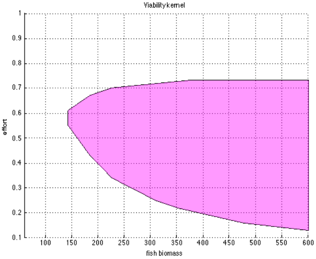

The model

- Harvest rate $h_y(t) = \alpha h_x(t)$; $0 < \alpha < 1$ measures how highly coupled the production relationships are (assumed $\alpha = 0.2$).

$$\dot{x}(t) = r_x x(t)\left(1 - \frac{x(t)}{L_x}\right) - q_x x(t) e(t)$$
$$\dot{y}(t) = r_y y(t)\left(1 - \frac{y(t)}{L_y}\right) - \alpha\, q_x x(t) e(t)$$

- Profit $\pi_{xy}(t) = p_x h_x(t) + p_y h_y(t) - ce(t) - C > 0$ (economic sustainability).

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- Constraint set $K \equiv \left\{ (x, y, e, u) : \left| \begin{array}{c} x(t) \geq \frac{L_x}{10} \\ y(t) \geq \frac{L_y}{10} \\ \pi_{xy}(t) \geq 0 \\ e(t) \in [e_{\min}, e_{\max}] \\ u(t) \in U \end{array} \right. \right\}.$

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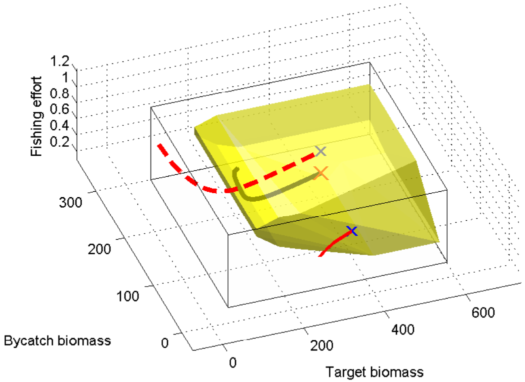
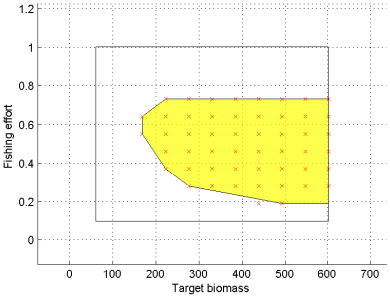
Viability kernel with viable and non-viable trajectories

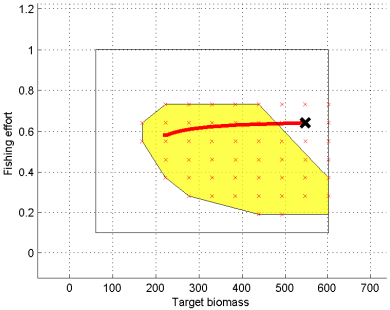


$[x(0), y(0), e(0)] =$
$[384,\ 165,\ 0.55] \in \mathcal{V}$; $[384,\ 57,\ 0.55] \notin \mathcal{V}$; $[384, 168, 0.91] \notin \mathcal{V}$

Kernel slices



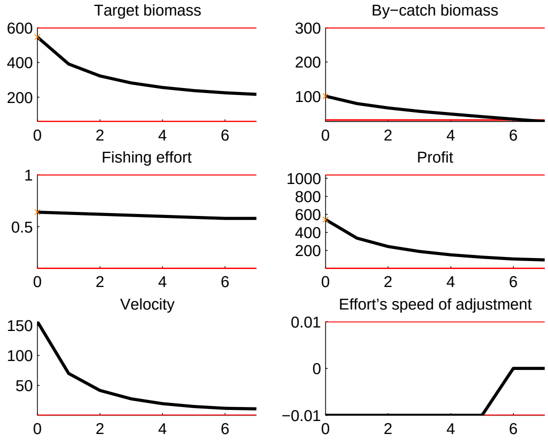
(a) High by-catch biomass



(b) Low by-catch biomass, with an example non-viable trajectory starting from $[546, 100, 0.64]$ shown in red

Slice (a) looks like a single species viability kernel.

Time profiles associated with the non-viable trajectory



The model

- Harvest rate $h_{2y}(t) = q_y e_2(t) y(t)$; $q_y > 0$ catchability.
- Stock y's equation of motion

$$\dot{y}(t) = r_y y(t)\left(1 - \frac{y(t)}{L_y}\right) - \alpha q_x x(t) e_1(t) - q_y e_2(t) y(t)\,.$$

- Effort adjustment $\dot{e}_2(t) = u_2(t) \in [\delta_2^-, \delta_2^+] \equiv U_2$.
- Profit $\pi_2(t) = p_y h_{2y}(t) - c_2 e_2(t) - C_2 > 0$ (economic sustainability).

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The model *cont*

- Constraint set

$$K_{aug} \equiv \left\{ (x, y, e_1, e_2, u_1, u_2) : \left| \begin{array}{c} x(t) \geq \frac{L_x}{10} \\ y(t) \geq \frac{L_y}{10} \\ \pi_{xy}(t) \geq 0, \pi_2(t) \geq 0 \\ e_1(t) \in [e_{1_{min}}, e_{1_{max}}] \\ e_2(t) \in [e_{2_{min}}, e_{2_{max}}] \\ u_1(t) \in U_1 \\ u_2(t) \in U_2 \end{array} \right. \right\}.$$

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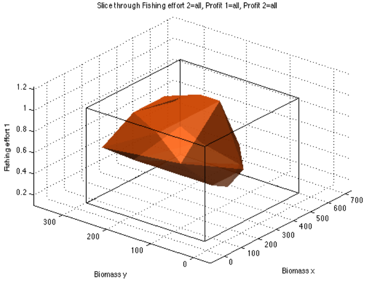
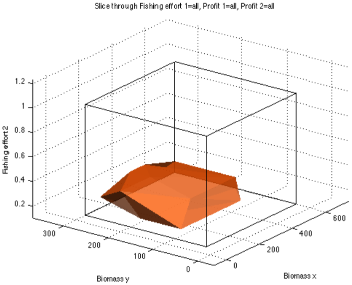
3D slices of 4D kernel for effort analysis

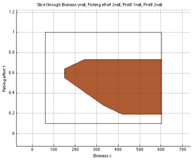


(c) First fleet's effort

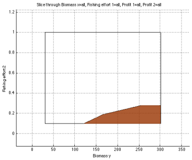


(d) Second fleet's effort

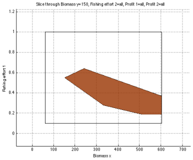
2D slices of 4D kernel



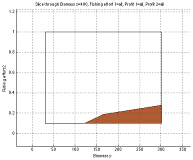
(e) First fleet's effort for all *y*-biomass values.



(f) Second fleet's effort for all *x*-biomass values.



(g) First fleet's effort for $y = 150$.



(h) Second fleet's effort for $x = 400$

Policy advice

- Satisficing solutions are generically non-unique and hence amenable to managers' own prioritisation.
- If fleets obey the $U = U_1 \times U_2$ choices and also respect the overall system's viability (i.e. the profitability of both fleets, and the non-extinction of both species), then the two-fleet case constitutes a constrained qualitative game between the fleets.
- Our viability kernel provides an overview for the space in which the game will be played.
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- Our analysis can help a regulator deciding whether or not to issue a permit for the second fleet to operate on the fishery.

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   - Differential inclusions
   - Kernel and policy
   - Viability vs optimality
2. Numerical deliverly
   - Algorithms
   - Vikaasa
3. Viability and VIKAASA in action: By-catch fishery
   - One-species fishery
   - A by-catch fishery problem
     - One fleet
     - Two fleets
     - Policy advice
   - Robustness of model
4. Concluding remarks and future research

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Robustness of model *cont.*

- A viability model is robust because it allows for the system's inertia.
- I.e., a viable policy avoids or steers the system away from areas of no return.
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- We have presented some basic notions of viability theory and shown how this theory can be useful for modeling and solution of sustainability problems.
- Attempts should be made to analyse sustainability problems (including disease management) from the viability-theory point of view. Given that these (ecologic-economic, health care, etc.) problems' models are essentially nonlinear, more research is needed on kernel existence for such models.
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